Chapter 4 The Semiconductor in Equilibrium

4.1 Charge Carriers in Semiconductors

Here, we have two types of charge carriers, electrons and holes in the conduction and valence bands respectively. We wish to find how many of each are available to conduct current.

4.1.1 Equilibrium Distribution of Electrons and Holes

From Chap. 3, we'll bring together the density of states $g(E)$ and the Fermi-Dirac probability function $f_F(E)$ to get

electrons in conduction band @ energy E $\equiv \underline{n(E) = g_c(E) f_F(E)}$ (4.1)

holes in valence band @ energy E $\equiv \underline{p(E) = g_v(E)[1 - f_F(E)]}$ (4.2)

- ✱ To get total # of electrons, integrate $n(E)$ over range of energies in cond. band
- ✱ To get total # of holes, integrate $p(E)$ over valence band energy range.

4.1.1 contd

Recall $$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}$$

$$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v-E}$$

(see Fig 3.27)

Recall $$f_F(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}}$$

(see Fig 3.34 for $f_F(E)$ & $1-f_F(E)$ or Fermi_example_2.pdf)

This is all brought together in Fig. 4.1

- → show $g_c(E)$ & $g_v(E)$
- → show $f_F(E)$ and its range @ bottom
- → show $n(E)$ & $p(E)$ lines @ right
- → show areas to be found to get n_0 (# of electrons) and p_0 (# of holes)

From *Semiconductor Physics and Devices: Basic Principles* (4th Edition), Donald A. Neamen, McGraw Hill, 2012, ISBN 978-0-07-352958-5.

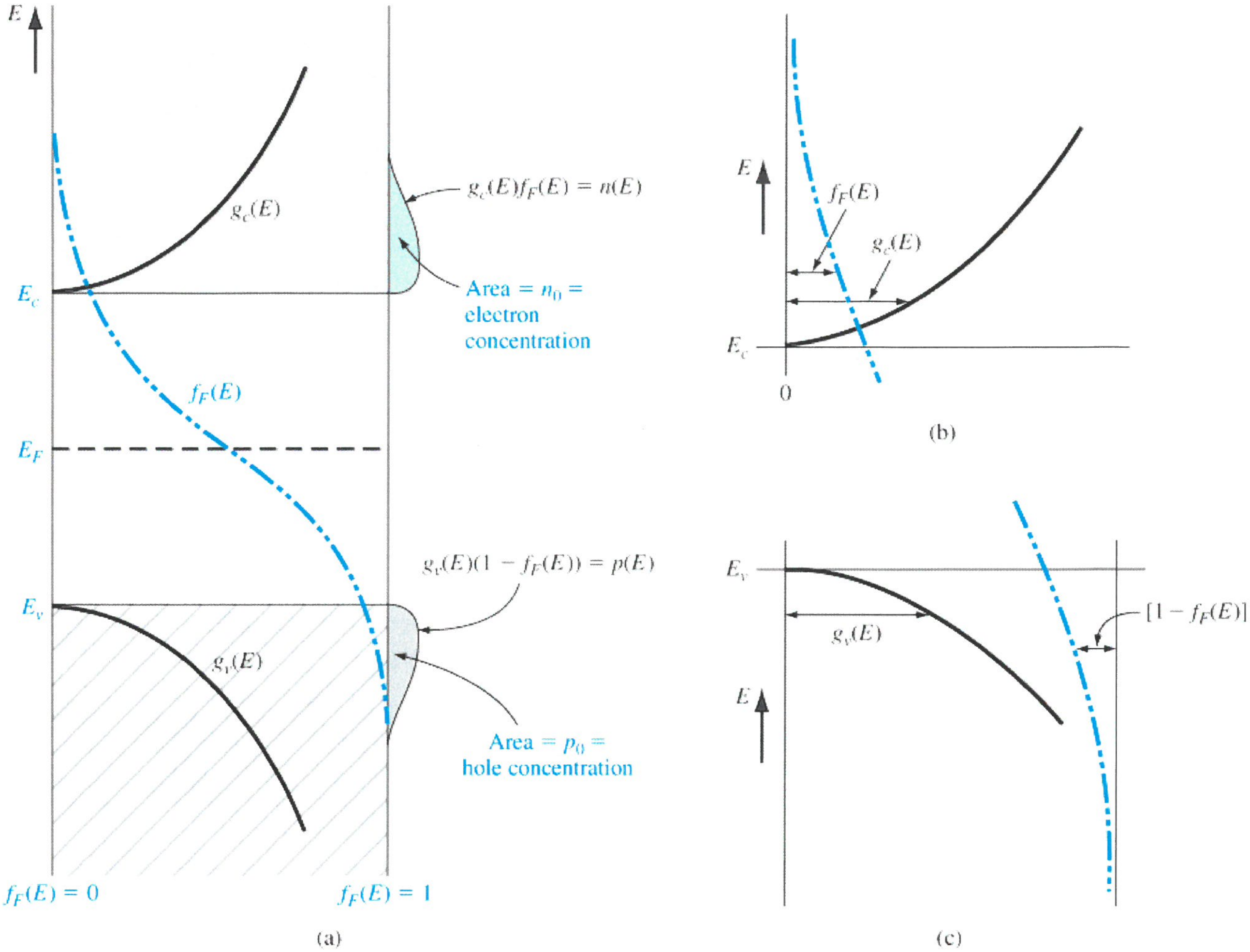


Figure 4.1 | (a) Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is near the midgap energy; (b) expanded view near the conduction-band energy; and (c) expanded view near the valence-band energy.

4.1.2 The n_0 and p_0 Equations

For intrinsic semiconductors, $E_F \simeq \frac{E_c + E_v}{2}$

$$\# \text{ of electrons} \equiv n_0 = \int_{E_c}^{E_{c,TOP}} g_c(E) f_F(E)\, dE \quad (4.3)$$

where E_c is energy of bottom of conduction band and $E_{c,TOP}$ is energy of top of conduction band. Since $f_F(E)$ falls off very rapidly, we can approximate $E_{c,top}$ w/ ∞ w/ very accurate results

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E)\, dE$$

Further, $(E_c - E_F) >> k_B T$ is typically true.

E.g., For GaAs $E_F \simeq 0.75\,eV$, $E_c \simeq 1.424\,eV$

so $E_c - E_F \simeq 0.674\,eV >> 300K\left(8.62 \times 10^{-5} \frac{eV}{K}\right) = 0.026\,eV$
(roughly 26x larger!!)

$\Rightarrow$ Can use Maxwell-Boltzmann approximation for $\underline{\underline{f_F(E)}}$!

