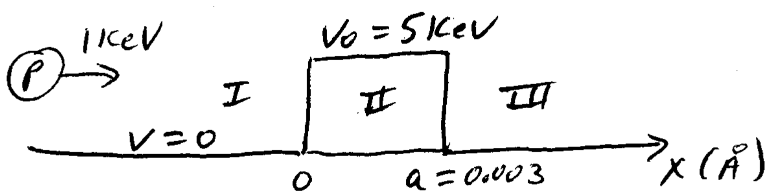
A proton with a kinetic energy of 1 keV traveling in the x-direction from -∞ in a region of zero potential energy (Region I) is incident on a finite potential barrier at $x = 0$ of 5 keV that is 0.003 Å wide (Region II) followed by another zero potential region (Region III). a) Find the voltage required to give a proton initially at rest this energy. b) Find the velocity of the proton. c) Find the de Broglie wavelength of the proton. d) Find the wave numbers in Regions I, II, and III. e) Find the exact and approximate transmission coefficient, i.e., tunneling probability (unitless and %). f) How wide would the barrier need to be for the tunneling probability to be 0.1%?



a) $E = qV = (1.6021766\times10^{-19}\,C)\,V = 1000\,eV\left(1.6021766\times10^{-19}\,\frac{J}{eV}\right)$

↳ $V = 1000\ V$

b) $K.E. = T = \frac{1}{2} m_p v^2 = 1000\,eV\,(1.60218\times10^{-19}\ J/eV)$

$$v = \sqrt{\frac{2(1000)(1.6021766\times10^{-19})}{1.672622\times10^{-27}}} = 437{,}694.7\ m/s$$

c) Using (2.3) $\lambda = \frac{h}{p} = \frac{h}{m_p v}$

$$\lambda = \frac{6.62607\times10^{-34}}{1.672622\times10^{-27}\,(437{,}694.7)}$$

$$\lambda = 9.0508\times10^{-13}\ m$$

d) Per (2.61a), $K_1 = K_3 = \sqrt{\dfrac{2mE}{\hbar^2}}$

$$K_1 = \sqrt{\frac{2(1.672622\times10^{-27})1000(1.6021766\times10^{-19})}{(1.054572\times10^{-34})^2}}$$

$$\underline{\underline{K_1 = K_3 = 6.942\times10^{12}\ \text{rad/m}}}$$

Per (2.61b), $K_2 = \sqrt{\dfrac{2m(V_0-E)}{\hbar^2}}$

$$K_2 = \sqrt{\frac{2(1.672622\times10^{-27})(5000-1000)(1.6021766\times10^{-19})}{(1.054572\times10^{-34})^2}}$$

$$\underline{\underline{K_2 = 1.38843\times10^{13}\ \text{Np/m}}}$$

e) Note: $K_2 a = 1.38843\times10^{13}(0.003\times10^{-10}) = 4.16528$

(Notes) $T_{exact} = \dfrac{1}{1+\dfrac{V_0^2\sinh^2(K_2a)}{4E(V_0-E)}} = \dfrac{1}{1+\dfrac{5000^2\sinh^2(4.1653)}{4(1000)(5000-1000)}}$

$$\underline{\underline{T_{exact} = 6.16972\times10^{-4} = 0.061697\ \%}}$$

Per (2.63), $T_{approx} = 16\dfrac{E}{V_0}\left(1-E/V_0\right)e^{-2K_2a}$

$$= 16\,\frac{1000}{5000}\left(1-\frac{1000}{5000}\right)e^{-2(4.16528)}$$

$$\underline{\underline{T_{approx} = 6.17055\times10^{-4} = 0.0617055\ \%}}$$

f) Set $0.1\% = 0.001 = \dfrac{1}{1+\dfrac{V_0^2\sinh^2(K_2a_{0.1})}{4E(V_0-E)}}$

using MathCad, $\underline{\underline{a_{0.1\%} = 0.002826\ \text{Å} = 2.826\times10^{-13}\ \text{m}}}$