EE 330 (Spring 2012) Examination 3

Name Key

Instructions: Show all work for full credit. Write answers in indicated places. Attach equation sheet to exam.

1) A 120 V_{rms}, 1.5 hp, 60 Hz, Y-connected, 4-pole, three-phase induction motor has a rated slip of 4%, 56 W of mechanical losses, 32 W of copper losses, 20 W of stray losses, and 44 W of core losses. Determine the synchronous, slip, and rated speeds (in RPM) as well as the frequency of the current on the rotor for this motor under rated conditions. Also, find the input power (W), output torque, induced torque, and efficiency (%) under rated conditions.

(6-1) $n_{sync} = \frac{120 f_{se}}{P} = \frac{120(60)}{4} = 1800$ RPM

(6-3) $n_{slip} = s\, n_{sync} = 0.04(1800) = 72$ RPM

(6-6) $n_{rated} = (1-s)\, n_{sync} = (1-0.04)1800 = 1728$ RPM

(6-8) $f_{re} = s\, f_{se} = 0.04(60) = 2.4$ Hz

$$P_{in} = P_{out} + losses = (1.5\,hp)\left(\frac{746\,W}{1\,hp}\right) + 56 + 32 + 20 + 44$$

$$= 1119 + 152 = 1271\ W$$

$$P_{out} = \tau_{out}\,\omega_m \Rightarrow \tau_{out} = \frac{(1.5)(746)}{1728(\pi/30)} = 6.18383\ N.m$$

$$P_{conv} = \tau_{ind}\,\omega_m = P_{out} + P_{stray} + P_{F\&W} = 1.5(746) + 20 + 56$$

$$\tau_{ind} = \frac{1119 + 20 + 56}{1728\,(\pi/30)} = 6.6038\ N.m$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{1.5(746)}{1271} \times 100\% = 88.0409\ \%$$

