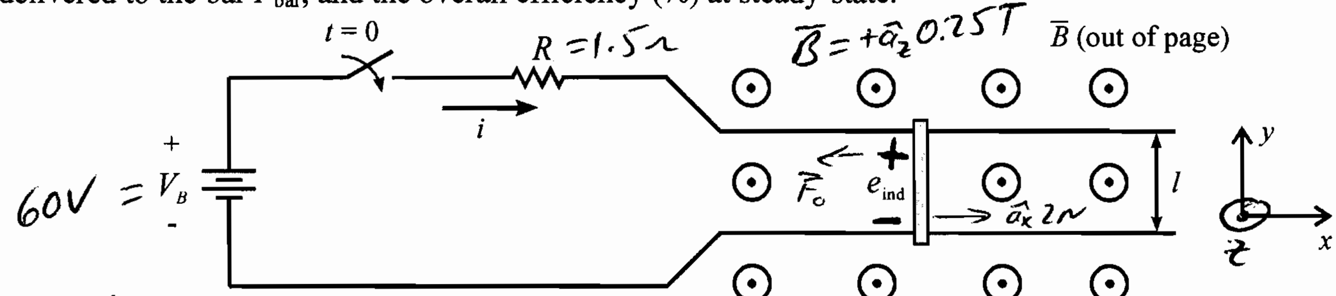
EE 330 (Spring 2012) Examination 1

Name Key A

Instructions: Show all work for full credit. Write answers in indicated places. Attach equation sheet to exam.

1) The linear DC machine shown has: $B = 250$ mT, $l = 0.5$ m, $R = 1.5\ \Omega$, $m_{bar} = 1$ kg, & $V_B = 60$ V. With the brass bar initially at rest, find the current i_0, induced voltage $e_{ind,0}$, and force $\bar{F}_0$ on the bar at $t = 0^+$. If a force of $2\hat{a}_x$ (N) is later applied to the moving bar, determine the **steady-state** current i_{ss} (direction per figure), vector velocity $\bar{v}_{SS}$, and induced voltage $e_{ind,ss}$ across the bar (show polarity on figure). Is the machine acting as a motor or a generator? Also, find the power supplied by the battery P_{batt}, power delivered to the bar P_{bar}, and the overall efficiency (%) at steady-state.



@ $t = 0^+$ $\quad e_{ind,0} = 0$ since $\bar{v}_0 = 0$

By KVL $-V_B + i_0 R + 0 = 0 \Rightarrow i_0 = V_B/R = \frac{60}{1.5} = 40A$

$\bar{F}_0 = i(\bar{l} \times \bar{B}) = 40(-\hat{a}_y 0.5 \times \hat{a}_z 0.25) = -\hat{a}_x 5\ N$

@ S.S. w/ $\bar{F}_{app} = \hat{a}_x 2N \Rightarrow$ @ S.S., $\bar{a} = 0$ so $\bar{F}_{ind} + \bar{F}_{app} = 0$ opposing

so $\bar{F}_{ind,ss} = -\hat{a}_x 2 = i_{ss}(-\hat{a}_y 0.5 \times \hat{a}_z (0.25)) = -\hat{a}_x 0.125\, i_{ss}$

$\hookrightarrow i_{ss} = \frac{2}{0.125} = 16A$ $\quad e_{ind,ss} = 60 - 16(1.5)$

By KVL, $-V_B + i_{ss}R + e_{ind,ss} = 0 \Rightarrow e_{ind,ss} = 36V$

Also $e_{ind,ss} = 36V = (\bar{v}_{ss} \times \bar{B}) \cdot \bar{l} = (\bar{v}_{ss} \times \hat{a}_z(0.25)) \cdot +\hat{a}_y 0.5 \Rightarrow \bar{v}_{ss} = -\hat{a}_x \frac{36}{0.25/0.5} = -\hat{a}_x 288 \frac{m}{s}$

