2.20 An electron is described by a wave function given by $\psi(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)$ for $\frac{-a}{2} < x < \frac{+a}{2}$. The wave function is zero elsewhere. Calculate the probability of finding the electron between (*a*) $0 < x < \frac{a}{4}$, (*b*) $\frac{a}{4} < x < \frac{a}{2}$, and (*c*) $\frac{-a}{2} < x < \frac{+a}{2}$.

Per (2.17), $|\Psi(x,t)|^2 = \psi(x)\,\psi^*(x) = \frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)$ is the probability density.

a) $$P_a = \int_0^{a/4} \frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)dx$$
$$= \frac{2}{a}\left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4(\pi/a)}\right]\Bigg|_{x=0}^{a/4}$$
$$= \frac{2}{a}\left[\left(\frac{a/4}{2} + \frac{\sin(\pi/2)}{4(\pi/a)}\right) - (0+0)\right]$$
$$\underline{\underline{P_a = 0.409155 = 40.9155\%}}$$

b) $$P_b = \int_{a/4}^{a/2} \frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)dx = \frac{2}{a}\left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4(\pi/a)}\right]\Bigg|_{x=a/4}^{a/2}$$
$$= \frac{2}{a}\left[\left(\frac{a/2}{2} + \frac{\overbrace{\sin\pi}^{0}}{4(\pi/a)}\right) - \left(\frac{a/4}{2} + \frac{\sin(\pi/2)}{4(\pi/a)}\right)\right]$$
$$\underline{\underline{P_b = 0.09084506 = 9.084506\%}}$$

c) $$P_c = \int_{-a/2}^{a/2} \frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)dx = \frac{2}{a}\left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4(\pi/a)}\right]\Bigg|_{x=-a/2}^{a/2}$$
$$= \frac{2}{a}\left[\left(\frac{a/2}{2} + \frac{\overbrace{\sin\pi}^{0}}{4(\pi/a)}\right) - \left(\frac{-a/2}{2} + \frac{\overbrace{\sin(-\pi)}^{0}}{4(\pi/a)}\right)\right]$$
$$\underline{\underline{P_c = 1 = 100\%}}$$