EE 330 (Spring 2012) Examination 2

Name Key

Instructions: Show all work for full credit. Write answers in indicated places. Attach equation sheet to exam.

1) A 3-phase, 6-pole, Y-connected AC generator outputs an open circuit terminal voltage of 500 V_{rms} at 50 Hz. There are 66 turns per coil and the coils are installed in 18 slots around the stator. For each phase, the coils are connected in series. Determine the necessary shaft rotation speed ω_m (rad/s) and n_m (RPM). Find the open circuit phase voltage V_ϕ and required magnetic flux. For an electrical load of 25 kVA with a 0.8 lagging power factor at the rated voltage, the electrical losses are 800 W, mechanical losses are 1.4 kW, and stray/core losses are 600 W. Determine the input power P_{in}, applied torque τ_{app}, and efficiency η. Use MKS units.

Per (3-34) $f_{se} = \frac{n_{sm} P}{120} \Rightarrow n_m = \frac{120(50)}{6} = \underline{1000 \text{ RPM}}$

$\omega_m = \left(1000 \frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 33.\bar{3}\pi = \underline{104.72 \text{ rad/s}}$

Y-connected, per (A-11) $V_{LL} = \sqrt{3}\, V_\phi \Rightarrow V_\phi = \frac{500}{\sqrt{3}} = \underline{288.675 \, V_{RMS}}$

Per (3-50), the induced RMS voltage for a coil (2-slots per coil)

is $E_A = \sqrt{2}\,\pi N_c \phi f$. With 18 slots, we have

$\frac{18 \text{ slots}}{3 \text{ phases}} = 6 \frac{\text{slots}}{\text{phase}}$. So, w/ open circuit conditions -

$$E_A = V_\phi = \frac{\sqrt{2}\,\pi N_c \phi f}{2 \text{ slots}} (6 \text{ slots}) = 3\sqrt{2}\,\pi N_c \phi f$$

$$\hookrightarrow \phi = \frac{288.675}{3\sqrt{2}\,\pi (66) 50} = \underline{0.0065631 \text{ wb}}$$

$P_{in} = P_{out} + \text{losses} = 25 \text{ kVA}(0.8) + 0.8 \text{ kW} + 1.4 \text{ kW} + 0.6 \text{ kW} = \underline{22.8 \text{ kW}}$

$P_{in} = P_{mech} = \tau_{app}\omega_m \Rightarrow \tau_{app} = \frac{22{,}800}{104.72} = \underline{217.72 \text{ N·m}}$

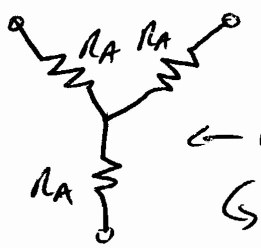
$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{25(0.8)}{22.8} \times 100\% = 87.72\%$

