A 16 kVA, 480 V_{rms}, 3-phase, Δ-connected, 4-pole, 50 Hz, synchronous generator has a per-unit synchronous reactance of 0.05 and a per-unit armature resistance of 0.016. If the generator is providing its rated voltage to a balanced 3-phase load with a power factor of 0.866 lagging drawing 9 kW, determine: a) synchronous reactance and armature resistance in ohms (sketch 3-phase equivalent circuit), b) armature current for phase *a* (assume output phase voltage is at an angle of 0°), c) induced voltage $\bar{E}_A$ for phase *a*, d) voltage regulation, e) total armature electrical (i.e., copper) losses, and f) machine speed in RPM.

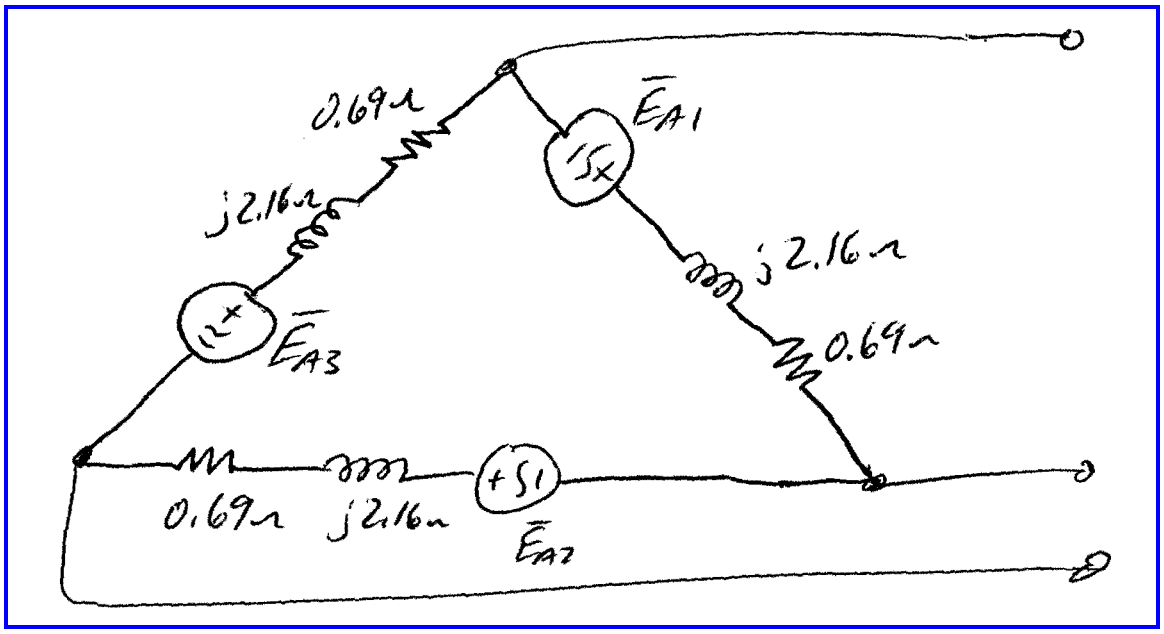
a) Use (3-102) to calculate the base impedance

$$Z_{base} = 3\,(V_{\phi,base})^2 / S_{rated} = 3(480)^2 / 16\times10^3 = 43.2\,\Omega.$$

$$X_S = X_{S,pu}\,(Z_{base}) = 0.05\,(43.2) \quad \Rightarrow \quad \underline{\boldsymbol{X_S = 2.16\ \Omega}}.$$

$$R_A = R_{A,pu}\,(Z_{base}) = 0.016\,(43.2) \quad \Rightarrow \quad \underline{\boldsymbol{R_A = 0.6912\ \Omega}}.$$

Per Figure 5-11b of text



b) Use (2-29) to find line current magnitude

$$I_L = \frac{P}{\sqrt{3}\,V_{LL}\cos\theta} = \frac{9000}{\sqrt{3}\,(480)0.866} = 12.5\ A_{rms}.$$

Using PF = cos $\theta \Rightarrow \theta = \theta_V - \theta_I = \cos^{-1}(0.866) = 30.0029° \Rightarrow \theta_I = -30.0029°$. So,

$$\Rightarrow \underline{\bar{I}_{A,a} = 12.50\angle -30°\ (A_{rms})}.$$

c) Per (5-9),

$$\begin{aligned}\overline{E}_A &= \overline{V}_\phi + jX_S\overline{I}_A + R_A\overline{I}_A = \overline{V}_\phi + \overline{I}_A(R_A + jX_S) \\ &= 480\angle 0^\circ + (12.5\angle -30^\circ)(0.6912 + j\,2.16)\end{aligned} \quad \Rightarrow \quad \underline{\overline{E}_{A,a} = 501.347\angle 2.179^\circ \ (\mathrm{V_{rms}})}.$$

d) The voltage regulation (4-56) is

$$\mathrm{VR} = (V_{\mathrm{NL}} - V_{\mathrm{FL}})/V_{\mathrm{FL}} \times 100\% = (501.347 - 480)/480 \times 100\% \Rightarrow \underline{\mathbf{VR = 4.45\%}}.$$

e) Per (4-53), the copper losses are $P_{\mathrm{Cu}} = 3\,I_A^2\,R_A = 3\,(12.5)^2\,0.6912 \quad \Rightarrow \quad \underline{\boldsymbol{P_{\mathrm{Cu}} = 324\ \mathrm{W}}}.$

f) Per (4-23), the machine speed in RPM is

$$n_m = 120 f_e / P = 120(50)/4 \quad \Rightarrow \quad \underline{\boldsymbol{n_m = 1500\ \mathrm{RPM}}}.$$