4.1.2 cont.

$$n_o = \int_{E_c}^{\infty} \left(\frac{4\pi (m_n^*)^3 \sqrt{E-E_c}}{h^3}\right)\left(e^{-\frac{(E-E_F)}{k_B T}}\right) dE$$

Factor out constants and make a change of variables where

☆ → $\underline{\eta = \frac{E-E_c}{k_B T}}$ $\Rightarrow \frac{d\eta}{dE} = \frac{1}{k_B T}$

↳ $\underline{E - E_c = \eta\, k_B T}$ ↳ $d\eta = \frac{dE}{k_B T} \Rightarrow \underline{dE = k_B T\, d\eta}$

↳ limit $E_c \to 0$

limit $\infty \to \infty$

Term $\sqrt{E-E_c} \to k_B T^{1/2} \sqrt{\eta}$

$$\text{Term } e^{-\frac{E-E_F}{k_B T}} = e^{-\frac{E_c - E_c + E - E_F}{k_B T}}$$

$$= e^{-\left(\frac{E_c - E_F}{k_B T}\right)} \underbrace{e^{-\frac{E-E_c}{k_B T}}}_{e^{-\eta}}$$

☆ → w/ the substitutions

$$n_o = \frac{4\pi (2 m_n^* k_B T)^{3/2}}{h^3} e^{-\left(\frac{E_c - E_F}{k_B T}\right)} \underbrace{\int_0^{\infty} \sqrt{\eta}\, e^{-\eta}\, d\eta}_{\text{gamma function } = \frac{\sqrt{\pi}}{2}}$$

4.1.2 cont.

$$n_o = \frac{4\pi(2m_n^* k_B T)^{3/2}}{h^3} e^{-\left(\frac{E_c - E_F}{k_B T}\right)} \frac{\sqrt{\pi}}{2}$$

$$\underline{n_o = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2} e^{-\left(\frac{E_c - E_F}{k_B T}\right)} \qquad (4.9)}$$

Define $N_c = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2} \equiv$ effective density of states function in cond. band

$$\underline{n_o = N_c e^{-\left(\frac{E_c - E_F}{kT}\right)}} \quad \left(\frac{\#\text{ electrons}}{\text{vol}}\right)$$

What about holes?

$$1 - f_F(E) = \frac{1}{1 + e^{\frac{E_F - E}{k_B T}}} \qquad (4.13a)$$

If $E_F - E_v >> k_B T$ (almost always true)

$$1 - f_F(E) \simeq e^{-\left(\frac{E_F - E}{k_B T}\right)}$$

Then, replacing 0 w/ $-\infty$ (great approx)

$$\#\text{ holes} = p_o = \int_{-\infty}^{E_v} \left(\frac{4\pi(2m_p^*)^{3/2}}{h^3}\right)\sqrt{E_v - E}\; e^{-\left(\frac{E_F - E}{k_B T}\right)} dE$$

4.1.2 cont.

Factoring out constants and doing a change of variables where

$$\eta' = \frac{E_V - E}{k_B T}$$

leads to

$$p_0 = \frac{-4\pi\left(2 m_p^* k_B T\right)^{3/2}}{h^3}\, e^{\frac{-(E_F - E_V)}{k_B T}} \underbrace{\int_{\infty}^{0} \sqrt{\eta'}\, e^{-\eta'}\, d\eta'}_{-\sqrt{\pi}/2}$$

$$\underline{p_0 = \underbrace{2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}}_{N_V} e^{-\frac{(E_F - E_V)}{k_B T}}} \qquad (4.17)$$

$$\underline{p_0 = N_V\, e^{-\frac{(E_F - E_V)}{k_B T}}} \qquad (4.19)$$

4.1.3 The Intrinsic Carrier Concentration

Intrinsic semiconductor ⇒ Pure semiconductor

⇒ Charge neutral

#protons = #electrons

Here, by Conservation of Charge/Matter, the number of electrons in conduction band (call it n_i) must equal the number of holes in valence band p_i

$$n_i = p_i$$

$$n_i = n_0 = N_c\, e^{-\frac{(E_c - E_{Fi})}{k_B T}} = N_v\, e^{-\frac{(E_{Fi} - E_v)}{k_B T}} = p_0 = p_i$$

Then

$$n_i^2 = n_i p_i = N_c N_v\, e^{-\frac{(E_c - E_{Fi})}{k_B T}}\, e^{-\frac{(E_{Fi} - E_v)}{k_B T}}$$

$$= N_c N_v\, e^{-\frac{(E_c - E_v)}{k_B T}} \quad \text{but } \overset{\overset{\text{gap}}{\downarrow}}{E_g} = E_c - E_v$$

$$\underline{n_i^2 = N_c N_v\, e^{-E_g/k_B T}} \qquad (4.23)$$

⇒ @ a constant temperature, $n_i = p_i$ = constant

⇒ However, $n_i = p_i$ is very temperature dependent for obvious reasons

From *Semiconductor Physics and Devices: Basic Principles* (4th Edition), Donald A. Neamen, McGraw Hill, 2012, ISBN 978-0-07-352958-5.

- Table 4.1 gives some typical values (@ 300 K?).

Table 4.1 | Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Table 4.2 | Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

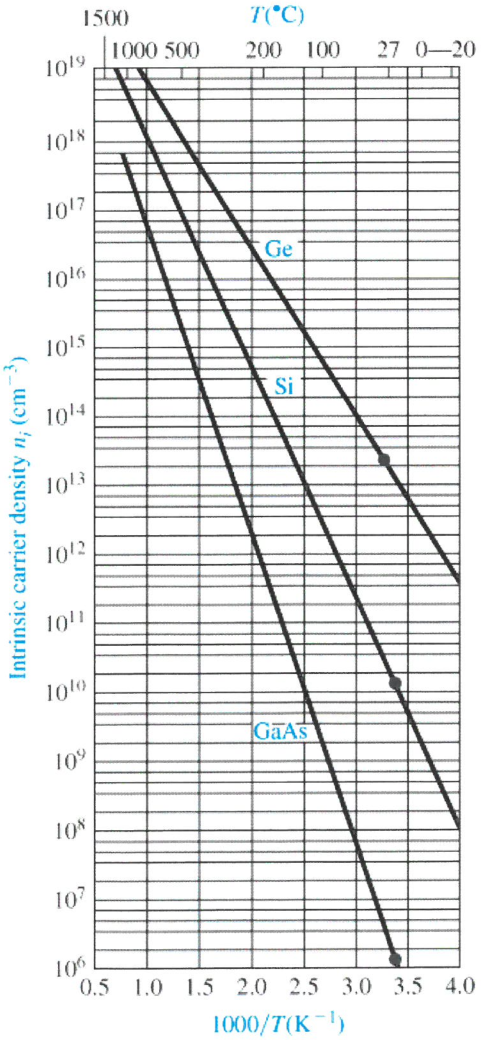


Figure 4.2 | The intrinsic carrier concentration of Ge, Si, and GaAs as a function of temperature.
(From Sze [14].)

Example- At 300 K for silicon, find effective density of states & carrier concentrations.