Batt: $P_{Batt} = 60(16) = 960W$ $\quad i^2R = 16^2(1.5) = 384W$

Bar: $P_{Bar} = F v_{ss} = (2N)(288) = 576W$ $\quad \eta = \frac{576}{960} \times 100\%$

$i_0 =$ 40A $\quad e_{ind,0} =$ 0 $\quad \bar{F}_0 =$ $-\hat{a}_x 5\ N$

$\bar{v}_{SS} =$ $-\hat{a}_x 288\ m/s$ $\quad i_{ss} =$ 16A $\quad e_{ind,ss} =$ 36V

(motor or generator? Circle correct answer) motor

$P_{batt} =$ 960W $\quad P_{bar} =$ 576W $\quad$ efficiency = 60%

2) A 60 Hz, 180-kVA, 240/2500-V_{rms} step-up transformer has **per-unit** equivalent circuit parameters of $R_{series} = 0.013$, $X_{series} = 0.06$, $X_m = 46$, and $R_C = 64$. Using this information, calculate and sketch a fully-labeled approximate transformer equivalent circuit (with units) referred to the **secondary** side. What is the turns ratio a? Next, a 160-kVA load with a power factor of 0.75 lagging is connected to the transformer. Assuming that the load is receiving the rated secondary voltage (use as reference), determine the phasor load current $\bar{I}_L$ & primary voltage $\bar{V}_P$, and voltage regulation VR (%). **Extra credit**: Determine the core losses P_{core}, resistive line losses P_{cu}, and efficiency η of the transformer.

$a = V_p/V_s = \frac{240}{2500} = 0.096$ $\quad Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{2500^2}{180\times10^3} = 34.7\bar{2}\,\Omega$

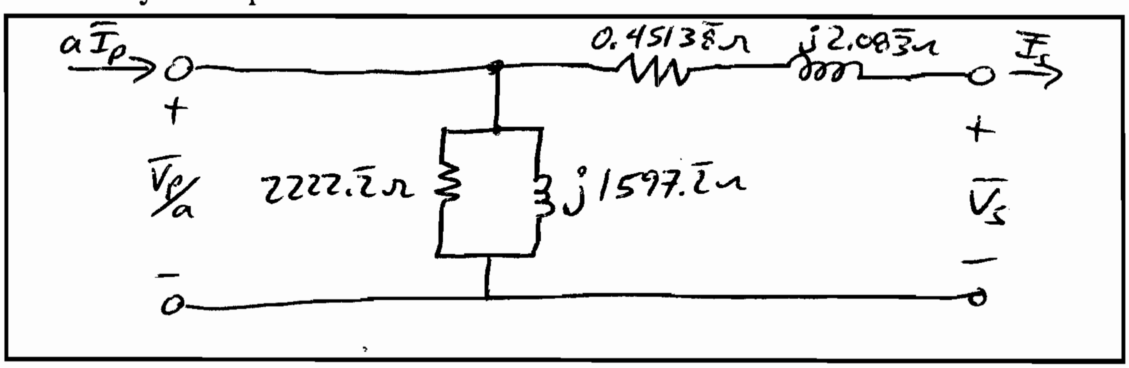
$R_{eq,s} = 0.013(34.7\bar{2}) = 0.4513\bar{8}\,\Omega$

$X_{eq,s} = 0.06(34.7\bar{2}) = 2.08\bar{3}\,\Omega$

$R_{c,s} = 64(34.7\bar{2}) = 2222.\bar{2}\,\Omega$

$X_{m,s} = 46(34.7\bar{2}) = 1597.\bar{2}\,\Omega$

Secondary-side equivalent circuit



$I_{LO} = \frac{S_{LO}}{V_{LO}} = \frac{160\times10^3}{2500} = 64\ A_{rms}$ $\qquad a =$ 0.096

$\bar{V}_{LO} = 2500\angle0°\ V_{rms}$ $\quad \theta = \theta_v - \theta_I = \cos^{-1}0.75 = 41.4096° \Rightarrow \theta_I = -41.41°$