n_{sync} = 1800 RPM

n_{slip} = 72 RPM

n_{rated} = 1728 RPM

f_{rotor} = 2.4 Hz

P_{in} = 1271 W

τ_{out} = 6.1838 N.m

τ_{ind} = 6.6038 N.m

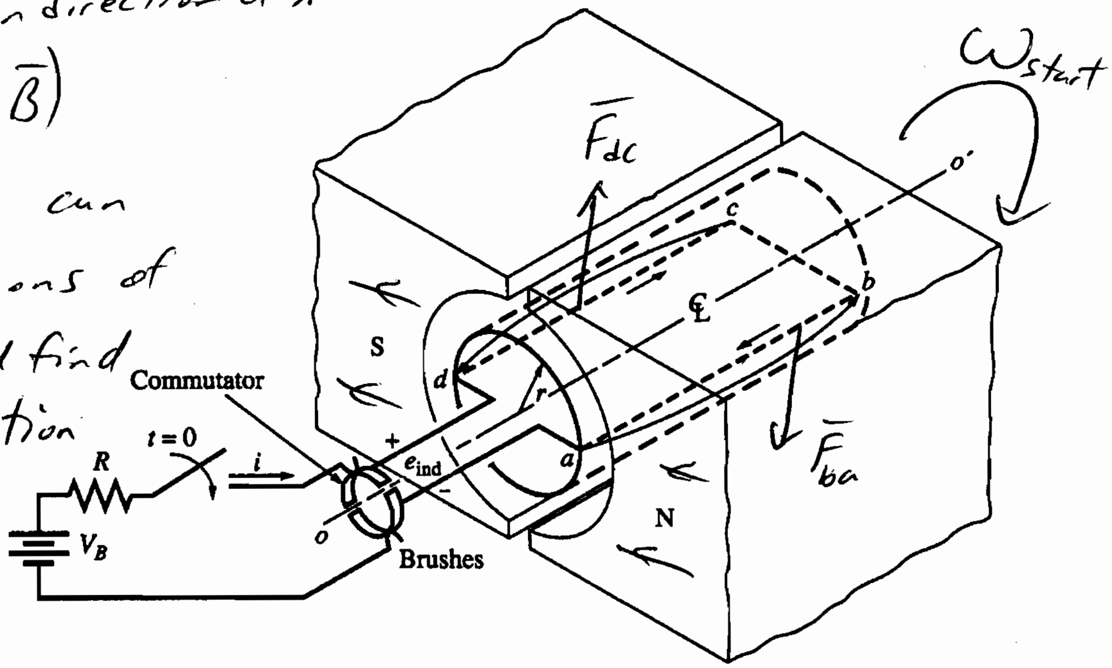
η = 88.04 %

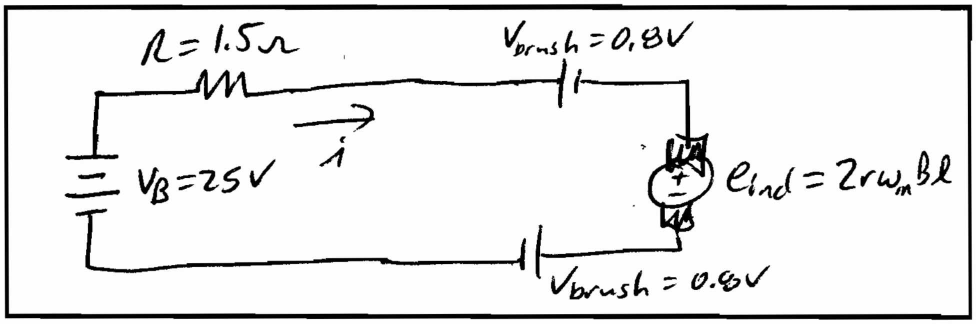
2) A simple rotating loop (see below) has: $B = 0.15$ T, $l = 100$ cm, $r = 35$ cm, $R = 1.5\ \Omega$, $V_B = 25$ V, and **brush voltage drops of 0.8 V/ea**. Sketch a labeled equivalent electrical circuit in the box provided. Given that the loop is initially at rest, determine the no-load starting induced emf, induced torque, and current. Show the initial direction of loop rotation with a curved arrow on the drawing by *o'*.

in direction of i

$$\bar{F} = i(\bar{l} \times \bar{B})$$

w/ this, we can find directions of $\bar{F}_{dc}$ + $\bar{F}_{ba}$ and find ω_{start} direction





← (7-5) to (7-6)

@ $t = 0^+$, $\omega_m = 0 \Rightarrow e_{ind} = 0$
(@ rest)

By KVL, $\overset{25}{\cancel{V_B}} = i_{start}(1.5) + 0.8 + \overset{0}{\cancel{e_{ind,start}}} + 0.8$

$$i_{start} = \frac{25 - 0.8 - 0.8}{1.5} = 15.6\,A$$

Per (8-15) $\tau_{ind,start} = 2 r i_{start} l B = 2(0.35)(15.6)(1)(0.15)$
$= 1.638$ N·m

$\tau_{ind,start} =$ 1.638 N.m $i_{start} =$ 15.6 A $e_{ind,start} =$ 0

2) cont. Later, this loop is rotating clockwise (defined with right thumb pointing toward *o'*) at a steady 1600 RPM. Is the loop acting as a generator or motor? Why? Find the steady-state induced emf, torque, current (base sign on arrow drawn on figure), electrical power into/out of battery, mechanical power into/out of the loop, and efficiency (%). Neglect mechanical losses.

$$e_{ind,ss} = 2r\omega Bl = 2(0.35)\left[1600\left(\frac{\pi}{30}\right)\right]0.15(1)$$

$$= 17.59292V < V_B \Rightarrow \underline{Motor!}$$

By KVL, $25 = i_{ss}(1.5) + 0.8 + 17.5929 + 0.8$

$$\hookrightarrow i_{ss} = \frac{25-0.8-17.5929-0.8}{1.5} = 3.87138743A$$

Per (8-15) $\tau_{ind,ss} = 2r i_{ss} lB = 2(0.35)(3.8714)(1)(0.15)$

$$= 0.4064956 \text{ N·m}$$

$$P_B = -25(3.87) = -96.784686 W$$

$$P_{mech} = \tau_{ind,ss}\omega_{ss} = (0.4065)1600\left(\frac{\pi}{30}\right) = 68.109 W$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{mech}}{P_B} \times 100\% = \frac{68.109}{96.785} \times 100\%$$

$$= 70.4\%$$

generator / [✓ motor]? Why? $e_{ind,ss} < V_B$

$e_{ind,SS}$ = 17.5929V $\tau_{ind,SS}$ = 0.4065 N·m i_{SS} = 3.8714 A

$|P_B|$ = 96.785W (into / [✓ out of battery]?) P_{mech} = 68.109 W (into / [✓ out of loop?])

