3.42 Consider the energy levels shown in Figure P3.42. Let $T = 300$ K. (*a*) If $E_1 - E_F = 0.30$ eV, determine the probability that an energy state at $E = E_1$ is occupied by an electron and the probability that an energy state at $E = E_2$ is empty.

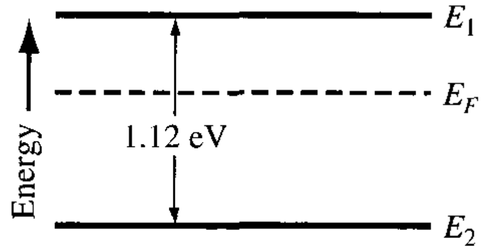

Figure P3.42 | Energy levels for Problem 3.42.

a) Use Fermi-Dirac probability function (3.79) to find the probability of an electron at $E = E_1$ as

$$f_F(E=E_1) = \frac{1}{1+e^{\frac{E_1-E_F}{k_BT}}} = \frac{1}{1+e^{\frac{0.3}{8.617333\times10^{-5}(300)}}} = \frac{1}{1+e^{\frac{0.3}{0.025852}}} \Rightarrow \underline{f_F(E=E_1) = 9.12468\times10^{-6}}.$$

Per example 3.7, the probability that an energy state is empty at $E = E_2$ can be found by subtracting the probability of electron at that energy from 1. To compute this, we need

$$E_F - E_2 = E_{gap} - (E_1 - E_F) = (E_1 - E_2) - (E_1 - E_F) \Rightarrow E_2 - E_F = 0.3 - 1.12 = \mathbf{-0.82\ eV}$$

$$1 - f_F(E=E_2) = 1 - \frac{1}{1+e^{\frac{E_2-E_F}{k_BT}}} = 1 - \frac{1}{1+e^{\frac{-0.82}{0.025852}}}.$$

The second term above is difficult to evaluate with a calculator due to round off error. Use the Binomial series expansion that $1/(1 + x) \approx 1 - x$ when $x \ll 1$ to get

$$1 - f_F(E=E_2) = 1 - \frac{1}{1+e^{\frac{-0.82}{0.025852}}} \approx 1 - \left(1 - e^{\frac{-0.82}{0.025852}}\right) = e^{\frac{-0.82}{0.025852}}$$

$$\Rightarrow \underline{1 - f_F(E=E_2) = 1.67728\times10^{-14}}.$$