3.45 Assume that the Fermi energy level is exactly in the center of the bandgap energy of a semiconductor at $T = 300$ K. (*a*) Calculate the probability that an energy state in the bottom of the conduction band is occupied by an electron for Si, Ge, and GaAs. (*b*) Calculate the probability that an energy state in the top of the valence band is empty for Si, Ge, and GaAs.

Per (3.79), $f_F(E) = \dfrac{1}{1 + e^{(E-E_F)/k_B T}}$

w/ $E_F = \dfrac{E_V + E_C}{2}$ & $k_B(300) = 0.025852$ eV

a) For $E = E_C$

$$E - E_F = E_C - \frac{E_V}{2} - \frac{E_C}{2} = \frac{E_C - E_V}{2} = E_g/2$$

Silicon ($E_g = 1.12$ eV)

$$f_F(E_c) = \frac{1}{1 + e^{1.12/2/0.025852}} = \underline{\underline{3.912 \times 10^{-10}}}$$

Germanium ($E_g = 0.66$ eV)

$$f_F(E_c) = \frac{1}{1 + e^{0.66/2/0.025852}} = \underline{\underline{2.859 \times 10^{-6}}}$$

GaAs ($E_g = 1.424$ eV)

$$f_F(E_c) = \frac{1}{1 + e^{1.424/2/0.025852}} = \underline{\underline{1.0938 \times 10^{-12}}}$$

b) For $E = E_V$, $E - E_F = E_V - \frac{E_V}{2} - \frac{E_c}{2} = \frac{E_V - E_c}{2}$

$= -E_g/2$

Empty (Example 3.7)

$$1 - f_E(E_V) = 1 - \frac{1}{1 + e^{-E_g/2/k_BT}}$$

Silicon

$$1 - f_F(E_V) = 1 - \frac{1}{1 + e^{-1.12/2/0.025852}}$$

$$\underline{\underline{1 - f_F(E_V) = 3.912 \times 10^{-10}}}$$

Germanium

$$1 - f_F(E_V) = 1 - \frac{1}{1 + e^{-0.66/2/0.025852}}$$

$$\underline{\underline{1 - f_F(E_V) = 2.859 \times 10^{-6}}}$$

GaAs

$$1 - f_F(E_V) = 1 - \frac{1}{1 + e^{-1.424/2/0.025852}}$$

$$\underline{\underline{1 - f_F(E_V) = 1.0938 \times 10^{-12}}}$$

$\Rightarrow$ Same as part a) answers!