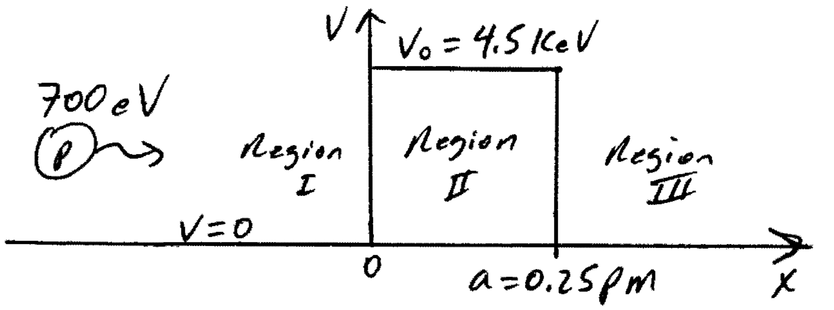
A proton with a kinetic energy of 700 eV, traveling in the x-direction from $-\infty$ in a region of zero potential energy (Region I), is incident on a finite potential barrier at $x = 0$ of 4.5 keV that is 0.25 pm wide (Region II) followed by another zero potential region (Region III). a) Find the voltage required to give a proton initially at rest this energy. b) Find the velocity of the proton. c) Find the de Broglie wavelength of the proton. d) Find the wave numbers in Regions I, II, and III. e) Find the exact and approximate transmission coefficient, i.e., tunneling probability (unitless and %). f) How wide would the barrier need to be for the tunneling probability to be 0.1%?



a) $E = qV = (1.6021766 \times 10^{-19} C)\, V = 700 eV (1.6021766 \times 10^{-19} \frac{J}{eV})$

$\hookrightarrow \underline{\underline{V = 700\,V}}$

b) $K.E. = T = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2T}{m}}$

$$v = \sqrt{\frac{2(700)\,1.6021766 \times 10^{-19}}{1.672622 \times 10^{-27}}} \Rightarrow \underline{\underline{v = 366{,}201.67\ m/s}}$$

c) (2.3) $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$= \frac{6.62607015 \times 10^{-34}}{1.672622 \times 10^{-27} (366{,}201.67)}$$

$$\underline{\underline{\lambda = 1.08178 \times 10^{-12} m = 1.08178\ pm}}$$

d) Per (2.61a), $\kappa_1 = \kappa_3 = \sqrt{\dfrac{2mE}{\hbar^2}}$

$$\kappa_1 = \kappa_3 = \sqrt{\frac{2(1.672622\times10^{-27})\,700\,(1.6021766\times10^{-19})}{(1.0545718\times10^{-34})^2}}$$

$$\underline{\underline{\kappa_1 = \kappa_3 = 5.80821\times10^{12}\ \text{rad/m}}}$$

Per (2.61b), $\kappa_2 = \sqrt{\dfrac{2m(V_0-E)}{\hbar^2}}$

$$\kappa_2 = \sqrt{\frac{2(1.672622\times10^{-27})(4500-700)(1.6021766\times10^{-19})}{(1.0545718\times10^{-34})^2}}$$

$$\underline{\underline{\kappa_2 = 1.35327\times10^{13}\ \text{Np/m}}}$$

e) $\kappa_2 a = 1.35327\times10^{13}(0.25\times10^{-12}) = 3.38318$

Notes (exact) $$T = \frac{1}{1+\dfrac{V_0^2\sinh^2(\kappa_2 a)}{4E(V_0-E)}} = \frac{1}{1+\dfrac{4500^2\sinh^2(3.38318)}{4(700)(4500-700)}}$$

$$\underline{\underline{T = 2.42067\times10^{-3} = 0.242\%}}$$

Approx (2.63) $$T \simeq 16\,\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\kappa_2 a}$$

$$\simeq 16\,\frac{700}{4500}\left(1-\frac{700}{4500}\right)e^{-2(3.38318)}$$

$$\underline{\underline{T \simeq 2.42095\times10^{-3} = 0.242\%}}$$

f) By trial-and-error using MathCad + exact eq'n

$T = 10^{-3} = 0.1\%$ when $\underline{\underline{a = 0.282666\ \text{pm}}}$