By KVL $\quad \bar{V}_p/a = \bar{I}_s(0.4514 + j\,2.08\bar{3}) + \bar{V}_s = 64\angle-41.41°(0.4514 + j\,2.08\bar{3}) + 2500\angle0°$

$= 2611.11168\angle1.7753°\ V_{rms} \Rightarrow \bar{V}_p = 0.096(\bar{V}_p/a) =$

$= 250.67\angle1.775°$

$VR = \frac{V_p/a - V_s}{V_s}\times100\% = \frac{2611.11 - 2500}{2500}\times100\% = 4.4445\%$

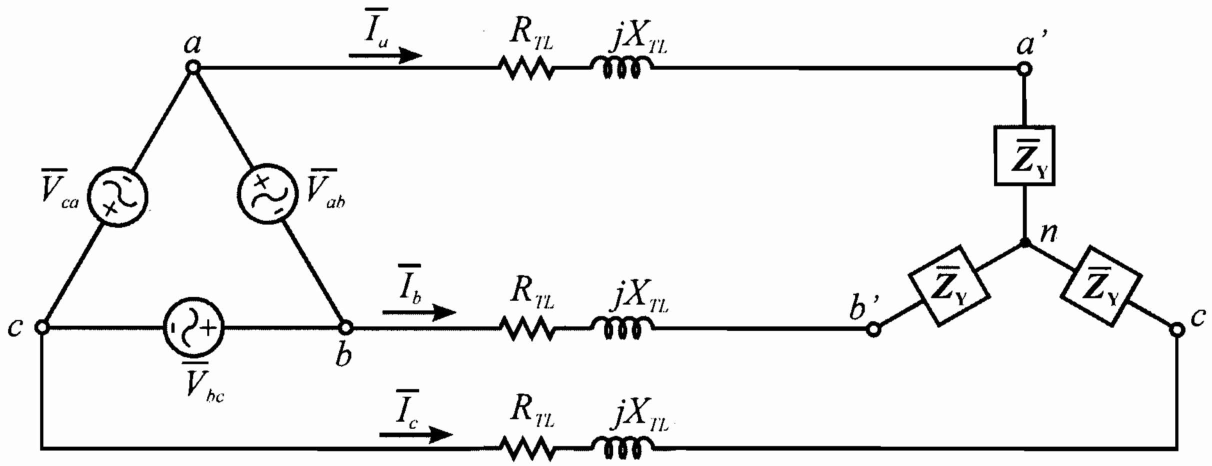
$P_{core} = \frac{(V_p/a)^2}{R_{c,s}} = \frac{2611.111^2}{2222.\bar{2}} = 3068.06\,W$ $\quad P_{cu} = I_s^2 R_{eq,s} = 64^2(0.4514) = 1848.\bar{8}\,W$

$\bar{I}_L =$ $64\angle-41.41°\ A_{rms}$ $\qquad \bar{V}_P =$ $250.67\angle1.775°\ V_{rms}$ $\qquad VR =$ 4.4445 %

Extra credit: $P_{core} =$ 3068.06 W $\qquad P_{cu} =$ 1848.89 W $\qquad \eta =$ 96.064%

$\eta = \frac{160\times10^3(0.75)}{160\times10^3(0.75) + 3068 + 1849}\times100\%$

3) For the 3ϕ circuit shown, $\overline{V}_{ab} = V_\phi \angle 0°$, $\overline{V}_{bc} = V_\phi \angle -120°$, $\overline{V}_{ca} = V_\phi \angle -240°$ with $V_\phi = 480\ \mathrm{V_{rms}}$, R_{TL} = 0.2 Ω, X_{TL} = 0.6 Ω, and $\overline{Z}_Y = 3 + j2\,\Omega$. Determine the magnitude of the line currents I_L, magnitude of the line-to-line $V_{LL,\mathrm{LD}}$ and phase $V_{\phi\mathrm{LD}}$ voltages for the loads, magnitude of the voltage drop on the transmission lines V_{line}, total complex power $\overline{S}_{\mathrm{load}}$ for the loads, total complex power $\overline{S}_{\mathrm{line}}$ for the lines, and the power factor *pf* seen by the source.



