5-15 A 100-MVA, 11.5-kV, 0.85 PF lagging, 50-Hz, two-pole, Y-connected synchronous generator has a per-unit synchronous reactance of 0.8 and a per-unit armature resistance of 0.012.

a. What are its synchronous reactance and armature resistance in ohms?

b. What is the magnitude of the internal generated voltage E_A at the rated conditions? What is its torque angle δ at these conditions?

c. Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at full load?

a) Use (3-102) to calculate the base impedance

$$Z_{\text{base}} = 3\,(V_{\phi,\text{base}})^2 / S_{\text{rated}} = 3(11{,}500/\sqrt{3})^2 / 100\times10^6 = 1.3225\ \Omega.$$

$$X_S = X_{S,\text{pu}}\,(Z_{\text{base}}) = 0.8\ (1.3225) \quad \Rightarrow \quad \underline{\boldsymbol{X_S = 1.058\ \Omega}}.$$

$$R_A = R_{A,\text{pu}}\,(Z_{\text{base}}) = 0.012\ (1.3225) \quad \Rightarrow \quad \underline{\boldsymbol{R_A = 0.01587\ \Omega}}.$$

b) At rated conditions, $V_T = 11{,}500\ \text{V}_{\text{rms}}$.

For a Y-connection, $V_\phi = 11500/\sqrt{3} = 6639.528\ \text{V}_{\text{rms}}$. Let $\angle V_\phi = 0°$.

Use (2-29), $P = \sqrt{3}\,V_{LL} I_L \cos\theta$ to find the line current magnitude

$$I_A = I_L = \frac{P}{\sqrt{3}\,V_{LL}\cos\theta} = \frac{100\times10^6(0.85)}{\sqrt{3}\,(11{,}500)0.85} \quad \Rightarrow I_L = 5020.4371\ \text{A}_{\text{rms}}.$$

Using PF = cos $\theta \Rightarrow \theta = \theta_V - \theta_I = \cos^{-1}(0.85) = 31.78833° \Rightarrow \theta_I = -31.78833°$.

$$\bar{E}_A = \bar{V}_\phi + jX_S\bar{I}_A + R_A\bar{I}_A$$

Per (5-9), $$= 6639.528\angle 0° + (5020.4371\angle -31.78833°)(0.01587 + j1.058)\ .$$

$$= 10{,}505.14\angle 25.20°\ \text{V}_{\text{rms}}$$

$$\Rightarrow \quad \underline{\boldsymbol{E_A = 10{,}505\ \text{V}_{\text{rms}}}}.$$

Torque angle δ is angle between phase voltage $\bar{V}_\phi$ & induced voltage $\bar{E}_A$

$$\Rightarrow \underline{\boldsymbol{\delta = 25.2°}}.$$

c) Ignoring losses, $P_{\text{in}} = P_{\text{out}} = S_{\text{rated}}\,\text{PF} = 100\times10^6\ (0.85) = 85\times10^6\ \text{W}$.

Per (4-23), $n_m = 120 f_e / P = 120(50) / 2 = 3000$ RPM. Then, the generator speed is $\omega_m = n_m\,(2\pi/60) = 3000\,(2\pi/60) = 314.1593$ rad/s.

The applied torque from the prime mover can now be found as

$$\tau_{\text{app}} = P_{\text{in}} / \omega_m = 85\times10^6 / 314.1593 \quad \Rightarrow \underline{\boldsymbol{\tau_{\text{app}} = 270{,}563.4\ \text{N-m}}}.$$