2.13 (*a*) The uncertainty in position is 12 Å for a particle of mass 9×10^{-31} kg. The nominal energy of the particle is 16 eV. Determine the minimum uncertainty in (*i*) momentum and (*ii*) kinetic energy of the particle. (*b*) Repeat part (*a*) for a particle of mass 5×10^{-28} kg.

a) Per (2.4), $\Delta p \, \Delta x \geq \hbar$

(i) $\Delta p \geq \frac{\hbar}{\Delta x} = \frac{1.054571817 \times 10^{-34}}{12 \times 10^{-10}}$

$\Delta p_{min} = 8.788099 \times 10^{-26}$ kg m/s

(ii) $\Delta KE = \frac{dKE}{dp} \Delta p = \frac{d\left(\frac{p^2}{2m}\right)}{dp} \Delta p = \frac{2p}{2m} \Delta p = \frac{p \, \Delta p}{m}$

$p = \sqrt{2m \, KE} = \sqrt{2(9 \times 10^{-31}) 16 (1.602176634 \times 10^{-19})}$

$= 2.148085 \times 10^{-24}$ kg m/s

$\Delta KE = \frac{2.148085 \times 10^{-24} (8.788099 \times 10^{-26})}{9 \times 10^{-31}}$

$\Delta KE = 2.09751 \times 10^{-19}$ J $= 1.30916$ eV

b) (i) $\Delta p_{min} = \frac{\hbar}{\Delta x} = 8.788099 \times 10^{-26}$ kg m/s

(ii) $p = \sqrt{2(5 \times 10^{-28}) 16 (1.602176634 \times 10^{-19})}$

$= 5.063085 \times 10^{-23}$ kg m/s

$\Delta KE = \frac{5.063085 \times 10^{-23} (8.788099 \times 10^{-26})}{5 \times 10^{-28}}$

$\Delta KE = 8.89898 \times 10^{-21}$ J $= 0.05554305$ eV