Example- An electrically isolated block of n-type GaAs ($N_d = 1.4 \times 10^{16}$ #/cm^3, $\tau_{p0} = 40$ ns, & $\mu_p = 300$ cm2/V·s) exists for $x \geq 0$ (assume air for $x < 0$). It is at thermal equilibrium at 300 K with no external electric field, but has excess charge carriers of the concentration 3×10^{14} #/cm^3 supplied at $x = 0$. Find the steady-state distribution of excess charge carriers.

For n-type GaAs, the minority carrier (i.e., holes/δp) transport equation is:

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}}$$

- "steady-state" $\Rightarrow \dfrac{\partial(\delta p)}{\partial t} = \dfrac{\partial(\delta n)}{\partial t} = 0$.
- "no external electric field" $\Rightarrow \bar{E}\dfrac{\partial(\delta n)}{\partial x} = \bar{E}\dfrac{\partial(\delta p)}{\partial x} = 0$.
- No mention of excess charge generation anywhere but at $x = 0 \Rightarrow g' = 0$.
- Per the Einstein relation $\dfrac{D_p}{\mu_p} = \dfrac{k_B T}{e} \quad \Rightarrow \quad D_p = \mu_p \dfrac{k_B T}{e}$.

$$D_p = 300 \frac{\text{cm}^2}{\text{V}\bullet\text{s}} (1\ \text{m}^2 / 100^2 \text{cm}^2) \frac{(1.38065 \times 10^{-23})300}{1.6021766 \times 10^{-19}}$$

$\Rightarrow$ $D_p = 0.00077556$ m^2/s $= 7.7556$ cm^2/s.

Therefore, our n-type minority carrier transport equation reduces to the second-order ODE

$$0 = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \frac{\delta p}{\tau_{p0}} \quad \Rightarrow \quad \frac{\partial^2(\delta p)}{\partial x^2} - \frac{1}{D_p \tau_{p0}} \delta p = 0.$$

In a fashion similar to second-order RLC circuits, assume a solution $\delta p(x) = Ae^{sx}$.

Substituting this into the second-order ODE yields a characteristic equation $s^2 - \dfrac{1}{D_p \tau_{p0}} = 0$. Defining length $L_n = \sqrt{D_p \tau_{p0}}$, we get $s^2 - \dfrac{1}{L_n^2} = 0 \quad \Rightarrow \quad s = \pm \dfrac{1}{L_n}$.

The general solution to our second-order ODE is then $\delta p(x) = A\,e^{-x/L_n} + B\,e^{x/L_n}$.

By conservation of charge, set $B = 0$ to avoid $\delta p(x) \to \infty$ as $x \to \infty$.

Next, apply boundary condition that $\delta p(x{=}0) = 3 \times 10^{14}$ #/cm^3 to find that the unknown constant $A = 3 \times 10^{14}$ #/cm^3.

Using the given minority carrier lifetime $\tau_{p0} = 40$ ns & $D_p = 0.00077556$ m^2/s , calculate length $L_n = \sqrt{0.00077556(40\times10^{-9})} = 5.56978\times10^{-6}$ m and get the steady-state distribution of excess charge carriers

$$\boxed{\delta p(x) = \delta n(x) = 3\times10^{14}\,e^{-x/5.57\times10^{-6}}\ \#/\text{cm}^3 \text{ for } x \ge 0}.$$

As a bonus, we can calculate the diffusion current density $J_{px|dif}$ using (5.34). Using all MKS units,

$$\begin{aligned} J_{px|dif} &= -eD_p\frac{\partial p}{\partial x} = -eD_p\frac{\partial(p_0+\delta p(x))}{\partial x} = -eD_p\frac{\partial\,\delta p(x)}{\partial x} \\ &= -1.6022\times10^{-19}(7.7556\times10^{-4})\frac{\partial}{\partial x}\left[3\times10^{20}\,e^{-x/5.57\times10^{-6}}\right] \\ &= -1.6022\times10^{-19}(7.7556\times10^{-4})\,3\times10^{20}\,\frac{-1}{5.57\times10^{-6}}e^{-x/5.57\times10^{-6}} \end{aligned}$$

$$\boxed{J_{px|dif} = 6692.915\,e^{-x/5.57\times10^{-6}}\ \text{A/m}^2 = 0.6693\,e^{-x/5.57\times10^{-6}}\ \text{A/cm}^2 \text{ for } x \ge 0.}$$