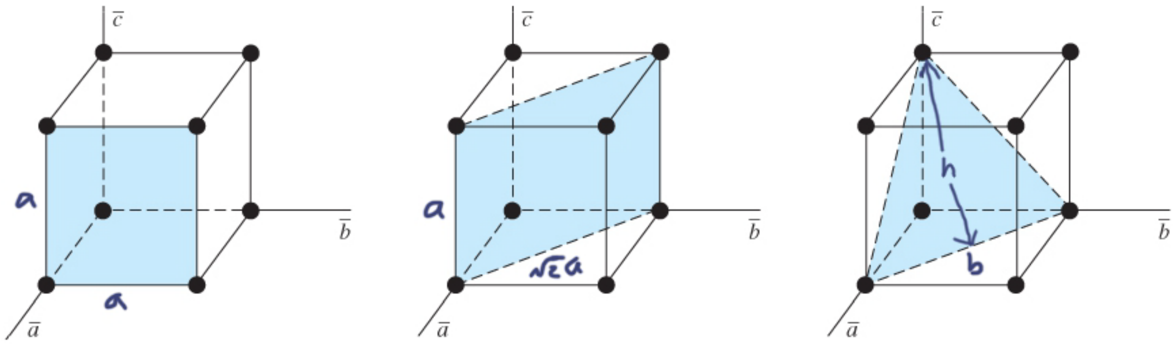
1.19 The lattice constant of a single crystal is 4.73 Å. Calculate the surface density (#/cm²) of atoms on the (i) (100), (ii) (110), and (iii) (111) plane for a (*a*) simple cubic, (*b*) body-centered cubic, and (*c*) face-centered cubic lattice.

a) simple cubic



(a) (100) plane (b) (110) plane (c) (111) plane.

(i) (100) plane area $= a^2$

$$\text{surf. density}_{100} = \frac{4\text{ corners}\left(\frac{1/4\text{ atom}}{\text{corner}}\right)}{a^2} = \frac{1}{(4.73\times10^{-10})^2}$$

$$\text{Surf. density}_{100} = 4.47\times10^{18}\ \frac{\text{atoms}}{m^2} = 4.47\times10^{14}\ \frac{\text{atoms}}{cm^2}$$

(ii) (110) plane area $= a\sqrt{a^2+a^2} = \sqrt{2}\,a^2$

$$\text{Surf. density}_{110} = \frac{4\text{ corners }(1/4\text{ atom/corner})}{\sqrt{2}\,a^2} = \frac{\text{Surf. density}_{100}}{\sqrt{2}}$$

$$\text{Surf. density}_{110} = 3.16\times10^{18}\ \frac{\text{atoms}}{m^2} = 3.16\times10^{14}\ \frac{\text{atoms}}{cm^2}$$

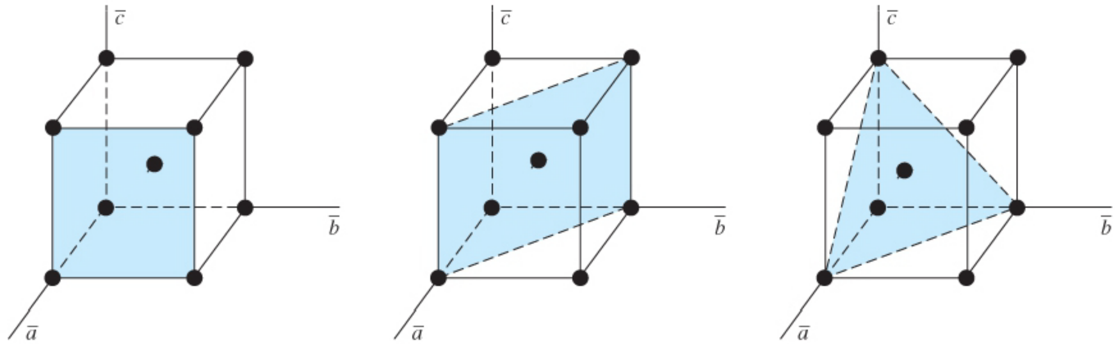
(iii) (111) plane area $= \frac{1}{2}bh$ where $b = \sqrt{2}\,a$ & $h = \sqrt{a^2 + \left(\frac{\sqrt{2}}{2}a\right)^2} = \sqrt{6/4\,a^2}$

$$\text{Surf. density}_{111} = \frac{3\text{ corners}\left(\frac{1/6\text{ atoms}}{\text{corner}}\right)}{\frac{1}{2}\sqrt{2}\,a\,\frac{\sqrt{6}}{2}a}$$ ← $60°/360° = 1/6$

$$= \frac{0.5\text{ atoms }(2)^2}{\sqrt{2}\sqrt{6}\,(4.73\times10^{-10})^2}$$

$$\text{Surf. density}_{111} = 2.58\times10^{18}\ \frac{\text{atoms}}{m^2} = 2.58\times10^{14}\ \frac{\text{atoms}}{cm^2}$$

b) body-centered cubic



(a) (100) plane (b) (110) plane (c) (111) plane.

