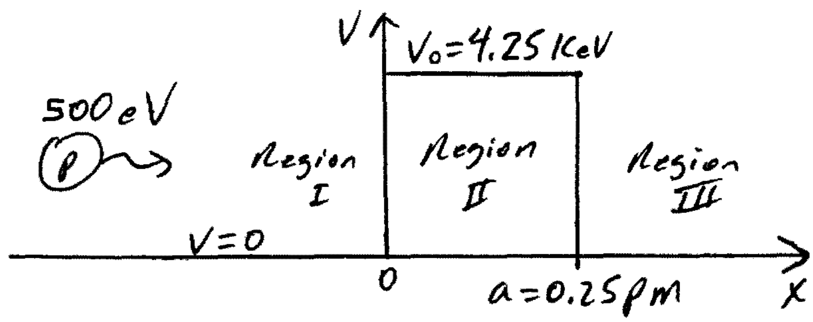
A proton with a kinetic energy of 500 eV, traveling in the x-direction from $-\infty$ in a region of zero potential energy (Region I), is incident on a finite potential barrier at $x = 0$ of 4.25 keV that is 0.3 pm wide (Region II) followed by another zero potential region (Region III). a) Find the voltage required to give a proton initially at rest this energy. b) Find the velocity of the proton. c) Find the de Broglie wavelength of the proton. d) Find the wave numbers in Regions I, II, and III. e) Find the exact and approximate transmission coefficient, i.e., tunneling probability (unitless and %). f) How wide would the barrier need to be for the tunneling probability to be 0.1%?



a) The charge of a proton is $q_p = e$.

From EE 381, $E = qV \Rightarrow V = E/q = 500\text{ eV}/e \Rightarrow$ $\underline{\underline{\mathbf{V = 500\ V}}}$.

b) From physics, K.E. = $T = 0.5mv^2$.

$$v = \sqrt{\frac{2T}{m}} = \sqrt{\frac{2(500)1.6021766\times10^{-19}}{1.6726219237\times10^{-27}}} \Rightarrow \underline{\underline{\mathbf{v = 309{,}496.9\ m/s = 3.095\times10^5\ m/s}}}.$$

c) Per (2.3),

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.62607015\times10^{-34}}{1.6726219237\times10^{-27}(309{,}496.9)} \Rightarrow \underline{\underline{\mathbf{\lambda = 1.27998\times10^{-12}\ m = 1.27998\ pm}}}.$$

d) Per (2.61a), the wave number in Regions I & III is

$$k_1 = k_3 = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{2(1.6726219237\times10^{-27})500(1.6021766\times10^{-19})}{(1.0545718\times10^{-34})^2}}$$

$$\Rightarrow \underline{\underline{\mathbf{k_1 = k_3 = 4.90883\times10^{12}\ rad/m}}}.$$

Per (2.61b), the wave number in Region II is

$$k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \sqrt{\frac{2(1.6726219237\times10^{-27})(4250-500)1.6021766\times10^{-19}}{(1.0545718\times10^{-34})^2}}$$

$$\Rightarrow \underline{\underline{\mathbf{k_2 = 1.34434\times10^{13}\ Np/m}}}.$$

e) From notes, the exact tunneling probability is

$$T_{exact} = \frac{1}{1+\frac{V_0^{\ 2}\sinh^2(k_2 a)}{4E(V_0 - E)}} = \frac{1}{1+\frac{4250^2\sinh^2(1.34434\times10^{13}\bullet0.3\times10^{-12})}{4(500)(4250-500)}}$$

$\Rightarrow$ **$T_{exact} = 5.21624\times10^{-4}$**.

Per (2.63, the approximate tunneling probability is

$$T_{approx} \approx 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2k_2 a} = 16\left(\frac{500}{4250}\right)\left(1-\frac{500}{4250}\right)e^{-2(1.34434\times10^{13}\bullet0.3\times10^{-12})}$$

$\Rightarrow$ **$T_{approx} = 5.21568\times10^{-4}$**.

f) Using the exact equation and Math Cad, find the finite barrier width $a_{0.1\%}$ for a 0.1% tunneling probability.

MathCad

By trial-n-error, guess $a01 := 0.275798\cdot10^{-12}$

$$T01 := \frac{1}{1+\frac{V0^2\cdot\sinh(k2\cdot a01)\cdot\sinh(k2\cdot a01)}{4\cdot E\cdot(V0-E)}}$$

$T01 = 0.001$ $\quad T01\cdot100 = 0.1$ %

$a01 = 2.75798\times10^{-13}$ m

$\Rightarrow$ **$a_{0.1\%} = 0.275798$ pm**.