At 300 K, a silicon npn bipolar junction transistor (BJT), with each region uniformly doped, has the following parameters:

Emitter (*n*-region)	**Base** (*p*-region)	**Collector** (*n*-region)
$N_E = 10^{18}$ cm^{-3}	$N_B = 10^{16}$ cm^{-3}	$N_C = 10^{15}$ cm^{-3}
$\mu_{nE} = 280$ cm^2/V-s	$\mu_{pB} = 420$ cm^2/V-s	$\mu_{nC} = 1200$ cm^2/V-s
$\tau_{E0} = 10$ ns	$\tau_{B0} = 50$ ns	$\tau_{C0} = 100$ ns
$x_E = 1.5$ µm	$x_B = 0.75$ µm	$x_C = 90$ µm

Also, $A_{BE} = 5 \times 10^{-3}$ cm^2, $V_{BE} = 0.64$ V and $V_{CE} = 5$ V.

We wish to find the excess minority carrier distributions.

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From Table B.4, $n_i = 1.5 \times 10^{10}$ cm^{-3}. At 300 K, (7.10) $V_t = k_B T/e = 0.025852$ V.

<u>Emitter region</u> (*n*-region ⇒ electrons are majority carriers, holes are minority)

Since $N_E >> n_i$ and there is no acceptor doping (4.60), $n_{E0} \cong N_E = 10^{18}$ cm^{-3}.

Using (4.43), $p_{E0} = n_i^2 / N_E = (1.5\times10^{10})^2/10^{18} = 225$ cm^{-3}.

To find the emitter diffusion length L_E, we need the diffusion coefficient. Using the Einstein relation (5.47), $D_E = \mu_{nE} V_t = 280(0.025852) = 7.24$ cm^2/s.

Then, using (12.18),

$$L_E = \sqrt{D_E \tau_{E0}} = \sqrt{7.24 \text{ cm}^2/\text{s}\,(10\bullet10^{-9}\text{s})} \quad \Rightarrow \; L_E = 2.69\times10^{-4} \text{ cm} = 2.69 \text{ µm}.$$

Per (12.21a),

$$\delta p_E(x') = \frac{p_{E0}\left[e^{V_{BE}/V_t} - 1\right]\sinh\left(\dfrac{x_E - x'}{L_E}\right)}{\sinh\left(x_E/L_E\right)} = \frac{225\left[e^{0.64/0.025852} - 1\right]\sinh\left(\dfrac{1.5\bullet10^{-6} - x'}{2.69\bullet10^{-6}}\right)}{\sinh\left(1.5\bullet10^{-6}/2.69\bullet10^{-6}\right)}$$

$$= \frac{225\left[5.643216\bullet10^{10} - 1\right]\sinh\left(\dfrac{1.5\bullet10^{-6} - x'}{2.69\bullet10^{-6}}\right)}{\sinh\left(0.55762\right)}$$

$$\underline{\delta p_E(x') = 2.16318 \bullet 10^{13} \sinh\left(\frac{1.5 \bullet 10^{-6} - x'}{2.69 \bullet 10^{-6}}\right) \text{ cm}^{-3} \qquad 0 \le x' \le 1.5\ \mu\text{m}}$$

Note that the '-1' term in $e^{V_{BE}/V_t} - 1$ is negligible. Also, note that

$$\sinh(x_E / L_E) = \sinh\left(1.5 \bullet 10^{-6} / 2.69 \bullet 10^{-6}\right) = \sinh(1.5 / 2.69)$$
$$= \sinh(0.55762) = 0.5870 \approx 0.55762 = x_E / L_E .$$

The total hole concentration is $p_E(x') = \delta p_E(x') + p_{E0} \qquad 0 \le x' \le x_E$

$$\underline{p_E(x') = 2.16318 \bullet 10^{13} \sinh\left(\frac{1.5 \bullet 10^{-6} - x'}{2.69 \bullet 10^{-6}}\right) + 225 \text{ cm}^{-3} \qquad 0 \le x' \le 1.5\ \mu\text{m}}$$

Obviously, the p_{E0} term is insignificant.

