6.12 Consider a silicon sample at $T = 300$ K that is uniformly doped with acceptor impurity atoms at a concentration of $N_a = 10^{16}$ cm^{-3}. At $t = 0$, a light source is turned on generating excess carriers uniformly throughout the sample at a rate of $g' = 8 \times 10^{20}$ cm^{-3} s^{-1}. Assume the minority carrier lifetime is $\tau_{n0} = 5 \times 10^{-7}$ s, and assume mobility values of $\mu_n = 900$ cm^2/V-s and $\mu_p = 380$ cm^2/V-s. (*a*) Determine the conductivity of the silicon as a function of time for $t \geq 0$. (*b*) What is the value of conductivity at (*i*) $t = 0$ and (*ii*) $t = \infty$?

- First, find the electron and hole concentrations at thermal equilibrium. Second, find the minority charge carrier concentration as a function of time. Then, do parts a) & b).

From Table B.4, $n_i = 1.5\times10^{10}$ cm^{-3} for silicon at 300 K.

Since $N_a >> n_i$, we have p-type silicon where $p_0 \cong N_a$ $\Rightarrow$ $\boxed{p_0 = 10^{16} \text{ cm}^{-3}}$.

Per (4.43), $n_0 = n_i^2/p_0 = (1.5\times10^{10})^2/10^{16}$ cm^{-3} $\Rightarrow$ $\boxed{n_0 = 22{,}500 \text{ cm}^{-3}}$.

Given: **$g' = 8\times10^{20}$ cm^{-3}s^{-1}, $\tau_{n0} = 5\times10^{-7}$ s, $\mu_n = 900$ cm^2/V-s and $\mu_p = 380$ cm^2/V-s.**

For p-type silicon, the minority carrier (i.e., electrons/δn) transport equation (6.55) is:

$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}}$$

- "uniformly throughout" implies uniform distribution

$$\Rightarrow \frac{\partial^2(\delta n)}{\partial x^2} = \frac{\partial^2(\delta p)}{\partial x^2} = 0 \text{ and } \bar{E}\frac{\partial(\delta n)}{\partial x} = \bar{E}\frac{\partial(\delta p)}{\partial x} = 0.$$

Therefore, our p-type minority carrier transport equation reduces to

$$\frac{\partial(\delta n)}{\partial t} = g' - \frac{\delta n}{\tau_{n0}} \quad \Rightarrow \quad \frac{\partial(\delta n)}{\partial t} + \frac{1}{\tau_{n0}}\delta n = g'.$$

This first-order ODE has a natural solution of $\delta n_n(t) = Ae^{-t/\tau_{n0}}$ and forced solution of $\delta n_f(t) = g'\tau_{n0} = 8\times10^{20}(5\times10^{-7}) = 4\times10^{14}$ cm^{-3}. Adding these solutions gives the general solution $\underline{\delta n(t) = Ae^{-t/\tau_{n0}} + g'\tau_{n0} = Ae^{-t/5\times10^{-7}} + 4\times10^{14} \text{ cm}^{-3}}$.

Applying the initial condition that $\delta n(0) = 0 = A + 4\times10^{14}$, leads to $A = -4\times10^{14}$ and

$$\boxed{\delta n(t) = \delta p(t) = 4\times10^{14}\left(1 - e^{-t/5\times10^{-7}}\right) \text{ cm}^{-3} \text{ for } t \geq 0}.$$

a) Per (5.20) and (6.5a) & (6.5b),

$$\sigma = e(\mu_n n + \mu_p p) = e[\mu_n (n_0 + \delta n) + \mu_p (p_0 + \delta n)] = e(\mu_n n_0 + \mu_p p_0) + e(\mu_n + \mu_p)\,\delta n$$

$$= 1.602176634\times10^{-19}\,[900\text{ cm}^2/\text{V-s}(22500\text{ cm}^{-3}) + 380\text{ cm}^2/\text{V-s}(10^{16}\text{ cm}^{-3})] + 1.602176634\times10^{-19}(900\text{ cm}^2/\text{V-s} + 380\text{ cm}^2/\text{V-s})\,4\times10^{14}\,(1 - e^{-t/5\text{e-}7})$$

$$\Rightarrow \boxed{\boldsymbol{\sigma(t) = 0.608827 + 0.0820335\,(1 - e^{-t/5\text{e-}7})\text{ S/cm for } t \geq 0}}.$$

b)

(i) $\sigma(t = 0) = 0.608827 + 0.0820335\,(1 - e^{-0})$ $\Rightarrow$ $\boxed{\boldsymbol{\sigma(t = 0) = 0.608827\ \text{S/cm}}}$.

(ii) $\sigma(t \to \infty) = 0.608827 + 0.0820335\,(1 - e^{-\infty})$ $\Rightarrow$ $\boxed{\boldsymbol{\sigma(t \to \infty) = 0.690861\ \text{S/cm}}}$.