Constants

$h := 6.62607015 \cdot 10^{-34}$ J-s $\quad m0 := 9.1093837015 \cdot 10^{-31}$ kg $\quad T := 300$ K

$kBeV := 8.617333 \cdot 10^{-5}$ eV/K $\quad kBeV \cdot T = 0.025852$ eV

$kB := 1.380649 \cdot 10^{-23}$ J/K $\quad kB \cdot T = 4.14195 \times 10^{-21}$ J $\quad EgSi := 1.12$ eV

From Table 4.1, the eff. electron & hole masses for Si $\quad mne := 1.08 \cdot m0 \quad mpe := 0.56 \cdot m0$

Conduction band eff. density of states function (4.10) $\quad Nc := 2 \cdot \left(\frac{2 \cdot \pi \cdot mne \cdot kB \cdot T}{h^2}\right)^{\frac{3}{2}}$

$\boxed{Nc = 2.816 \times 10^{25}}$ #/m³ $\quad \boxed{\frac{Nc}{100^3} = 2.816 \times 10^{19}}$ #/cm³

From Table 4.1, $\boldsymbol{N_c = 2.8 * 10^{19}}$ **#/cm³**

Valence band eff. density of states function (4.18) $\quad Nv := 2 \cdot \left(\frac{2 \cdot \pi \cdot mpe \cdot kB \cdot T}{h^2}\right)^{\frac{3}{2}}$

$\boxed{Nv = 1.052 \times 10^{25}}$ #/m³ $\quad \boxed{\frac{Nv}{100^3} = 1.052 \times 10^{19}}$ #/cm³

From Table 4.1, $\boldsymbol{N_v = 1.04 * 10^{19}}$ **#/cm³**

Assuming $E_c - E_F \sim 0.5 E_g$, the concentration of electrons is (4.11)

$n0 := Nc \cdot e^{-\left(\frac{0.5 \cdot EgSi}{kBeV \cdot T}\right)}$ $\quad \boxed{n0 = 1.102 \times 10^{16}}$ #/m³ $\quad \boxed{\frac{n0}{100^3} = 1.102 \times 10^{10}}$ #/cm³

Assuming $E_F - E_v \sim 0.5 E_g$, the concentration of holes is (4.19)

$p0 := Nv \cdot e^{-\left(\frac{0.5 \cdot EgSi}{kBeV \cdot T}\right)}$ $\quad \boxed{p0 = 4.114 \times 10^{15}}$ #/m³ $\quad \boxed{\frac{p0}{100^3} = 4.114 \times 10^{9}}$ #/cm³

Or, we can use (4.23), to get n_i^2, and then calculate $n_i = n_0 = p_0$.

$ni2 := Nc \cdot Nv \cdot e^{-\left(\frac{EgSi}{kBeV \cdot T}\right)}$ $\quad ni := \sqrt{ni2}$ $\quad \boxed{ni = 6.733 \times 10^{15}}$ #/m³ $\quad \boxed{\frac{ni}{100^3} = 6.733 \times 10^{9}}$ #/cm³

From Table 4.2, $\boldsymbol{n_i = n_0 = p_0 = 1.5 * 10^{10}}$ **#/cm³**

4.1.3 cont.

Why are the numbers from the Tables different from calculations?

⇒ Effective mass, m_n^* + m_p^*, values are found experimentally (very difficult) @ low temperatures & may change w/ temp.

⇒ Density of states function was found *assuming* & generalizing from 3D infinite potential well!!

4.1.4 The Intrinsic Fermi-Level Position

Already said $E_F = E_{Fi} \cong \frac{E_c + E_v}{2}$. Let's apply $n_i = p_i$ to see how close we come.

$$N_c\, e^{\frac{-(E_c - E_{Fi})}{k_B T}} = N_v\, e^{\frac{-(E_{Fi} - E_v)}{k_B T}}$$

$$e^{\frac{-(E_c - E_{Fi})}{k_B T}} = \frac{N_v}{N_c}\, e^{\frac{-(E_{Fi} - E_v)}{k_B T}}$$

ln ↓ use $\ln(AB) = \ln A + \ln B$

$$\frac{-(E_c - E_{Fi})}{k_B T} = \ln\left(\frac{N_v}{N_c}\right) - \frac{(E_{Fi} - E_v)}{k_B T}$$

$$-E_c + E_{Fi} = k_B T \ln\left(\frac{N_v}{N_c}\right) - E_{Fi} + E_v$$

4.14 cont.

$$2E_{F_i} = k_B T \ln\left(N_v/N_c\right) + E_v + E_c$$

$$E_{F_i} = \tfrac{1}{2} k_B T \ln\left(\frac{N_v}{N_c}\right) + \left(\frac{E_c + E_v}{2}\right) \leftsquigarrow E_{\text{midgap}}$$

using $\dfrac{N_v}{N_c} = \dfrac{2\left(\dfrac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}}{2\left(\dfrac{2\pi m_n^* k_B T}{h^2}\right)^{3/2}} = \left(\dfrac{m_p^*}{m_n^*}\right)^{3/2}$

$$E_{F_i} = \tfrac{3}{2}\left(\tfrac{1}{2}\right) k_B T \ln\left(\frac{m_p^*}{m_n^*}\right) + E_{\text{midgap}}$$

offset from midgap $= \underline{E_{F_i} - E_{\text{midgap}} = \frac{3}{4} k_B T \ln\left(\frac{m_p^*}{m_n^*}\right)}$

ex. Look @ GaAs @ 300K

$E_{F_i} = 0.7486\text{ eV}$ from graph + $E_g = 1.424\text{ eV}$

$m_n^* = 0.067\, m_0$ & $m_p^* = 0.48\, m_0$ from Table 4.1

$k_B T = k_B(300) = 0.025852\text{ eV}$

$$E_{F_i} - E_{\text{midgap}} = 3/4\,(0.025852\text{ eV}) \ln\Big(\underbrace{\frac{0.48\, m_0}{0.067\, m_0}}_{\to\, 1.969}\Big)$$

$$= \underline{0.0382\text{ eV}}$$

tiny compared to $E_g/2 = 0.712\text{ eV}$
(5%)

4.2 Dopant Atoms and Energy Levels

* From experience, the number of electrons/holes available in intrinsic semiconductors are NOT enough to carry useful amounts of current

4.2.1 Qualitative Description

* Solution? Use intentional impurities (AKA dopants) to either introduce extra electrons over those needed for covalent bonds or have too few electrons leading to holes

⇒ extrinsic semiconductors ⇐

For example Si & Ge are group IV elements. Group III elements are one electron short (e.g., boron, B). Group V elements have one electron extra (e.g., phosphorous, P)

dopants w/ extra electrons are donors (Type n)

dopant causing extra holes are acceptors (Type p)

Show Figs 4.3 → 4.7

From *Semiconductor Physics and Devices: Basic Principles* (4th Edition), Donald A. Neamen, McGraw Hill, 2012, ISBN 978-0-07-352958-5.

For a **donor dopant** (AKA type *n*)

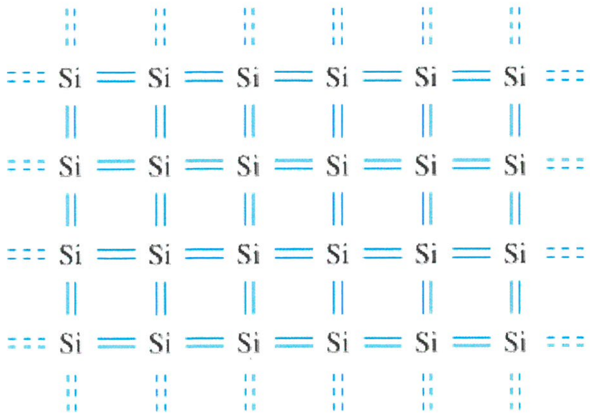


Figure 4.3 | Two-dimensional representation of the intrinsic silicon lattice.