(i) (100) plane → No change from simple cubic

$$\text{Surface density}_{100} = 4.47 \times 10^{18} \frac{\text{atoms}}{m^2} = 4.47 \times 10^{14} \frac{\text{atoms}}{cm^2}$$

(ii) (110) plane → 1 more atom than simple cubic

$$\text{surf. density}_{100,bcc} = 2\ \text{surf. density}_{100} = 2(3.16 \times 10^{18})$$

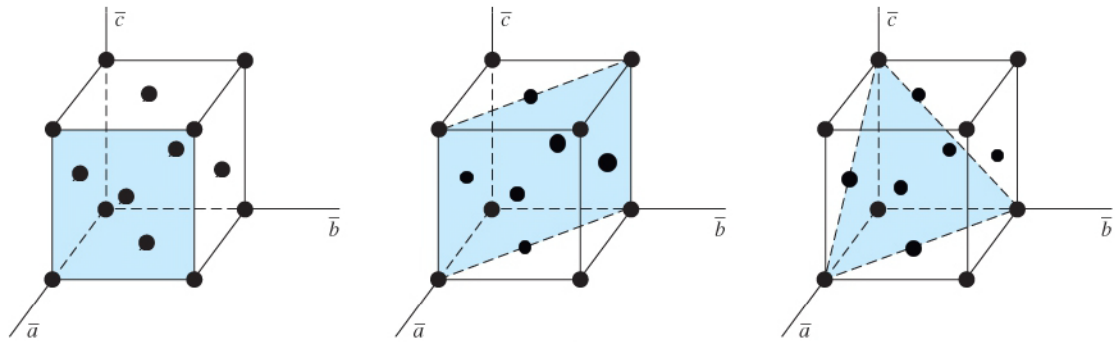
$$\text{surf. density}_{100,bcc} = 6.32 \times 10^{18} \frac{\text{atoms}}{m^2} = 6.32 \times 10^{14} \frac{\text{atoms}}{cm^2}$$

(iii) The (111) plane does not pass through center of cube. Since, we are not given the radius of the atoms, assume (111) plane does not intersect atom at center. Therefore,

$$\text{surf. density}_{111,bcc} = \text{surf. density}_{111}$$

$$\text{Surf. density}_{111,bcc} = 2.58 \times 10^{18} \frac{\text{atoms}}{m^2} = 2.58 \times 10^{14} \frac{\text{atoms}}{cm^2}$$

c) face-centered cubic



(a) (100) plane (b) (110) plane (c) (111) plane.

(i) (100) plane → 1 more atom than simple cubic

surface $density_{100,fcc}$ = 2 Surf. $density_{100}$ = $2(4.47 \times 10^{18})$

Surf. $density_{100,fcc} = 8.94 \times 10^{18} \frac{atoms}{m^2} = 8.94 \times 10^{14} \frac{atoms}{cm^2}$

(ii) (110) plane → $\frac{1}{2} + \frac{1}{2} = 1$ more atom than simple cubic

Surf. $density_{110,fcc}$ = 2 Surf. $density_{110}$ = $2(3.16 \times 10^{18})$

Surf. $density_{110,fcc} = 6.32 \times 10^{18} \frac{atoms}{m^2} = 6.32 \times 10^{14} \frac{atoms}{cm^2}$

(iii) (111) plane

$\# \text{ atoms} = 3 \text{ corners} \left(\frac{1/6 \text{ atom}}{\text{corner}}\right) + 3 \text{ faces} \left(\frac{1/2 \text{ atom}}{\text{face}}\right)$

$= \frac{1}{2} + \frac{3}{2} = 2 \text{ atoms}$

Since the simple cubic (111) plane had 0.5 atoms,

Surf. $density_{111,fcc}$ = 4 Surf. $density_{111}$ = $4(2.58 \times 10^{18})$

Surf. $density_{111,fcc} = 1.03 \times 10^{19} \frac{atoms}{m^2} = 1.03 \times 10^{15} \frac{atoms}{cm^2}$