Therefore, we can expect that the approximate expression (12.21b) will be quite accurate

$$\delta p_E(x') \approx \frac{p_{E0}}{x_E}\left[e^{V_{BE}/V_t} - 1\right]\left(x_E - x'\right) = \frac{225}{1.5 \bullet 10^{-6}}\left[e^{0.64/0.025852} - 1\right]\left(1.5 \bullet 10^{-6} - x'\right)$$
$$= \frac{225}{1.5 \bullet 10^{-6}}\left[e^{0.64/0.025852} - 1\right]\left(1.5 \bullet 10^{-6} - x'\right)$$

$$\underline{\delta p_E(x') \approx 8.46482 \bullet 10^{18}\left(1.5 \bullet 10^{-6} - x'\right) \text{ cm}^{-3} \qquad 0 \le x' \le 1.5\ \mu\text{m}}$$

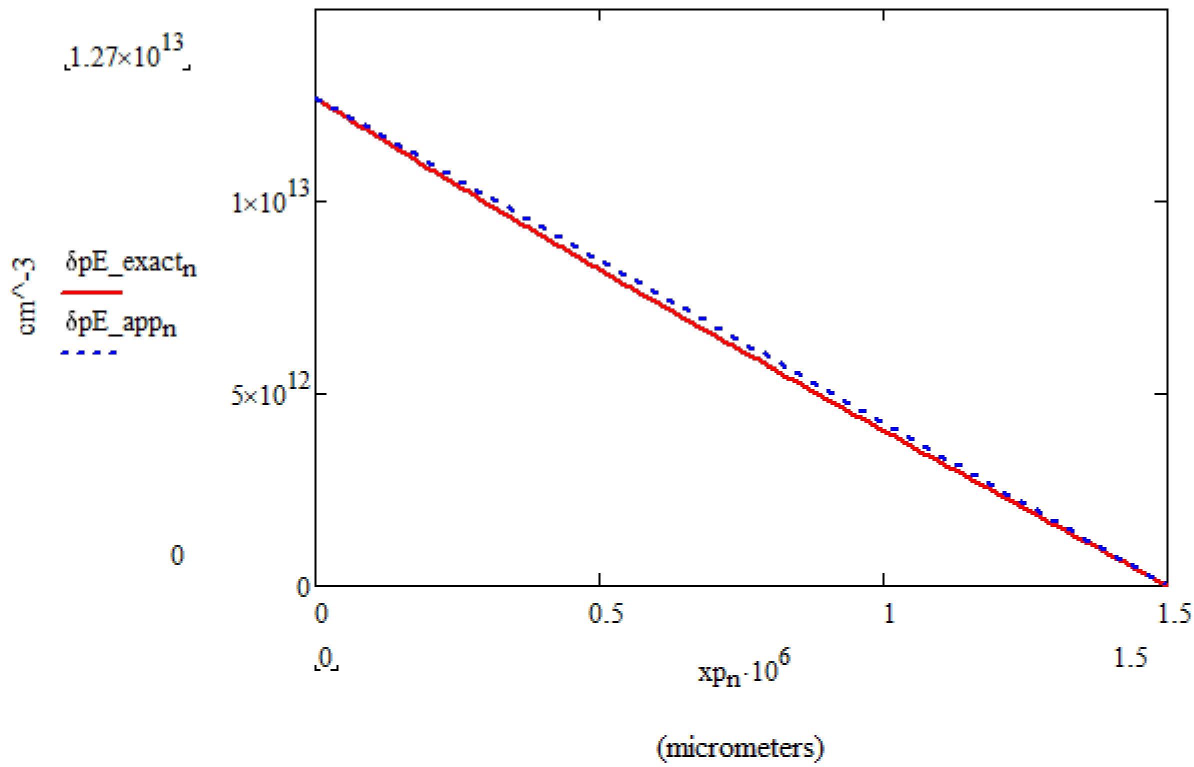
and $\underline{p_E(x') \approx 8.46482 \bullet 10^{18}\left(1.5 \bullet 10^{-6} - x'\right) + 225 \text{ cm}^{-3} \qquad 0 \le x' \le 1.5\ \mu\text{m}}$.