ω_m = 104.72 rad/s n_m = 1000 RPM V_ϕ = 288.675 V_{rms} ϕ = 6.563 mWb

P_{in} = 22.8 kW τ_{app} = 217.72 N·m η = 87.72%

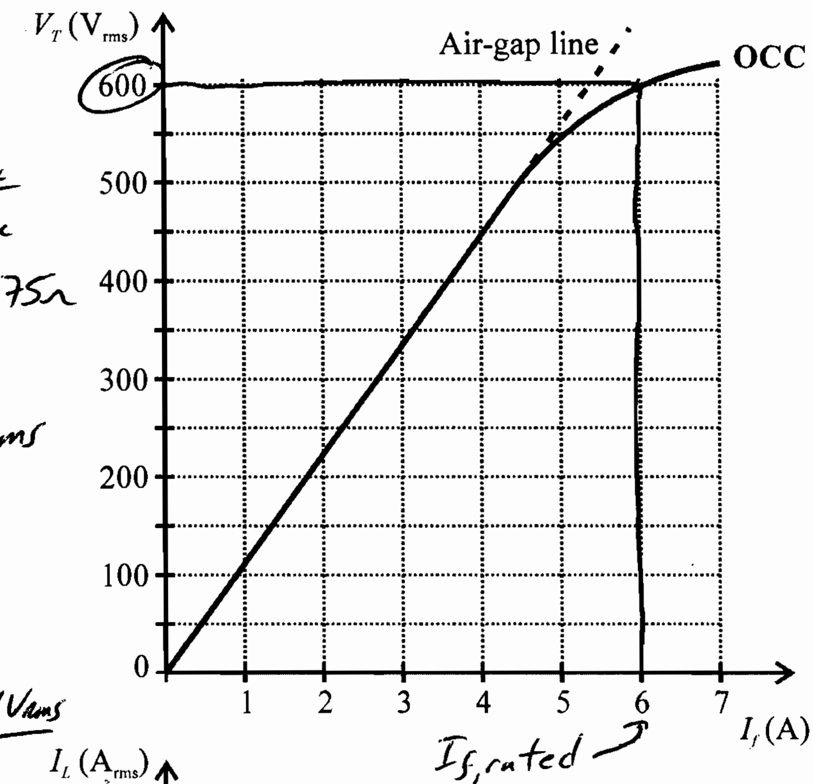
2) A Y-connected 3-phase, synchronous generator has the OCC and SCC shown. The rated field current is 6 A. A 6 V_{dc} voltage applied cross two terminals results in a current of 4 A_{dc}. Find the per-phase armature resistance R_A and rated synchronous reactance X_S. Sketch, with labels, the per-phase equivalent armature circuit. At the rated field current, a load draws a line current of 12 A_{rms} with a lagging power factor of 0.82. Find the **phasor** armature voltage $\overline{E}_A$, armature current $\overline{I}_A$, and phase voltage $\overline{V}_\phi$ (assume at angle $0°$).