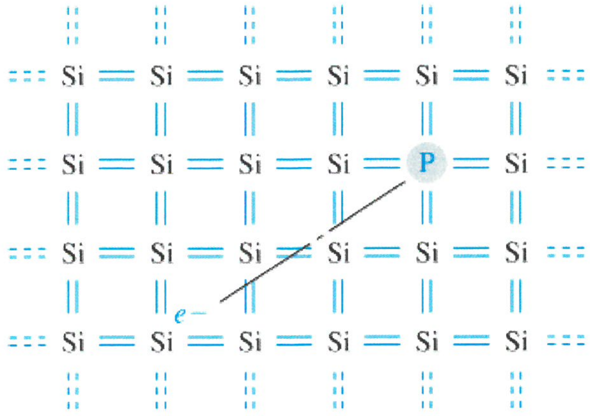
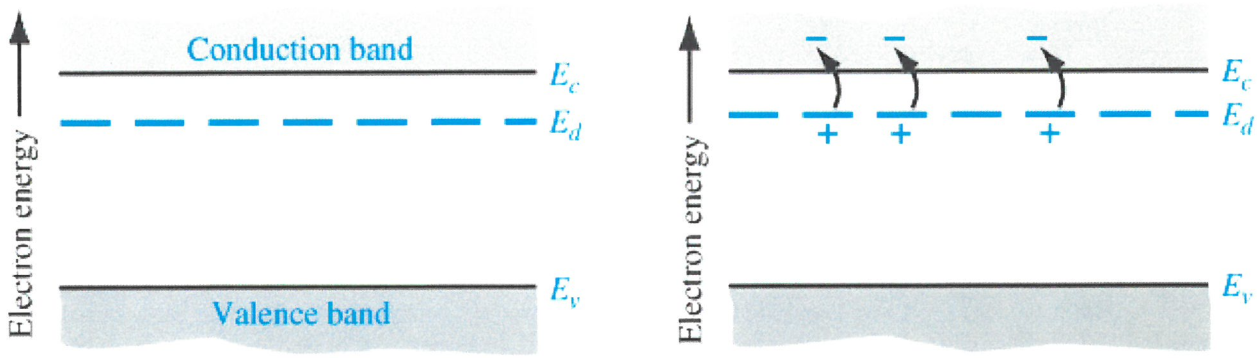


Figure 4.4 | Two-dimensional representation of the silicon lattice doped with a phosphorus atom.

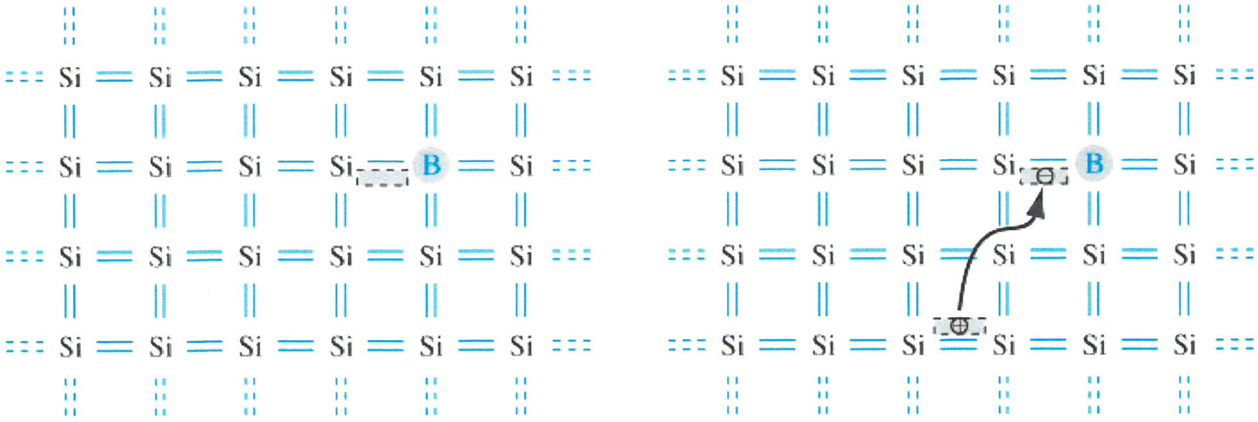


(a) (b)

Figure 4.5 | The energy-band diagram showing (a) the discrete donor energy state and (b) the effect of a donor state being ionized.

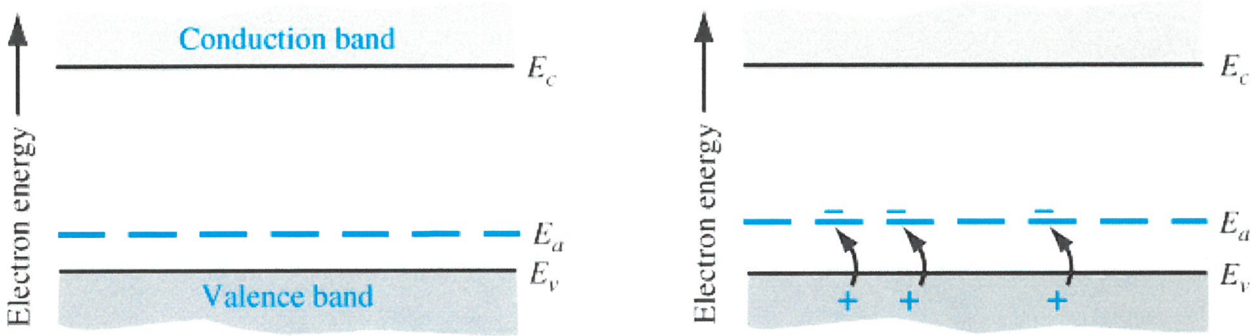
- Intuitively assume that the extra electron not needed for covalent bonding by the donor will not need as much energy to be lifted into conduction band.

For an **acceptor dopant** (AKA type *p*)



(a) (b)

Figure 4.6 | Two-dimensional representation of a silicon lattice (a) doped with a boron atom and (b) showing the ionization of the boron atom resulting in a hole.



(a) (b)

Figure 4.7 | The energy-band diagram showing (a) the discrete acceptor energy state and (b) the effect of an acceptor state being ionized.

- Intuitively assume that the extra electron needed for covalent bond by the acceptor will not need as much energy to be lifted from valence band, creating a hole.

4.2.2 Ionization Energy

Amount of energy needed to move electrons up to conduction band (doniors) or up from valence band (acceptor) called the ionization energy.

Table 4.3 shows some ionization energies for Si ($E_g = 1.12$ eV) and Ge ($E_g = 0.66$ eV) that are Group IV semiconductors

4.2.3 Group III-V semiconductors

GaAs (1.424eV = E_g) is a Group III-V semiconductor which needs group II acceptors or group VI domors as shown in Table 4.4.

From *Semiconductor Physics and Devices: Basic Principles* (4th Edition), Donald A. Neamen, McGraw Hill, 2012, ISBN 978-0-07-352958-5.

