From *Semiconductor Physics and Devices: Basic Principles* (4th Edition), Donald A. Neamen, McGraw Hill, 2012, ISBN 978-0-07-352958-5.

Summary (extended Table 12.3)

Emitter injection efficiency

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}} \right) \approx \frac{1}{1 + \dfrac{N_B}{N_E} \dfrac{D_E}{D_B} \dfrac{x_B}{x_E}} \quad \text{where } x_B \ll L_B \text{ and } x_E \ll L_E$$

Base transport factor

$$\alpha_T = \left(\frac{J_{nC}}{J_{nE}} \right) \approx \frac{1}{1 + \dfrac{1}{2} \left(\dfrac{x_B}{L_B} \right)^2} \quad \text{where } x_B \ll L_B$$

Recombination factor

$$\delta = \left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \right) \approx \frac{1}{1 + \dfrac{J_{r0}}{J_{s0}} e^{-V_{BE}/2V_t}} \quad \text{when } J_{pE} \ll J_{nE}$$

Common-base current gain

$$\alpha = \frac{i_C}{i_E} = \frac{J_{nC}}{J_{nC} + J_R + J_{pE}} = \left(\frac{J_{nE}}{J_{nE} + J_{pE}} \right) \left(\frac{J_{nC}}{J_{nE}} \right) \left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \right) = \gamma \, \alpha_T \, \delta$$

$$\approx \frac{1}{1 + \dfrac{N_B}{N_E} \dfrac{D_E}{D_B} \dfrac{x_B}{x_E} + \dfrac{1}{2} \left(\dfrac{x_B}{L_B} \right)^2 + \dfrac{J_{r0}}{J_{s0}} e^{-V_{BE}/2V_t}}$$

Common-emitter current gain

$$\beta = \frac{i_C}{i_B} = \frac{\alpha}{1 - \alpha} \approx \frac{1}{\dfrac{N_B}{N_E} \dfrac{D_E}{D_B} \dfrac{x_B}{x_E} + \dfrac{1}{2} \left(\dfrac{x_B}{L_B} \right)^2 + \dfrac{J_{r0}}{J_{s0}} e^{-V_{BE}/2V_t}}$$