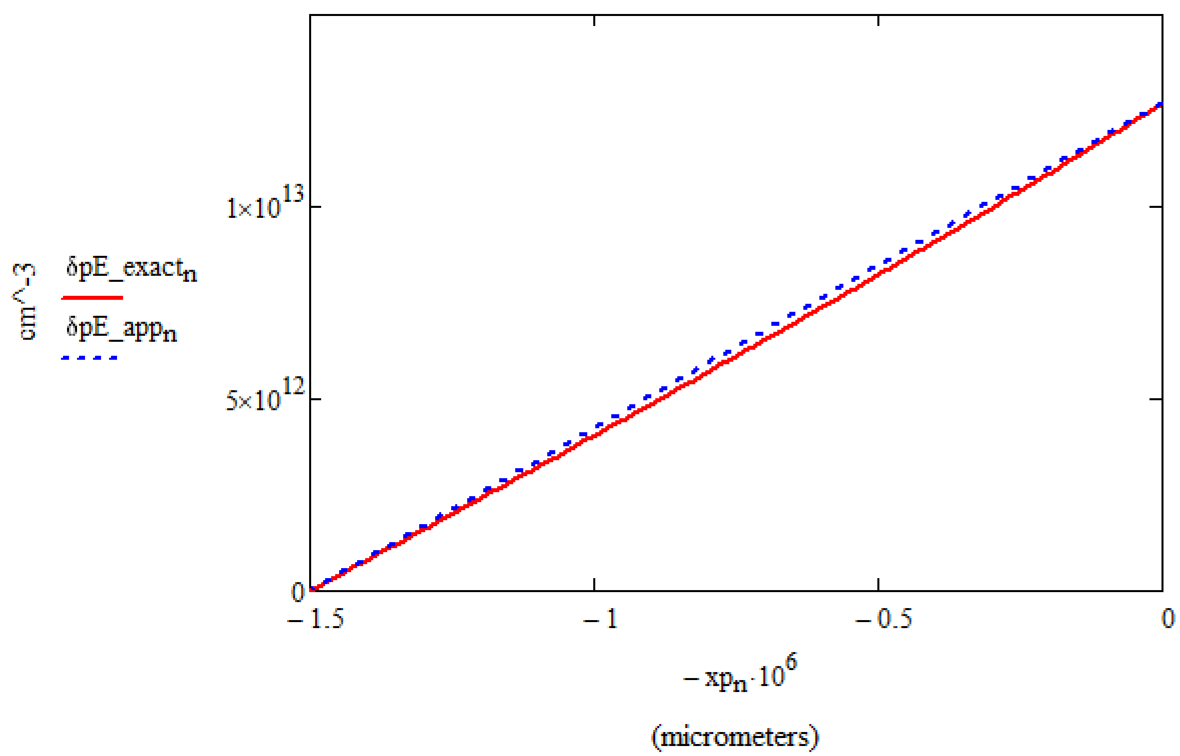
Obviously, the p_{E0} term is insignificant.

MathCAD was used to plot these **emitter region** excess minority carrier (hole) concentrations.

At *x*' = 0, they will both start at $\underline{\delta p_E(x' = 0) = 1.269724 \bullet 10^{13} \text{ cm}^{-3}}$**.**

For the second MathCAD plot, *x*' is multiplied by -1 to match the convention in Figure 12.13.





Note that the exact (12.21a) and approximate (12.21b) expressions yield very close results even though the emitter length is a significant fraction of the diffusion length, i.e., $x_E/L_E = 0.5576$.

