

2.21 Repeat Problem 2.20 if the wave function is given by $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right)$.

2.20 An electron is described by a wave function given by $\psi(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{\pi x}{a}\right)$ for $-\frac{a}{2} < x < \frac{a}{2}$. The wave function is zero elsewhere. Calculate the probability of finding the electron between (a) $0 < x < \frac{a}{4}$, (b) $\frac{a}{4} < x < \frac{a}{2}$, and (c) $-\frac{a}{2} < x < \frac{a}{2}$.

Per (2.17), the probability density is

$$|\Psi(x,t)|^2 = \Psi(x) \Psi^*(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right) \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right) = \frac{2}{a} \sin^2\left(\frac{2\pi x}{a}\right).$$

a)

$$\begin{aligned} P_a &= \int_0^{a/4} |\Psi(x,t)|^2 dx = \int_0^{a/4} \frac{2}{a} \sin^2\left(\frac{2\pi x}{a}\right) dx \\ &= \frac{2}{a} \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right]_0^{a/4} = \frac{2}{a} \left[\left(\frac{a/4}{2} - \frac{\sin\left(\frac{\pi a}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right) - \left(0 - \frac{\sin(0)}{4\left(\frac{2\pi}{a}\right)} \right) \right] \Rightarrow \underline{\underline{P_a = 0.25}} \end{aligned}$$

b)

$$\begin{aligned} P_b &= \int_{a/4}^{a/2} \frac{2}{a} \sin^2\left(\frac{2\pi x}{a}\right) dx = \frac{2}{a} \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right]_{a/4}^{a/2} \\ &= \frac{2}{a} \left[\left(\frac{a/2}{2} - \frac{\sin\left(\frac{2\pi a}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right) - \left(\frac{a/4}{2} - \frac{\sin\left(\frac{\pi a}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right) \right] = 0.5 - 0.25 \Rightarrow \underline{\underline{P_b = 0.25}} \end{aligned}$$

c)

$$\begin{aligned} P_c &= \int_{-a/2}^{a/2} \frac{2}{a} \sin^2\left(\frac{2\pi x}{a}\right) dx = \frac{2}{a} \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right]_{-a/2}^{a/2} \\ &= \frac{2}{a} \left[\left(\frac{a/2}{2} - \frac{\sin\left(\frac{2\pi a}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right) - \left(\frac{-a/2}{2} - \frac{\sin\left(\frac{-2\pi a}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right) \right] = 0.5 - (-0.5) \Rightarrow \underline{\underline{P_c = 1}} \end{aligned}$$