

Example- Let's examine various quantities when an electron with a kinetic energy of 1 eV in a region of zero potential is incident on a finite potential barrier at $x = 0$ of 12 eV that is 4 Angstroms wide followed by another zero potential region.

Define some constants

$$h := 6.62607015 \cdot 10^{-34} \text{ J-s} \quad h_{\text{mod}} := \frac{h}{2 \cdot \pi} \quad h_{\text{mod}} = 1.05457 \times 10^{-34} \text{ J-s}$$

$$\underline{m} := 9.1093837015 \cdot 10^{-31} \text{ kg} \quad a := 4 \cdot 10^{-10} \text{ m}$$

Calculate some quantities and/or define equations

$$E_{\text{eV}} := 1 \text{ eV} \quad E := E_{\text{eV}} \cdot 1.602176634 \cdot 10^{-19} \quad \boxed{E = 1.60218 \times 10^{-19}} \text{ J}$$

$$V0_{\text{eV}} := 12 \text{ eV} \quad V0 := V0_{\text{eV}} \cdot 1.602176634 \cdot 10^{-19} \quad \boxed{V0 = 1.92261 \times 10^{-18}} \text{ J}$$

Per (2.61a), the wave number k_1 (rad/m) for regions I & III is

$$k_1 := \sqrt{\frac{2 \cdot m \cdot E}{h_{\text{mod}}^2}} \quad \boxed{k_1 = 5.12317 \times 10^9} \text{ rad/m}$$

Per (2.61b), the wave number k_2 (Np/m) in region II is

$$k_2 := \sqrt{\frac{2 \cdot m \cdot (V0 - E)}{h_{\text{mod}}^2}} \quad \boxed{k_2 = 1.69916 \times 10^{10}} \text{ Np/m}$$

From notes, the exact transmission coefficient T is

$$T_{\text{exact}} := \frac{1}{1 + \frac{V0^2 \cdot \sinh(k_2 \cdot a) \cdot \sinh(k_2 \cdot a)}{4 \cdot E \cdot (V0 - E)}} \quad \boxed{T_{\text{exact}} = 1.52636 \times 10^{-6}}$$

Pretty small!

$$\text{For } k_2 \cdot a = 6.7966 \quad \sinh(k_2 \cdot a) \cdot \sinh(k_2 \cdot a) = 2.001859 \times 10^5$$

$$\text{which can be approximated as } 0.25 \cdot e^{2 \cdot k_2 \cdot a} = 2.001864 \times 10^5$$

Per (2.63), the approximate transmission coefficient T is

$$T_{\text{approx}} := 16 \cdot \frac{E}{V0} \cdot \left(1 - \frac{E}{V0}\right) \cdot e^{-2 \cdot k_2 \cdot a} \quad \boxed{T_{\text{approx}} = 1.52635 \times 10^{-6}}$$

Very good approximation!