Base region (p-region ⇒ holes are majority carriers, electrons are minority)

Since $N_B >> n_i$ and there is no donor doping (4.62), $p_{B0} \cong N_B = 10^{16}$ cm^{-3}.

Using (4.43), $n_{B0} = n_i^2 / N_B = (1.5\times10^{10})^2/10^{16} = 22{,}500$ cm^{-3}.

Using the Einstein relation (5.47), $D_B = \mu_{pB} V_t = 420(0.025852) = 10.86$ cm^2/s.

Then, using (12.11),

$$L_B = \sqrt{D_B \tau_{B0}} = \sqrt{10.86 \text{ cm}^2/\text{s}\ (50 \bullet 10^{-9}\text{s})} \quad \Rightarrow L_B = 7.37\times10^{-4} \text{ cm} = 7.37\ \mu\text{m}.$$

Per (12.15a),

$$\delta n_B(x) = \frac{n_{B0}\left\{\left[e^{V_{BE}/V_t} - 1\right]\sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}}{\sinh\left(x_B / L_B\right)}$$

$$= \frac{22{,}500\left\{\left[e^{0.64/0.025852} - 1\right]\sinh\left(\frac{0.75 \bullet 10^{-6} - x'}{7.37 \bullet 10^{-6}}\right) - \sinh\left(\frac{x}{7.37 \bullet 10^{-6}}\right)\right\}}{\sinh\left(0.75 \bullet 10^{-6} / 7.37 \bullet 10^{-6}\right)}$$

$$= 220{,}662.07\left\{\left[5.643216 \bullet 10^{10} - 1\right]\sinh\left(\frac{0.75 \bullet 10^{-6} - x'}{7.37 \bullet 10^{-6}}\right) - \sinh\left(\frac{x}{7.37 \bullet 10^{-6}}\right)\right\}$$

$$\delta n_B(x) = 1.24524 \bullet 10^{16} \sinh\left(\frac{0.75 \bullet 10^{-6} - x}{7.37 \bullet 10^{-6}}\right) - 220{,}662.07 \sinh\left(\frac{x}{7.37 \bullet 10^{-6}}\right) \text{ cm}^{-3}$$

for 0 ≤ x ≤ 0.75 μm.

Note that the '-1' term in $e^{V_{BE}/V_t} - 1$ is negligible. Also, note that

$$\sinh(x_B / L_B) = \sinh(0.10176) = 0.10194 \approx 0.10176 = x_B / L_B.$$

The total electron concentration is $n_B(x) = \delta n_B(x) + n_{B0} \quad 0 \le x \le x_B$

$$n_B(x) = 1.24524 \bullet 10^{16} \sinh\left(\frac{0.75 \bullet 10^{-6} - x}{7.37 \bullet 10^{-6}}\right) - 220{,}662 \sinh\left(\frac{x}{7.37 \bullet 10^{-6}}\right) + 22{,}500 \text{ cm}^{-3}$$

for 0 ≤ x ≤ 0.75 μm.

Obviously, the n_{B0} term is insignificant.

The approximate expression (12.15b) should be very accurate

$$\delta n_B(x) \approx \frac{n_{B0}}{x_B}\left[\left(e^{V_{BE}/V_t}-1\right)\left(x_B - x\right) - x\right]$$

