6.13 An n-type GaAs semiconductor at $T = 300$ K is uniformly doped at $N_d = 5 \times 10^{15}$ cm^{-3}. The minority carrier lifetime is $\tau_{p0} = 5 \times 10^{-8}$ s. Assume mobility values of $\mu_n = 7500$ cm^2/V-s and $\mu_p = 310$ cm^2/V-s. A light source is turned on at $t = 0$ generating excess carriers uniformly at a rate of $g' = 4 \times 10^{21}$ cm^{-3} s^{-1} and turns off at $t = 10^{-6}$ s.
(a) Determine the excess carrier concentrations versus time over the range $0 \le t \le \infty$.
(b) Calculate the conductivity of the semiconductor versus time over the same time period as part (a).

- First, find the electron and hole concentrations at thermal equilibrium. [Hint: Remember first-order RL and RC circuit problems where a source turns on and then switches off?]

From Table B.4, $n_i = 1.8\times10^6$ cm^{-3} for GaAs at 300 K.

Given: $\mathbf{g' = 4\times10^{21}}$ **cm^{-3}s^{-1}, $\tau_{p0} = 5\times10^{-8}$ s, $\mu_n = 7500$ cm^2/V-s and $\mu_p = 310$ cm^2/V-s.**

Since $N_d >> n_i$, we have n-type GaAs where $n_0 \cong N_d$ $\Rightarrow$ $\boxed{n_0 = 5\times10^{15}\ \text{cm}^{-3}}$.

Per (4.43), $p_0 = n_i^2 / n_0 = (1.8\times10^6)^2 / 5\times10^{15}$ $\Rightarrow$ $\boxed{p_0 = 0.000648\ \text{cm}^{-3}}$.

a) For n-type GaAs, the minority carrier (i.e., holes/δp) ambipolar transport equation (6.56) is:

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_n E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}}$$

"uniformly" implies uniform distribution wrt x $\Rightarrow$ $\frac{\partial^2(\delta p)}{\partial x^2} = 0$ & $E\frac{\partial(\delta p)}{\partial x} = 0$.

Therefore, our ambipolar minority carrier transport equation reduces to

$$\frac{\partial(\delta p)}{\partial t} = g' - \frac{\delta p}{\tau_{p0}} \quad \Rightarrow \quad \frac{\partial(\delta p)}{\partial t} + \frac{1}{\tau_{p0}}\delta p = g'.$$

This first-order ODE has a natural solution of $\delta p_n(t) = Ae^{-t/\tau_{p0}}$ and a forced solution of $\delta p_f(t) = g'\tau_{p0} = 4\times10^{21}(5\times10^{-8}) = 2\times10^{14}$ cm^{-3}.

Adding these solutions gives the general solution

$$\underline{\delta p(t) = Ae^{-t/\tau_{p0}} + g'\tau_{p0} = Ae^{-t/5\times10^{-8}} + 2\times10^{14}\ \text{cm}^{-3}.}$$

Applying the initial condition that $\delta p(0) = 0 = A + 2\times10^{14}$, leads to $A = -2\times10^{14}$ and

$$\Rightarrow \boxed{\delta p(t) = \delta n(t) = 2\times10^{14}\,(1 - e^{-t\,/\,5\text{e-}8})\ \text{cm}^{-3}\ \ \text{for } 0 \le t \le 10^{-6}\ \text{s}}.$$

At $t = 10^{-6}$ s, the light turns off, i.e., $g' = 0$. Therefore, our ambipolar minority carrier transport equation reduces to

$$\frac{\partial(\delta p)}{\partial t} = -\frac{\delta p}{\tau_{p0}} \qquad \Rightarrow \qquad \frac{\partial(\delta p)}{\partial t} + \frac{1}{\tau_{p0}}\delta p = 0.$$

The time-shifted first-order ODE has a solution of

$$\delta p_n(t) = Ae^{-(t-t_0)/\tau_{p0}} = Ae^{-(t-10^{-6})/5\times10^{-8}}.$$

At $t = 10^{-6}$ s, use prior solution to get $\delta p(10^{-6}) = 2\times10^{14}\,(1 - e^{-1\text{e-}6/5\text{e-}8}) = 2\times10^{14}\ \text{cm}^{-3} = A$. Therefore,

$$\Rightarrow \boxed{\delta p(t) = \delta n(t) = 2\times10^{14}\, e^{-(t\,-\,1\text{e-}6)\,/\,5\text{e-}8}\ \text{cm}^{-3}\ \ \text{for } t \ge 10^{-6}\ \text{s}}.$$

b) Per (5.20) and (6.5a) & (6.5b),

$$\sigma = e\,(\mu_n n + \mu_p p) = e\,[\mu_n\,(n_0 + \delta n) + \mu_p\,(p_0 + \delta n)] = e(\mu_n n_0 + \mu_p p_0) + e(\mu_n + \mu_p)\,\delta n.$$

For the initial period,

$$\sigma(t) = 1.602176634\times10^{-19}\,[7500\ \text{cm}^2/\text{V-s}\,(5\times10^{15}\ \text{cm}^{-3}) + 310\ \text{cm}^2/\text{V-s}\,(0.000648\ \text{cm}^{-3})]$$
$$+\ 1.602176634\times10^{-19}(7500\ \text{cm}^2/\text{V-s} + 310\ \text{cm}^2/\text{V-s})\ 2\times10^{14}\,(1 - e^{-t/5\text{e-}8})$$

$$\Rightarrow \boxed{\sigma(t) = 6.00816 + 0.25026\,(1 - e^{-t\,/\,5\text{e-}8})\ \text{S/cm}\ \ \text{for } 0 \le t \le 10^{-6}\ \text{s}}.$$

For the final period,

$$\sigma(t) = 1.602176634\times10^{-19}\,[7500\ \text{cm}^2/\text{V-s}\,(5\times10^{15}\ \text{cm}^{-3}) + 310\ \text{cm}^2/\text{V-s}\,(0.000648\ \text{cm}^{-3})]$$
$$+\ 1.602176634\times10^{-19}(7500\ \text{cm}^2/\text{V-s} + 310\ \text{cm}^2/\text{V-s})\ 2\times10^{14}\, e^{-(t\,-\,1\text{e-}6)/5\text{e-}8}$$

$$\Rightarrow \boxed{\sigma(t) = 6.00816 + 0.25026\, e^{-(t\,-\,1\text{e-}6)\,/\,5\text{e-}8}\ \text{S/cm}\ \ \text{for } t \ge 10^{-6}\ \text{s}}.$$