7.8 (*a*) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 25 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.710$ V. Determine (*i*) N_a, (*ii*) N_d, (*iii*) x_n, (*iv*) x_p, and (*v*) $|E_{max}|$.

a) From Table B.4, $n_i = 1.5 \times 10^{10}\ cm^{-3}$ @ 300 K

$\epsilon_r = 11.7$

Per (7.30), $W = x_n + x_p$

Given: $x_n = 0.25\,W = 0.25(x_n + x_p)$

$\hookrightarrow (1 - 0.25)\,x_n = 0.25\,x_p \Rightarrow \frac{x_p}{x_n} = 3$

Per (7.17) $N_a x_p = N_d x_n \Rightarrow \frac{x_p}{x_n} = \frac{N_d}{N_a} = 3$

(i) Per (7.10), $V_{bi} = \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$V_t = \frac{8.617333 \times 10^{-5}\ eV/K\ (300K)}{e} = 0.025852\ V$

$V_{bi} = 0.710\ V = (0.025852\ V) \ln\left(\frac{N_a (3N_a)}{n_i^2}\right)$

$N_a = \left\{ \frac{n_i^2}{3} e^{0.71/0.025852} \right\}^{1/2}$

$= \left\{ \frac{(1.5 \times 10^{10})^2}{3} e^{0.71/0.025852} \right\}^{1/2}$

$N_a = 7.9665 \times 10^{15}\ cm^{-3}$

(ii) $N_d = 3N_a = 2.39 \times 10^{16}\ cm^{-3}$

(iii) $$x_n = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2} \quad (7.28)$$

$$= \left\{ \frac{2(11.7)8.8542\times10^{-12}(0.71)}{1.602176634\times10^{-19}} \left(\frac{1}{3}\right) \frac{1}{7.9665\times10^{21} + 2.39\times10^{22}} \right\}^{1/2}$$

$$\underline{\underline{x_n = 9.80012\times10^{-8}\ m = 98.0012\ nm}}$$

(iv) $$x_p = 3x_n = 3(9.80012\times10^{-8})$$

$$\underline{\underline{x_p = 2.94\times10^{-7}\ m = 294\ nm}}$$

Bonus: $W = x_n + x_p = 392\ nm$

(v) Per (7.14), $$|E_{max}| = \left| -\frac{eN_a}{\epsilon_s} x_p \right|$$

$$|E_{max}| = \left| \frac{1.602176634\times10^{-19}(7.9665\times10^{21})}{(11.7)8.8541878\times10^{-12}} 2.94\times10^{-7} \right|$$

$$\underline{\underline{|E_{max}| = 3.6224\times10^{6}\ V/m = 3.6224\ \frac{MV}{m}}}$$