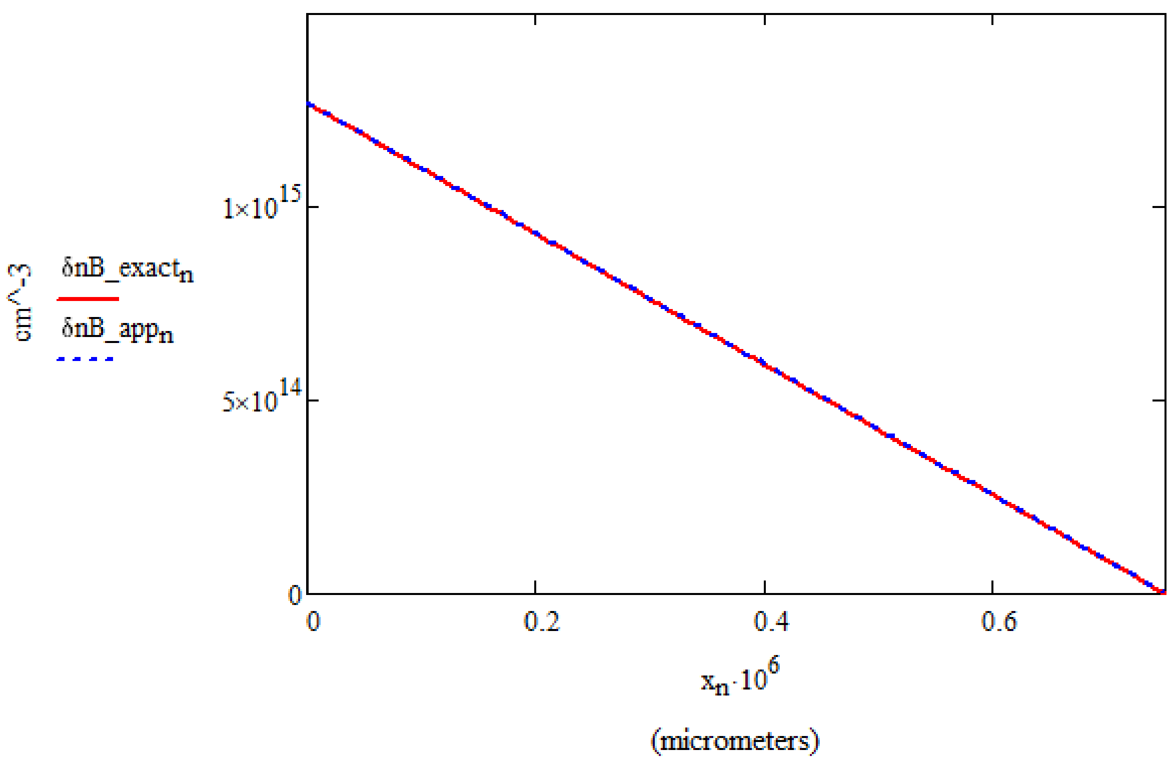
$$= \frac{22,500}{0.75 \cdot 10^{-6}}\left[\left(e^{0.64/0.025852}-1\right)\left(0.75 \cdot 10^{-6} - x\right) - x\right]$$

$$\underline{\delta n_B(x) \approx 3 \cdot 10^{10}\left[5.64322 \cdot 10^{10}\left(0.75 \cdot 10^{-6} - x\right) - x\right] \text{ cm}^{-3} \quad 0 \le x \le 0.75 \ \mu\text{m}}$$

and

$$\underline{n_B(x) \approx 3 \cdot 10^{10}\left[5.64322 \cdot 10^{10}\left(0.75 \cdot 10^{-6} - x\right) - x\right] + 22,500 \text{ cm}^{-3} \quad 0 \le x \le 0.75 \ \mu\text{m}}.$$

MathCAD was used to plot these **base region** excess minority carrier (electron) concentrations. At $x = 0$, they both start at $\underline{\delta n_B(x=0) = 1.269724 \cdot 10^{15} \text{ cm}^{-3}}$.



To the eye, the traces for the exact (12.15a) and approximate (12.15b) expressions for the excess minority carrier (electron) concentrations match exactly.

<u>Collector region</u> (n-region ⇒ electrons are majority carriers, holes are minority)

Since $N_C >> n_i$ and there is no acceptor doping (4.60), $n_{C0} \cong N_C = 10^{15}$ cm^{-3}.

Using (4.43), $p_{C0} = n_i^2 / N_C = (1.5\times10^{10})^2/10^{15} = 225{,}000$ cm^{-3}.

Using the Einstein relation (5.47), $D_C = \mu_{nC} V_t = 1200(0.025852) = 31.02$ cm^2/s.