For the Group IV elemental semiconductor Si (E_g = 1.12 eV) and Ge (E_g = 0.66 eV), we use Group III elements as acceptors and Group V elements as donors.

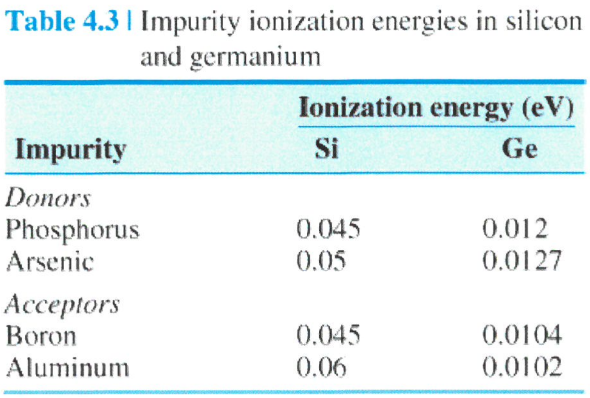


Table 4.3 | Impurity ionization energies in silicon and germanium

Impurity	**Ionization energy (eV)** Si	Ge
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

- ➢ Note that the ionization energies are indeed far smaller than the gap energies for silicon and germanium.

For a Group III-V semiconductor such as GaAs (E_g = 1.424 eV), we use Group II elements as acceptors and Group VI elements as donors.

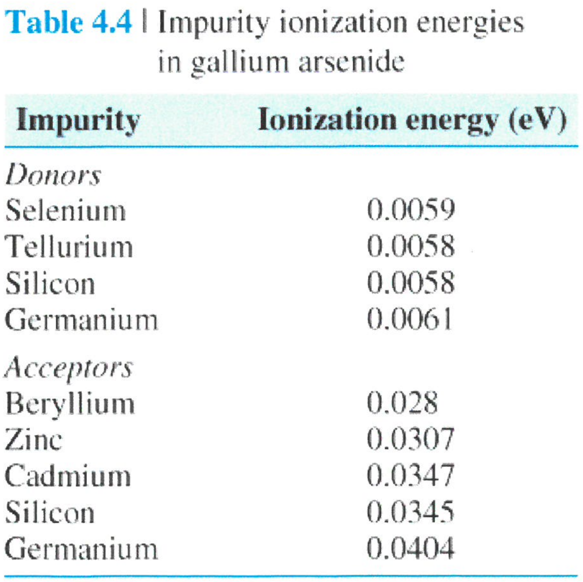


Table 4.4 | Impurity ionization energies in gallium arsenide

Impurity	**Ionization energy (eV)**
Donors	
Selenium	0.0059
Tellurium	0.0058
Silicon	0.0058
Germanium	0.0061
Acceptors	
Beryllium	0.028
Zinc	0.0307
Cadmium	0.0347
Silicon	0.0345
Germanium	0.0404

- ➢ Note that the ionization energies are indeed far smaller than the gap energy for gallium arsenide.

4.3 The Extrinsic Semiconductor

w/ extrinsic semiconductors, we have dopants introducing extra electrons or holes. Now, the thermal-equilibrium electron & hole concentrations are skewed from intrinsic (pure) semiconductors (where $n_0 = p_0 = n_i$).

4.3.1 Equilibrium Distribution of Electrons and Holes

Since $n_0 \neq p_0$, it follows that the Fermi energy will no longer be $\sim \frac{E_c + E_v}{2} = E_{midgap}$

$\Rightarrow$ For type n, $E_F > E_{Fi}$ (see Fig. 4.8).

$\Rightarrow$ For type p, $E_F < E_{Fi}$ (see Fig 4.9).

From *Semiconductor Physics and Devices: Basic Principles* (4th Edition), Donald A. Neamen, McGraw Hill, 2012, ISBN 978-0-07-352958-5.

Type *n* (more electrons)

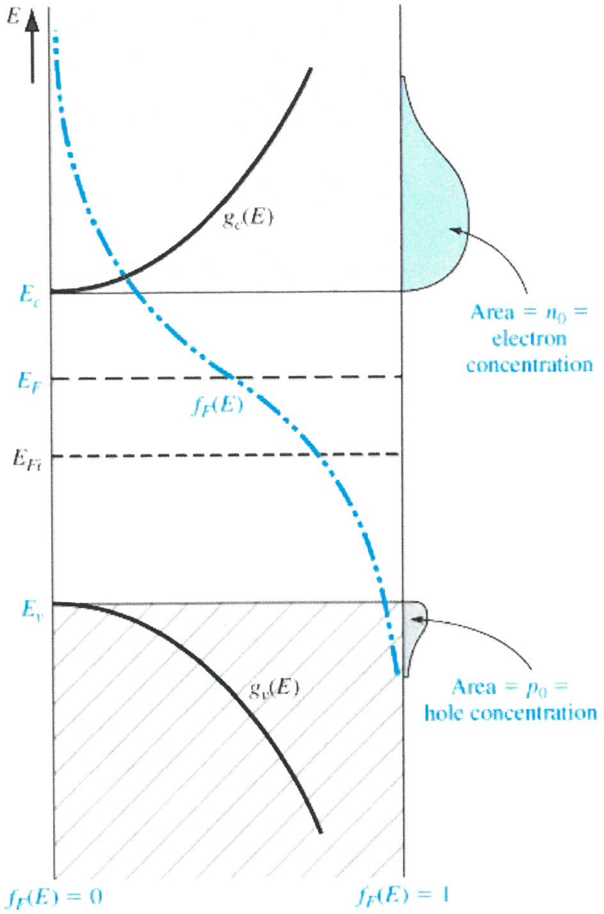


Figure 4.8 | Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is above the intrinsic Fermi energy.

Type *p* (more holes)

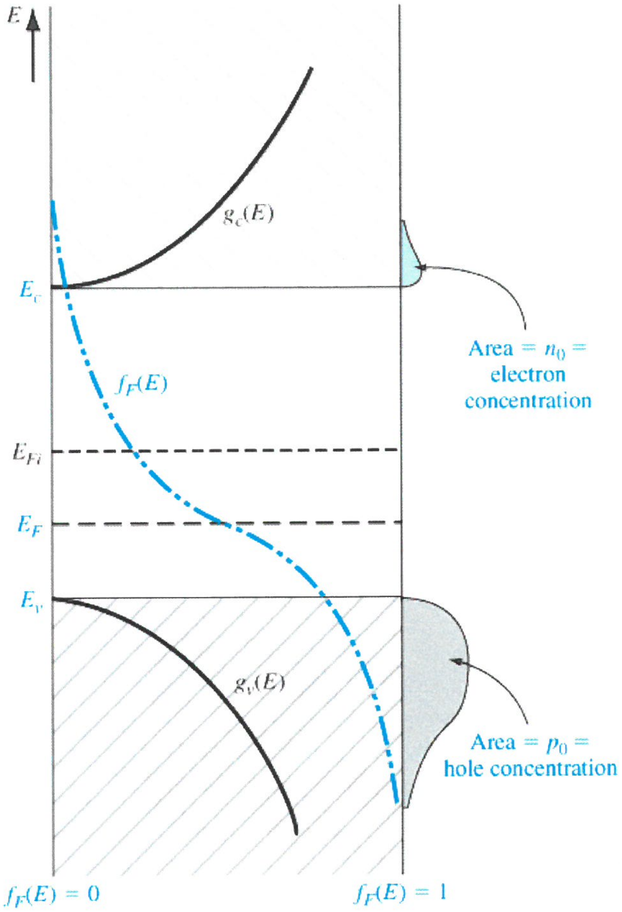


Figure 4.9 | Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is below the intrinsic Fermi energy.