Y-connection



$\leftarrow R_T = 2R_A = \frac{V_{dc}}{I_{dc}}$

$\hookrightarrow R_A = \frac{6}{2(4)} = 0.75\,\Omega$

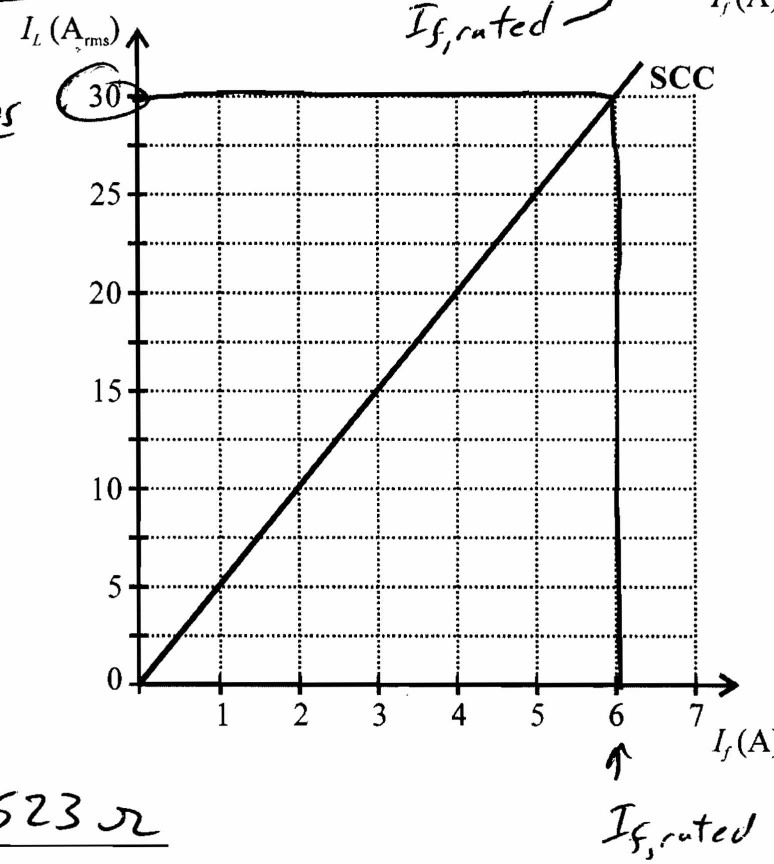


From OCC, $V_{T,oc} = 600\,V_{rms}$

Per (A-11) $V_{LL} = \sqrt{3}\,V_\phi$

$\hookrightarrow V_{\phi,oc} = E_{A,oc} = \frac{600}{\sqrt{3}}$

$= 346.41\,V_{rms}$



From SCC, $I_{L,sc} = I_{\phi,sc} = 30\,A_{rms}$

Per (4-25), $\sqrt{R_A^2 + X_S^2} = \frac{E_{A,oc}}{I_{A,sc}}$

$\hookrightarrow X_S = \sqrt{\left(\frac{346.41}{30}\right)^2 - 0.75^2}$

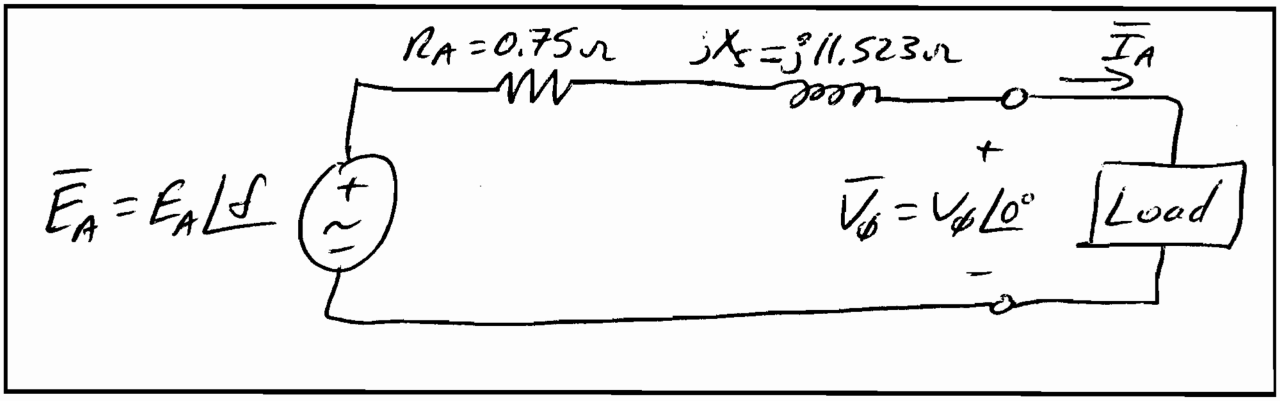
$= \sqrt{11.547^2 - 0.75^2}$

$= 11.5226227\,\Omega$

$R_A =$ 0.75 Ω $X_S =$ 11.523 Ω

2) cont.

Per-phase equivalent armature circuit



We are given $I_A = I_L = 12\,A_{rms}$. For a $pf = 0.82$ lagging,

$$\theta = \overset{0^\circ}{\cancel{\phi_{V_\phi}}} - \theta_{I_A} = \cos^{-1}(0.82) = 34.9152^\circ \Rightarrow \theta_{I_A} = -34.9152^\circ$$

$$\hookrightarrow \underline{\bar{I}_A = 12\angle -34.915^\circ\ A_{rms}}$$

By KVL, $\bar{E}_A = \bar{I}_A(R_A + jX_S) + \bar{V}_\phi$ @ rated I_F, $\underline{E_A = \frac{600}{\sqrt{3}}\ V_{rms}}$

$$\frac{600}{\sqrt{3}}\angle\delta = (12\angle -34.915^\circ)(0.75 + j11.523) + V_\phi \angle 0^\circ$$

$$\frac{600}{\sqrt{3}}\cos\delta + j\frac{600}{\sqrt{3}}\sin\delta = 86.52154658 + j108.2313354 + V_\phi$$

Equate imaginary parts $\rightarrow \frac{600}{\sqrt{3}}\sin\delta = 108.2313354$