(For questions, **circle** correct answers)

+2 Extra credit → $\underline{\eta = 70.4\%}$ Motor delivers power

3) A 480 V_{rms}, 110 hp, 50 Hz, six-pole, Y-connected, three-phase induction motor has per-phase equivalent circuit parameters: $R_1 = R_2 = 0.09\ \Omega$, $X_1 = X_2 = 0.3\ \Omega$, $X_M = 9\ \Omega$, **and $R_C = 75\ \Omega$.** With a slip of 4.5%, sketch a fully-labeled per-phase equivalent circuit (assume line voltage at 0°). Then, find the magnitude of the line current, input power, and input power factor.

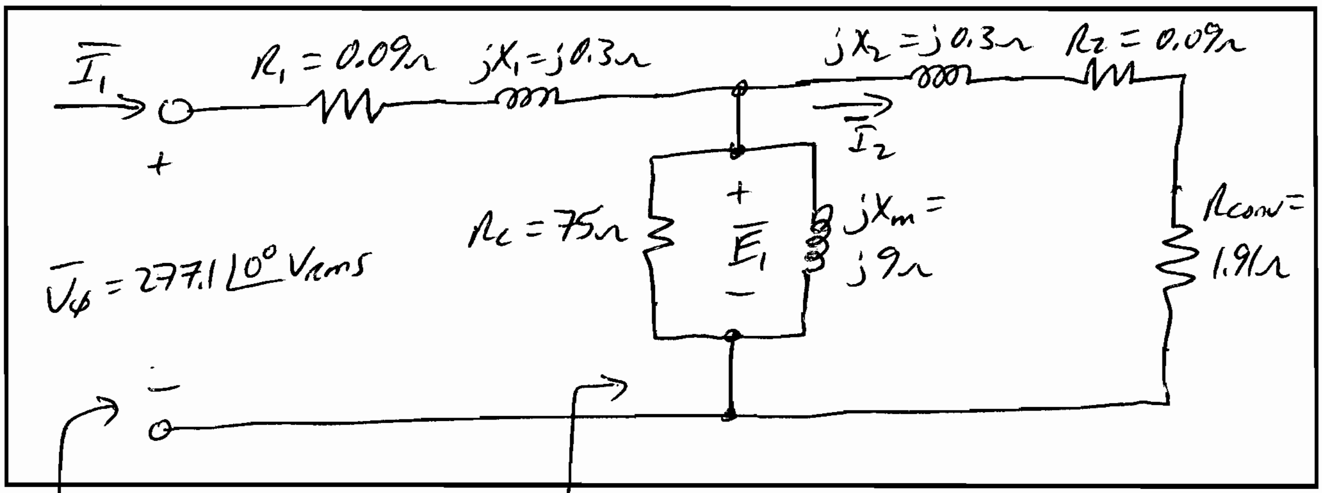
$$\bar{V}_\phi = \frac{480}{\sqrt{3}}\angle 0^\circ\ V_{rms} = 277.13\angle 0^\circ\ V_{rms}$$

$$n_{sync} = \frac{120 f_{se}}{P} = \frac{120(50)}{6} = 1000\ RPM \quad \omega_{sync} = n_{sync}(\pi/30) = 104.719755\ \frac{rad}{s}$$

$$R_2/s = \frac{0.09}{0.045} = 2\ \Omega,\ R_2 = 0.09\ \Omega,\ R_{conv} = R_2\left(\frac{1-s}{s}\right) = 0.09\left(\frac{1-0.045}{0.045}\right) = 1.91\ \Omega$$