Then, using (12.24),

$$L_C = \sqrt{D_C \tau_{C0}} = \sqrt{31.02 \text{ cm}^2/\text{s}\,(100\bullet10^{-9}\text{s})} \quad \Rightarrow \quad L_C = 1.761\times10^{-3} \text{ cm} = 17.61 \text{ µm}.$$

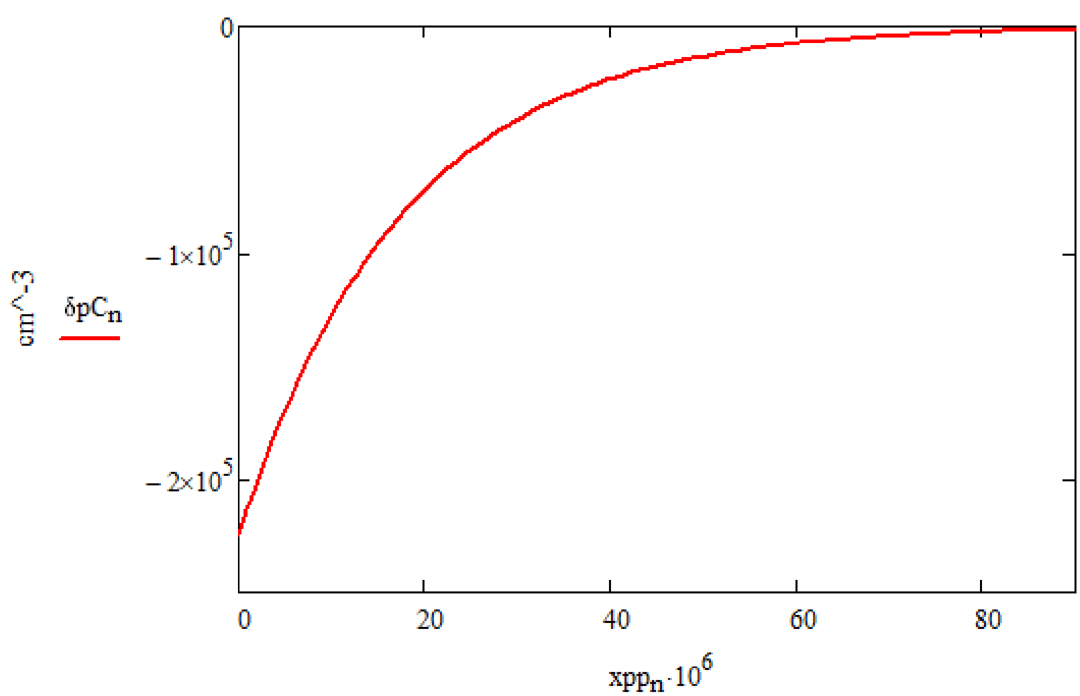
(12.24) $\delta p_C(x'') = -p_{C0} e^{-x''/L_C}$

$$\underline{\boldsymbol{\delta p_C(x'') = -225{,}000\, e^{-x''/17.61e-6} \text{ cm}^{-3} \quad 0 \le x'' \le 90 \text{ µm}}}$$

The total hole concentration is $p_C(x'') = \delta p_C(x'') + p_{C0} \qquad 0 \le x'' \le x_C$

$$\underline{\boldsymbol{p_C(x'') = 225{,}000 - 225{,}000\, e^{-x''/17.61e-6} \text{ cm}^{-3} \quad 0 \le x'' \le 90 \text{ µm}}}.$$

MathCAD was used to plot the **collector region** excess minority carrier (hole) concentration. At $x'' = 0$, it starts at **<u>$\delta p_C(0) = -225{,}000$ cm^{-3}</u>.**



At x_C, **<u>$\delta p_C(90 \text{ µm}) = -1{,}358$ cm$^{-3} \approx 0$</u>**, i.e., 0.6% of $\delta p_C(0)$. So, the long collector approximation is close for $x_C / L_C = 90/17.61 = 5.11$.