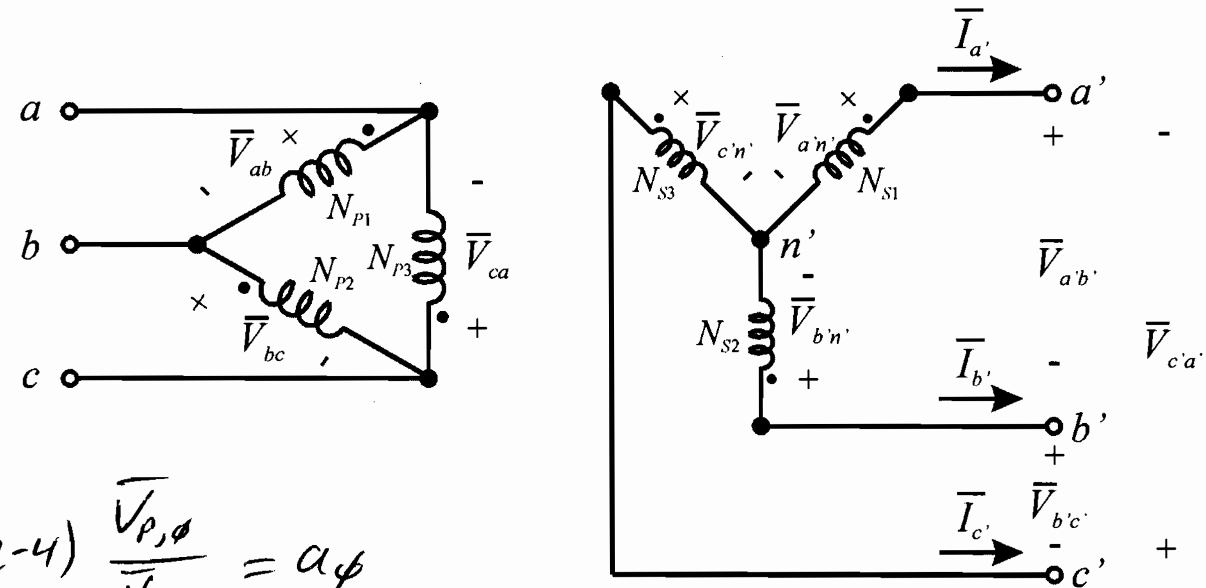
$$\hookrightarrow \delta = \underline{18.206154^\circ}$$

Equate real parts $\rightarrow \frac{600}{\sqrt{3}}\cos(18.206154^\circ) = 86.52155 + V_\phi$

$$\hookrightarrow V_\phi = 242.5468\ V_{rms}$$

$\bar{I}_A = \underline{12\angle -34.915^\circ\ A_{rms}}$ $\bar{E}_A = \underline{346.41\angle 18.206^\circ\ V_{rms}}$ $\bar{V}_\phi = \underline{242.55\angle 0^\circ\ V_{rms}}$

3) A 200 kVA, 480/2,400 V_{rms}, Δ-Y connected three-phase transformer is connected as shown. Given $\bar{V}_{ab} = V_{LL,P}\angle 0°$, $\bar{V}_{bc} = V_{LL,P}\angle 120°$, and $\bar{V}_{ca} = V_{LL,P}\angle 240°$, determine the secondary line voltages $\bar{V}_{a'b'}$, $\bar{V}_{b'c'}$, & $\bar{V}_{c'a'}$, phase voltages $\bar{V}_{a'n'}$, $\bar{V}_{b'n'}$, & $\bar{V}_{c'n'}$, and phase turns ratio a_ϕ.. **Bonus-** If the rated load with a power factor of 0.7 lagging is connected, find the line currents $\bar{I}_{a'}$, $\bar{I}_{b'}$, & $\bar{I}_{c'}$.



Per (2-4) $\dfrac{\bar{V}_{P,\phi}}{\bar{V}_{S,\phi}} = a_\phi$

So $\dfrac{\bar{V}_{ab}}{\bar{V}_{a'n'}} = a_\phi \Rightarrow \bar{V}_{a'n'} = \dfrac{\bar{V}_{ab}}{a_\phi} = \dfrac{480}{a_\phi}\angle 0°$ Vrms ← given

Similarly $\bar{V}_{b'n'} = \dfrac{\bar{V}_{bc}}{a_\phi} = \dfrac{480}{a_\phi}\angle 120°$ Vrms