- Note that $E_F > E_{Fi}$ for the type *n* extrinsic semiconductor with donors.
- Note that $E_F < E_{Fi}$ for the type *p* extrinsic semiconductor with acceptors.

4.3.1 cont.

How will calculate our new electron n_0 and hole p_0 concentrations? Adapt the previous expression for intrinsic semiconductors.

$$n_0 = n_i = N_c^{-\left(\frac{E_c - E_{Fi}}{k_B T}\right)} \quad \text{intrinsic}$$

Now

$$n_0 = N_c\, e^{-\left(\frac{E_c - E_{Fi} + E_{Fi} - E_F}{k_B T}\right)} \quad \text{extrinsic}$$

$$= \underbrace{N_c\, e^{-\left(\frac{E_c - E_{Fi}}{k_B T}\right)}}_{n_i} e^{-\left(\frac{E_{Fi} - E_F}{k_B T}\right)}$$

$$\underline{n_0 = n_i\, e^{\frac{E_F - E_{Fi}}{k_B T}}} \qquad (4.39)$$

Similarly, we had

$$p_0 = n_i = N_v\, e^{-\left(\frac{E_{Fi} - E_V}{k_B T}\right)} \quad \text{intrinsic}$$

which becomes

$$\underline{p_0 = n_i\, e^{-\left(\frac{E_F - E_{Fi}}{k_B T}\right)}} \qquad (4.40)$$

⇒ If $E_F > E_{Fi}$, ⇒ $n_0 > n_i$ while $p_0 < n_i$ (n-type)

⇒ If $E_F < E_{Fi}$ ⇒ $n_0 < n_i$ while $p_0 > n_i$ (p-type)

4.3.2 The $n_0 p_0$ Product

Taking the product of the extrinsic n_0 and p_0, we get

$$n_0 p_0 = n_i e^{\frac{E_F - E_{Fi}}{k_B T}} \, n_i e^{-\left(\frac{E_F - E_{Fi}}{k_B T}\right)}$$

constant $$\underline{n_0 p_0 = n_i^2 = N_c N_v e^{-E_g/k_B T}}$$ (4.23) + (4.43) same!

(This assumed Boltzmann approx works.)

$\Rightarrow$ If $n_0 \uparrow$ then $p_0 \downarrow$ and vice versa.

$\Rightarrow$ If $n_0 > p_0$, we call the extrinsic semiconductor n-type.

$\Rightarrow$ If $p_0 > n_0$, we call the extrinsic semiconductor p-type.

4.3.3 The Fermi-Dirac Integral

→ skip

4.3.4 Degenerate + Nondegenerate Semiconductors

→ Assumed dopant atom #s << semiconductor atom #s

so that dopant atoms do NOT interact w/ each other

→ Too many donors → $E_F > E_c$ $\#e^- > N_c$ degenerate ☹

→ Too many acceptors → $E_F < E_v$ #holes $> N_v$ ☹

4.4 Statistics of Donors and Acceptors

Reconsider Fermi-Dirac in light of donors & acceptors.

4.4.1 Probability Function

Pauli exclusion principle still applies to donors & acceptors. This results in a distribution function for donor electrons that's a little different than Fermi-Dirac

probability function of electrons occupying a donor state/level $\equiv n_d = \dfrac{N_d}{1+\frac{1}{2}e^{\frac{E_d-E_F}{k_BT}}}$

$\equiv$ ← spin or $\frac{1}{g}$ factor

$g =$ deg. factor

where $E_d \equiv$ energy of donor level

$N_d \equiv$ doping concentration of donors ($\#/cm^3$ or $\#/m^3$)

we can also write

$$n_d = N_d - N_d^+$$

where $N_d^+ \equiv$ # of ionized dopant atoms (ones that went to conduction band)

Similarly, for acceptors

prob. function of holes in acceptor states $\equiv p_a = \dfrac{Na}{1+\frac{1}{g}e^{\frac{E_F-E_a}{k_BT}}} = Na - Na^-$

$g \equiv$ may be 2 or 4

$Na \equiv$ doping concentration of acceptors

$Na^- \equiv$ # of ionized acceptors (caused holes in valence band)

$E_a \equiv$ energy of acceptor level

4.4.2 Complete Ionization and Freeze-Out

If $E_d - E_F >> k_B T$, then

$$n_d \simeq 2 N_d \, e^{-\left(\frac{E_d - E_F}{k_B T}\right)} \qquad (4.53)$$

and

$$n_0 \simeq N_c \, e^{-\left(\frac{E_c - E_F}{k_B T}\right)}$$

We can use these expressions to compare the number of electrons in donor state energies to those in total in both the donor states & conduction band states

$$\frac{n_d}{n_d + n_0} = \frac{2 N_d \, e^{-\left(\frac{E_d - E_F}{k_B T}\right)}}{2 N_d \, e^{-(E_d - E_F)/k_B T} + N_c \, e^{-(E_c - E_F)/k_B T}}$$

$$= \frac{1}{1 + \frac{N_c}{2 N_d} \, e^{-\left(\frac{E_c - E_d}{k_B T}\right)}} \qquad (4.55)$$

Note! $E_c - E_d$ = ionization energy for donor electrons!

Remember $N_c = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2}$ (4.10)

4.4.2 cont.

ex. Let's look at Arsensic doping of germanium @ 295 K. Assume the doping level is 0.025 % of N_c

From Table 4.1, $m_n^* = 0.55\, m_0$

$$N_c = 2\left(\frac{2\pi(0.55)9.1094\times10^{-31}(1.380649\times10^{-23})295}{(6.62607015\times10^{-34})^2}\right)^{3/2}$$

$$\underline{N_c = 9.981\times10^{24}\ \#/m^3 = 9.981\times10^{18}\ \#/cm^3}$$

$$k_BT = 8.617333\times10^{-5}\ eV/K\,(295) = \underline{0.025421\ eV}$$

From Table 4.3, the ionization energy for As in Ge is 0.0127 eV.

$$N_d = (0.025/100)\,9.981\times10^{24} = 2.495\times10^{21}\ \#/m^3$$

$$= \underline{2.495\times10^{15}\ \#/cm^3}$$

$$\frac{n_d}{n_0+n_d} = \frac{1}{1+\frac{N_c}{2N_d}e^{-(E_c-E_d)/k_BT}}$$

$$= \frac{1}{1+\frac{9.981\times10^{18}}{2(2.495\times10^{15})}e^{-0.0127/0.025421}}$$

$$\frac{n_d}{n_0+n_d} = 8.233\times10^{-4} = 0.08233\ \%$$

$\Rightarrow$ Essentially <u>all</u> of donors are ionized.