Δ → Y Source conversion :

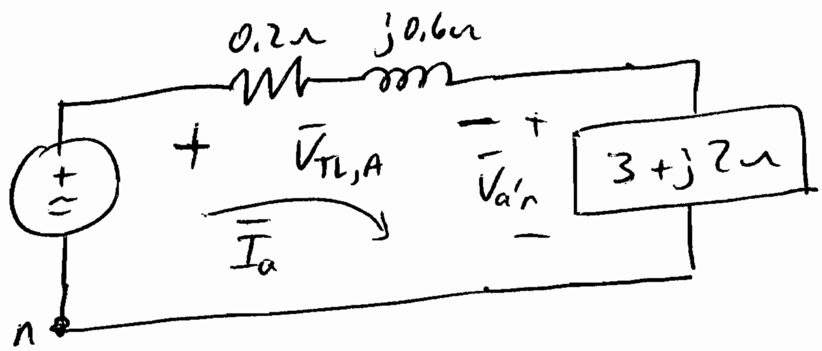
$$\overline{V}_{an} = \frac{480}{\sqrt{3}}\angle -30° = 277.13\angle -30°\ V_{rms}$$

$$\overline{V}_{bn} = \frac{480}{\sqrt{3}}\angle -150° = 277.13\angle -150°\ V_{rms}$$

$$\overline{V}_{cn} = \frac{480}{\sqrt{3}}\angle -270° = 277.13\angle -270°\ V_{rms}$$

Using phase A, the per-phase equiv. circuit is :

277.13 ∠-30° Vrms



$$\overline{I}_a = \frac{277.13\angle -30°}{(0.2 + j0.6) + (3 + j2)} = 67.213444\angle -69.093859°\ A_{rms}$$

↑

I_L

3) continued

$$\bar{V}_{TL,A} = \bar{I}_a(0.2+j0.6) = (67.2\angle -69.1°)(0.2+j0.6)$$
$$= \underbrace{42.5095145}_{V_{line}}\angle 2.47119229° \text{ Vrms}$$

$$\bar{V}_{a'n} = \bar{I}_a(3+j2) = (67.2\angle -69.1°)(3+j2) = \underbrace{242.34152}_{V_{\phi,LD}}\angle -35.03791°$$

$Y \rightarrow \Delta$

$$V_{LL,LD} = \sqrt{3}\, V_{\phi,LD} = \underline{419.7478 \text{ Vrms}}$$

$$\bar{S}_{load} = 3\,\bar{S}_{LD,A} = 3\,\bar{V}_{a'n}\,\bar{I}_a^* = 3(242.34\angle -35.04°)(67.21\angle +69.1°)$$
$$= \underline{48{,}865.8\angle 33.69° \text{ VA} = 40658.8 + j\,27{,}105.9 \text{ VA}}$$

$$\bar{S}_{line} = 3\,\bar{S}_{line,A} = 3\,\bar{V}_{TL,A}\,\bar{I}_a^* = 3(42.5095\angle 2.47°)(67.21\angle +69.1°)$$
$$= 8571.6\angle 71.565° \text{ VA} = 2710.6 + j\,8131.8 \text{ VA}$$

$$\bar{S}_{TOT} = \bar{S}_{Load} + \bar{S}_{line} = 55880.2\angle 39.094°$$

$$pf = \cos 39.093859° = 0.7761 \text{ lagging}$$

$I_L =$ 67.213 Arms $V_{LL,LD} =$ 419.75 Vrms $V_{\phi LD} =$ 242.34 Vrms $V_{line} =$ 42.51 Vrms

$\bar{S}_{load} =$ $48.866\angle 33.69°$ kVA
$40.66 + j27.11$ kVA

$\bar{S}_{line} =$ $8.57\angle 71.565°$ kVA
$2.71 + j8.13$ kVA

$pf =$ 0.7761 lagging