$\bar{V}_{c'n'} = \dfrac{\bar{V}_{ca}}{a_\phi} = \dfrac{480}{a_\phi}\angle 240°$ Vrms

By KVL, $\bar{V}_{a'b'} = \bar{V}_{a'n'} - \bar{V}_{b'n'} = \dfrac{480}{a_\phi}\left[1\angle 0° - 1\angle 120°\right]$

given → $2400\angle\theta_{a'b'} = \dfrac{480\sqrt{3}}{a_\phi}\angle -30°$

$\Leftrightarrow$ $\underline{\theta_{a'b'} = -30°}$ $\Rightarrow \underline{\bar{V}_{a'b'} = 2400\angle -30°\ V_{rms}}$

and

$a_\phi = \dfrac{480\sqrt{3}}{2400} = \underline{0.34641}$

Note: $a_{\Delta\text{-}Y} = \dfrac{480}{2400} = 0.2 = \sqrt{3}\, a_\phi$

3) cont.

Then $\overline{V}_{a'n'} = \frac{480}{a_\phi}\angle 0^\circ = \frac{480}{0.3464}\angle 0^\circ = \underline{1385.64\angle 0^\circ \text{ V}_{rms}}$

$\underline{\overline{V}_{b'n'} = 1385.64\angle 120^\circ \text{ V}_{rms}}$ & $\underline{V_{c'n'} = 1385.64\angle 240^\circ \text{ V}_{rms}}$

$\overline{V}_{b'c'} = \overline{V}_{b'n'} - \overline{V}_{c'n'} = 1385.64\angle 120^\circ - 1385.64\angle 240^\circ = \underline{2400\angle 90^\circ \text{ V}_{rms}}$

$\overline{V}_{c'a'} = \overline{V}_{c'n'} - \overline{V}_{a'n'} = 1385.64\angle 240^\circ - 1385.64\angle 0^\circ = 2400\angle -150^\circ \text{ V}_{rms}$

or

$\underline{2400\angle 210^\circ \text{ V}_{rms}}$

Bonus: Per (A-31) $I_L = I_{rated} = \frac{S_{rated}}{\sqrt{3}\, V_{LL,rated}} = \frac{200\times 10^3}{\sqrt{3}\, 2400} = \underline{48.1125 \text{ A}_{rms}}$

$\theta = \theta_V - \theta_I = \cos^{-1} 0.7 = 45.573^\circ \Rightarrow \theta_I = \theta_V - 45.573^\circ$

So $\theta_{I_{a'}} = \theta_{V_{a'n'}} - 45.573^\circ = 0 - 45.573^\circ = -45.573^\circ$

$\theta_{I_{b'}} = \theta_{V_{b'n'}} - 45.573^\circ = 120^\circ - 45.573^\circ = 74.427^\circ$

$\theta_{I_{c'}} = \theta_{V_{c'n'}} - 45.573^\circ = 240^\circ - 45.573^\circ = 194.427^\circ$

$= -165.573^\circ$

where

$I_{a'} = I_{b'} = I_{c'} = 48.1125 \text{ A}_{rms}$

$a_\phi = \underline{0.34641}$

$\overline{V}_{a'b'} = \underline{2400\angle -30^\circ \text{ V}_{rms}}$ $\overline{V}_{b'c'} = \underline{2400\angle 90^\circ \text{ V}_{rms}}$ $\overline{V}_{c'a'} = \underline{2400\angle 210^\circ \text{ V}_{rms}}$ or -150°

$\overline{V}_{a'n'} = \underline{1385.64\angle 0^\circ \text{ V}_{rms}}$ $\overline{V}_{b'n'} = \underline{1385.64\angle 120^\circ \text{ V}_{rms}}$ $\overline{V}_{c'n'} = \underline{1385.64\angle 240^\circ \text{ V}_{rms}}$

Bonus: $\overline{I}_{a'} = \underline{48.11\angle -45.57^\circ \text{ A}_{rms}}$ $\overline{I}_{b'} = \underline{48.11\angle 74.43^\circ \text{ A}_{rms}}$ $\overline{I}_{c'} = \underline{48.11\angle 194.43^\circ \text{ A}_{rms}}$