4.4.2 cont.

For typical doping levels, we can expect nearly 100% ionization at room temps ($\sim 295K$) ← complete ionization

Similarly, for acceptors w/ $g=4$

$$\frac{p_a}{p_0+p_a} = \frac{1}{1+\frac{N_v}{4N_a} e^{-(E_a-E_v)/k_BT}}$$

where $N_v = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}$ (4.18)

$E_a - E_v \equiv$ ionization energy

As $T \to 0K$, the Fermi energy for:

n-type is half way between E_d & E_c

p-type is halfway between E_a & E_v

and no donor electrons go to cond. band

" valence " go to acceptor (no holes in valence band)

⇒ freeze-out!

4.5 Charge Neutrality

- In equilibrium, the semiconductor (intrinsic or extrinsic) is charge neutral overall (electron + hole distributions may vary among various energy levels).
- If a semiconductor has both donors and acceptors, it is called a compensated semiconductor. If $N_d > N_a$, it is n-type. If $N_a > N_d$, it is p-type. If $N_d = N_a$, it will act similarly to an intrinsic semiconductor. (Can happen near interfaces of p + n layers in diodes + transistors.)
- See Fig 4.14 for this case

~~$N_d \equiv$ donor doping conc. (#/vol.)~~
~~$N_a \equiv$ acceptor " " (#/vol.)~~
~~$N_d^+ \equiv$ conc. of ionized donors (positive) (#/vol.)~~
~~$N_a^- \equiv$ conc. of ionized acceptor (neg.) (#/vol)~~
~~$n_0 \equiv$ electron conc. in cond. band (#/vol)~~
~~$p_0 \equiv$ hole conc. in val. band (#/vol.)~~

From *Semiconductor Physics and Devices: Basic Principles* (4th Edition), Donald A. Neamen, McGraw Hill, 2012, ISBN 978-0-07-352958-5.

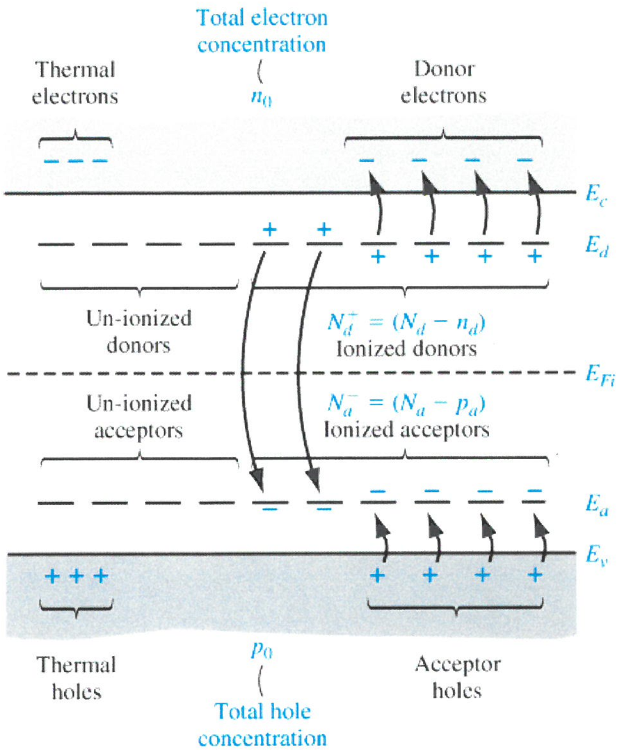


Figure 4.14 | Energy-band diagram of a compensated semiconductor showing ionized and un-ionized donors and acceptors.

$N_d \equiv$ donor doping concentration (#/m^3 or #/cm^3). These are initially charge neutral.

$N_a \equiv$ acceptor doping concentration (#/m^3 or #/cm^3) . These are initially charge neutral.

$N_d^+ \equiv$ concentration of ionized donors (#/m^3 or #/cm^3). These are **positive** as they lost their electron to conduction band or (less likely) they lost an electron to acceptors.

$N_a^- \equiv$ acceptor doping concentration (#/m^3 or #/cm^3) . These are **negative** as they gained an electron from valence band or (less likely) they gained an electron from donors.

$n_d \equiv$ concentration of donors that did not ionize (#/m^3 or #/cm^3). These are neutral.

$p_a \equiv$ concentration of acceptors that did not ionize (#/m^3 or #/cm^3). These are neutral.

$n_0 \equiv$ total concentration of electrons in conduction band (#/m^3 or #/cm^3). These are **negative**.

$p_0 \equiv$ total concentration of holes in valence band (#/m^3 or #/cm^3). These are **positive**.

4.5.2 Equilibrium Electron and Hole Concentrations

Conservation of Charge/Charge Neutrality requires that

density of neg. charges = density of positive charges

$$n_o + N_a^- = p_o + N_d^+$$

$$n_o + (N_a - p_a) = p_o + (N_d - n_d)$$

As we saw from $\frac{n_d}{n_o + n_d}$ calculation example, essentially all donors ionize at reasonable temperatures. (Similar result for $\frac{p_a}{p_o + p_a}$).

$$\text{Therefore} \quad \left. \begin{aligned} N_a^- &= N_a - p_a \simeq N_a \\ N_d^+ &= N_d - n_d \simeq N_d \end{aligned} \right) \text{Assume complete ionization}$$

$$\Downarrow$$

$$n_o + N_a = p_o + N_d$$

Electron Concentration

From section 4.3.2, the $n_o p_o$ product gave $n_o p_o = n_i^2$ (4.43)

$$\hookrightarrow p_o = n_i^2 / n_o$$

4.5.2 cont.

Substituting for p_0 yields

$$n_0 + N_a = \frac{n_i^2}{n_0} + N_d$$

$\times n_0$

$$n_0^2 + N_a n_0 = n_i^2 + N_d n_0$$

$$n_0^2 - (N_d - N_a) n_0 - n_i^2 = 0 \qquad (4.59b)$$

$\Downarrow$ use quadractic formula

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \qquad (4.60)$$

Used only '+' sol'n since as $N_a = N_d = 0$, $n_0 = +n_i$.

If $N_d > N_a$, we'll get the electron concentration for n-type semiconductors. We can find $p_0 = n_i^2/n_0$ from $n_0 p_0$ product.

$\Rightarrow$ Formula still works if $N_a = 0$ (No acceptor doping).

4.5.2 cont.

ex. Let's consider n-type silicon

* From Table B.4/ Chapter 1, the atomic density of Si is $5 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}$

* From Table 4.2, $n_i = 1.5 \times 10^{10} \frac{\#}{\text{cm}^3}$

* Assuming doping concentrations of $N_d = 2 \times 10^{13} \frac{\text{donors}}{\text{cm}^3}$ & $N_a \simeq 0$

(Note: Typical Si doping concentrations range from $10^{13} \frac{\#}{\text{cm}^3}$ to $10^{18} \frac{\#}{\text{cm}^3}$)

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$= \frac{2 \times 10^{13}}{2} + \sqrt{\left(\frac{2 \times 10^{13}}{2}\right)^2 + (1.5 \times 10^{10})^2}$$

$$= \underline{2.0000011\underset{\overset{\wedge}{25}}{} \times 10^{13} \frac{\#}{\text{cm}^3} \simeq 2 \times 10^{13} \frac{\#}{\text{cm}^3} = N_d}$$

$\Rightarrow$ Even w/ light doping $n_0 \simeq N_d$!

