2.19 The solution to Schrodinger's wave equation for a particular situation is given by $\psi(x) = \sqrt{2/a_0} \cdot e^{-x/a_0}$. Determine the probability of finding the particle between the following limits: (*a*) $0 \leq x \leq a_0/4$, (*b*) $a_0/4 \leq x \leq a_0/2$, and (*c*) $0 \leq x \leq a_0$.

- First, find the probability of finding the particle between $0 < x < \infty$.

Per (2.17), $|\Psi(x,t)|^2 = \Psi(x)\Psi^*(x) = \sqrt{\frac{2}{a_0}}\, e^{-x/a_0} \sqrt{2/a_0}\, e^{-x/a_0}$

$$= \frac{2}{a_0} e^{-2x/a_0}$$

$$Prob(0<x<\infty) = \int_{x=0}^{\infty} |\Psi(x,t)|^2 dx = \frac{2}{a_0}\int_{x=0}^{\infty} e^{-\frac{2x}{a_0}} dx$$

$$= \frac{2}{a_0} \left.\frac{e^{-2x/a_0}}{-2/a_0}\right|_{x=0}^{\infty} = -1(0-e^0)$$

$$\underline{\underline{Prob(0<x<\infty) = 1 \text{ or } 100\%}}$$

a) $$Prob(0 \leq x \leq a_0/4) = \frac{2}{a_0}\int_{x=0}^{a_0/4} e^{-2x/a_0} dx$$

$$= (-1)\left. e^{-2x/a_0}\right|_{x=0}^{a_0/4} = -1\left[e^{-1/2} - 1\right]$$

$$\underline{\underline{Prob(0 \leq x \leq \tfrac{a_0}{4}) = 0.39347 \text{ or } 39.347\%}}$$

b) $$Prob(\tfrac{a_0}{4} \leq x\ \tfrac{a_0}{2}) = (-1)\left. e^{-\frac{2x}{a_0}}\right|_{x=a_0/4}^{a_0/2} = -1(e^{-1} - e^{-1/2})$$

$$\underline{\underline{Prob(\tfrac{a_0}{4} \leq x \leq \tfrac{a_0}{2}) = 0.23865 \text{ or } 23.865\%}}$$

c) $$Prob(0 \leq x \leq a_0) = (-1)\left. e^{-2x/a_0}\right|_{x=0}^{a_0} = -1\left[e^{-2} - 1\right]$$

$$\underline{\underline{Prob(0 \leq x \leq a_0) = 0.86466 \text{ or } 86.466\%}}$$