$$\omega_m = (1-s)\omega_{sync} = (1-0.045)104.72 = 100.007\ \frac{rad}{s}$$

Per-phase equivalent circuit



$$\bar{Z}_{eq,p} = 75 // j9 // (2 + j0.3) = 1.75418 + j0.64415\ \Omega$$

$$\bar{Z}_{in} = (R_1 + jX_1) + \bar{Z}_{eq,p} = (0.09 + j0.3) + (1.75418 + j0.64415) = 1.84418 + j0.94415\ \Omega$$

$$\bar{I}_1 = \frac{\frac{480}{\sqrt{3}}\angle 0^\circ}{1.844 + j0.944} = 133.7609029\angle -27.1107608^\circ\ A$$

$$I_L = |\bar{I}_1| = 133.761\ A_{rms}$$

(R-L impedance)

$$pf_{in} = \cos\theta_{Z_{in}} = \cos(0 + 27.11076^\circ) = 0.890127\ \text{lagging}$$

$$P_{in} = \sqrt{3}\, V_{LL} I_L \cos\theta = \sqrt{3}(480)\,133.761\cos(27.11076^\circ)$$
$$= 98{,}988.13555\ W$$

I_L = 133.761 A_{rms} P_{in} = 98.988 kW pf_{in} = 0.8901 lagging

3) cont. Next, determine the stator copper losses, core losses, air gap power, & rotor copper losses, **Extra credit:** Find the converted power (from electrical to mechanical), and induced torque. If the friction & windage losses are 3.3 kW, determine the stray losses and efficiency (%) of the motor.

$$(6\text{-}25)\quad P_{SCL} = 3|\bar{I}_1|^2 R_1 = 3(133.761)^2 0.09 = \underline{4830.834W}$$

$$\text{By voltage division, } \bar{E}_1 = \bar{V}_\phi \frac{\bar{Z}_{eq,P}}{\bar{Z}_{in}} = \left(\frac{480}{\sqrt{3}}\angle 0^\circ\right)\frac{1.754+j0.644}{1.844+j0.944}$$

$$= 249.960527\angle -6.947039^\circ\ V_{rms}$$

$$(6\text{-}26)\quad P_{core} = \frac{3|\bar{E}_1|^2}{R_c} = \frac{3(249.96)^2}{75} = \underline{2499.211W}$$

$$\text{By Ohm's Law, } \bar{I}_2 = \frac{\bar{E}_1}{2+j0.3} = \frac{249.96\angle -6.95^\circ}{2+j0.3} = 123.59753\angle -15.4778^\circ A_{rms}$$

$$(6\text{-}27)\ \&\ (6\text{-}28)\quad P_{AG} = P_{in} - P_{SCL} - P_{core} = 91,658.09\ W \quad \text{same!}$$

$$= 3|\bar{I}_2|^2 \frac{R_2}{s} = 3(123.6)^2 \frac{0.09}{0.045} = \underline{91,658.1\ W}$$

$$(6\text{-}30)\quad P_{RCL} = 3|\bar{I}_2|^2 R_2 = 3(123.6)^2 0.09 = \underline{4124.61\ W}$$

$$\text{Extra Credit: } (6\text{-}33)\quad P_{conv} = (1-s)P_{AG} = (1-0.045)91,658.09 = \underline{87,533.5\ W}$$

$$(6\text{-}35)\quad P_{conv} = \tau_{ind}\,\omega_m \Rightarrow \tau_{ind} = \frac{87533.5}{100.0074} = \underline{875.27\ Nm}$$

$$P_{stray} = P_{in} - P_{SCL} - P_{core} - P_{RCL} - \overset{0\ (\text{Not given})}{\cancel{P_{mech}}} - P_{OUT}$$

$$= 98,988.136 - 4830.834 - 2499.211 - 4124.61 - 110\,hp\left(\frac{746W}{hp}\right) = \underline{5473.48W}$$

$$\eta = \frac{P_{OUT}}{P_{in}} \times 100\% = \frac{110(746)}{98988} \times 100\% = \frac{82060}{98988} \times 100\% = \underline{82.9\%}$$

P_{SCL} = 4.831 KW P_{core} = 2.499 KW P_{AG} = 91.658 KW P_{RCL} = 4.125 KW

Extra credit: P_{conv} = 87.533 KW τ_{ind} = 875.27 N·m

+2/ea

P_{stray} = 5473.5W η = 82.9 %

$+ P_{mech}$ (actually)