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{2.0000011 \times 10^{13}} = \underline{1.125 \times 10^{7} \frac{\#}{\text{cm}^3}}$$

$\Rightarrow$ hole concentration roughly 6 orders of magnitude smaller

4.5.2 cont.

Similarly, if we use $n_0 p_0 = n_i^2 \rightarrow n_0 = n_i^2 / p_0$, we get

$$\frac{n_i^2}{p_0} + N_a = p_0 + N_d$$

which leads to

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \quad (4.62)$$

for our p-type semiconductors. Here, we can find $n_0 = \frac{n_i^2}{p_0}$.

4.6 Position of Fermi Energy Level

Now that we have (4.60) + (4.62) to find n_0 + p_0, we can find the Fermi energy E_F relative to the intrinsic Fermi energy level E_{Fi}, E_c +/or E_v

4.6.1 Mathematical Derivation

n-type

$$n_0 = N_c \, e^{-\left(\frac{E_c - E_F}{k_B T}\right)} \underset{\text{cross divide}}{\rightarrow} e^{\frac{E_c - E_F}{k_B T}} = \frac{N_c}{n_0}$$

4.6.1 cont.

Taking ln() of both sides leads to

$$\underline{E_c - E_F = k_B T \ln\left(\frac{N_c}{n_0}\right)} \qquad (4.63)$$

going back to (4.60)

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

and assuming $N_d >> N_a$ & $N_d >> n_i$ (True for n-type semiconductors), we get

$$n_0 \approx \frac{N_d}{2} + \frac{N_d}{2} = N_d$$

leading us to simplify (4.63) to

$$\underline{E_c - E_F \simeq k_B T \ln\left(\frac{N_c}{N_d}\right)}$$

If $N_d \not>> N_a$ but still $N_d >> n_i$

$$\underline{E_c - E_F \simeq k_B T \ln\left(\frac{N_c}{N_d - N_a}\right)}$$

4.6.1 cont

w/ (4.39) $n_o = n_i e^{\frac{E_F - E_{Fi}}{k_B T}}$ which leads to

$$\underline{E_F - E_{Fi} = k_B T \ln\left(\frac{n_o}{n_i}\right)}$$

again if $N_d \gg N_a$ & $N_d \gg n_i \Rightarrow n_o \simeq N_d$

$$\underline{E_F - E_{Fi} = k_B T \ln\left(\frac{N_d}{n_i}\right)}$$

Also, if $N_d = N_a$, $\underline{E_F - E_{Fi} = 0} \Rightarrow \underline{E_F \cong E_{Fi}}$

Since $n_o = n_i$ (completely compensated)

p-type Similarly for p-type semiconductors

$$E_F - E_V = k_B T \ln\left(\frac{N_V}{p_o}\right) \quad (4.66)$$

going back to $p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$ (4.62)

if $N_a \gg N_d$ & $N_a \gg n_i \Rightarrow p_o \simeq N_a$

$$\underline{E_F - E_V \cong k_B T \ln\left(\frac{N_V}{N_a}\right)}$$

Also

$$\underline{E_{Fi} - E_F = k_B T \ln\left(\frac{p_o}{n_i}\right)} \quad (4.68)$$

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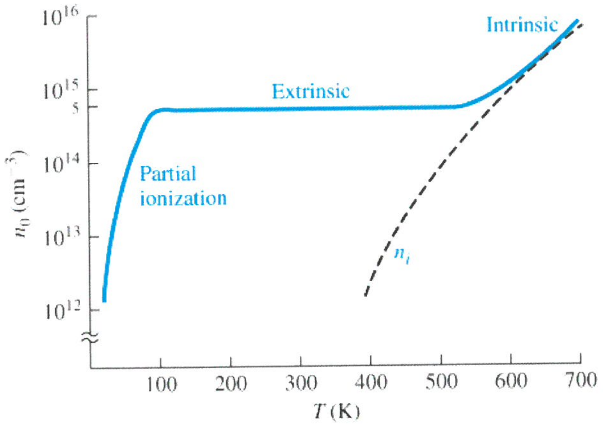

Figure 4.16 | Electron concentration versus temperature showing the three regions: partial ionization, extrinsic, and intrinsic.

- Note: At ‘normal temperatures’, the extrinsic electrons dominate.

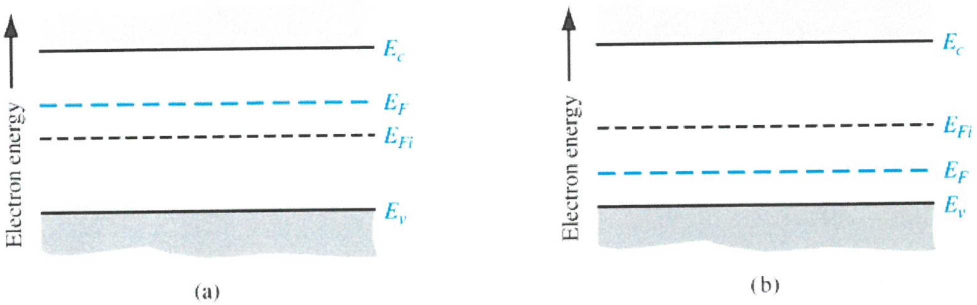

Figure 4.17 | Position of Fermi level for an (a) n-type ($N_d > N_a$) and (b) p-type ($N_a > N_d$) semiconductor.

- $E_F > E_{Fi} \approx E_{\text{midgap}}$ for *n*-type semiconductors while $E_F < E_{Fi} \approx E_{\text{midgap}}$ for *p*-type semiconductors.

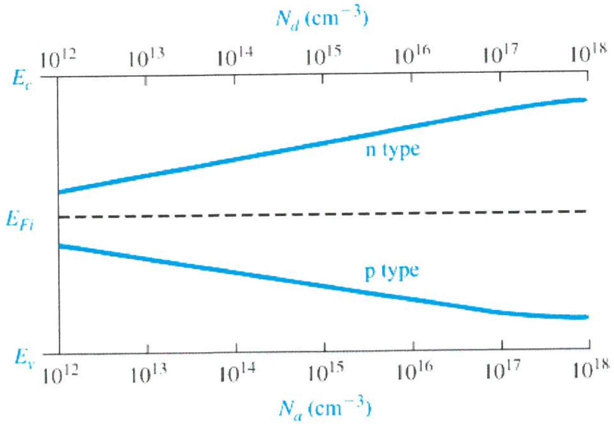

Figure 4.18 | Position of Fermi level as a function of donor concentration (n type) and acceptor concentration (p type).

- $E_F > E_{Fi} \rightarrow E_c$ as N_d increases for *n*-type semiconductors while $E_F < E_{Fi} \rightarrow E_v$ as N_a increases for *p*-type semiconductors.