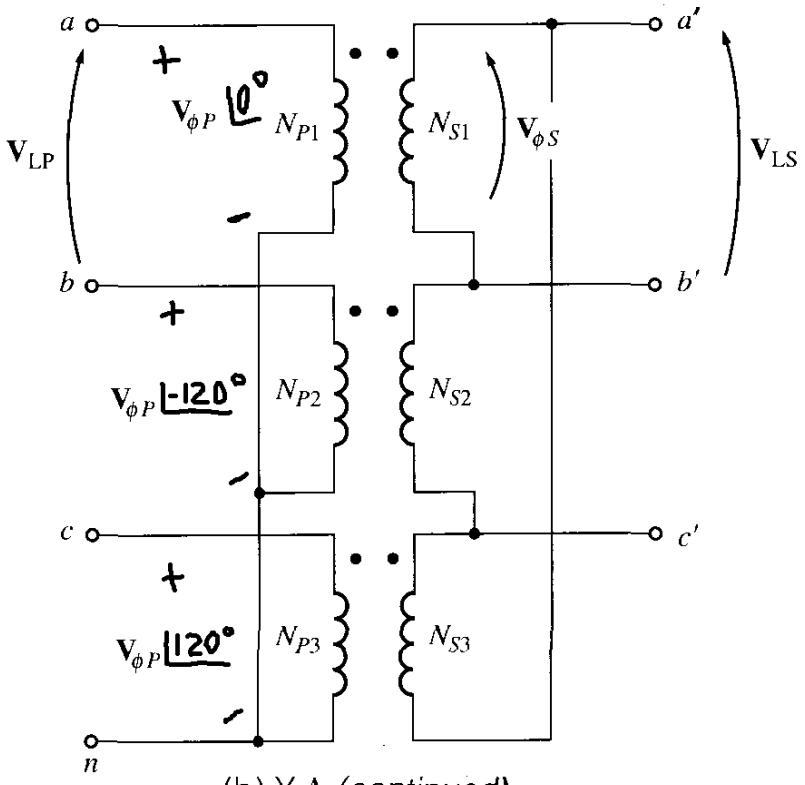
3–19 Prove that the three-phase system of voltages on the secondary of the Y–Δ transformer shown in Figure 3–37b lags the three-phase system of voltages on the primary of the transformer by 30°.

➢ To get consistent results, assume that the primary phase voltages are $\overline{V}_{an} = V_{\phi P}\angle 0°$, $\overline{V}_{bn} = V_{\phi P}\angle -120°$, and $\overline{V}_{cn} = V_{\phi P}\angle 120°$. As part of demonstrating the 30° lag, find the corresponding line-to-line secondary voltages $\overline{V}_{a'b'}$, $\overline{V}_{b'c'}$, and $\overline{V}_{c'a'}$ in terms of $V_{\phi P}$ and turns ratio a of the individual transformers and compare them to the primary line-to-line voltages $\overline{V}_{ab}$, $\overline{V}_{bc}$, and $\overline{V}_{ca}$.

Figure 3–37 | *(continued)*



(b) Y-Δ *(continued)*

By KVL on the primary transformer circuit-

$$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn} = V_{\phi P}\angle 0° - V_{\phi P}\angle 120° = V_{\phi P}\left(1\angle 0° - 1\angle -120°\right) = \sqrt{3}\, V_{\phi P}\angle 30°,$$

$$\overline{V}_{bc} = \overline{V}_{bn} - \overline{V}_{cn} = V_{\phi P}\angle -120° - V_{\phi P}\angle 120° = V_{\phi P}\left(1\angle -120° - 1\angle 120°\right) = \sqrt{3}\, V_{\phi P}\angle -90°, \text{ and}$$

$$\overline{V}_{ca} = \overline{V}_{cn} - \overline{V}_{an} = V_{\phi P}\angle 120° - V_{\phi P}\angle 0° = V_{\phi P}\left(1\angle 120° - 1\angle 0°\right) = \sqrt{3}\, V_{\phi P}\angle 150°.$$

Per (3-5), the turns ratio is $\frac{\overline{V}_P}{\overline{V}_S} = a$ for the phase voltages (i.e., coil pairs).

For coil set #1, $\overline{V}_{S1} = \overline{V}_{a'b'} = \frac{\overline{V}_{P1}}{a} = \frac{\overline{V}_{an}}{a} = \frac{V_{\phi P}\angle 0°}{a} = \frac{V_{\phi P}}{a}\angle 0°$.

For coil set #2, $\overline{V}_{S2} = \overline{V}_{b'c'} = \frac{\overline{V}_{P2}}{a} = \frac{\overline{V}_{bn}}{a} = \frac{V_{\phi P}\angle -120°}{a} = \frac{V_{\phi P}}{a}\angle -120°$.

For coil set #3, $\overline{V}_{S3} = \overline{V}_{c'a'} = \frac{\overline{V}_{P3}}{a} = \frac{\overline{V}_{cn}}{a} = \frac{V_{\phi P}\angle 120°}{a} = \frac{V_{\phi P}}{a}\angle 120°$.

Comparing primary and secondary line-to-line voltages-

$\overline{V}_{ab} = \sqrt{3}\, V_{\phi P}\angle 30°$ **but** $\overline{V}_{a'b'} = \frac{V_{\phi P}}{a}\angle 0° \Rightarrow$ **30° lag**,

$\overline{V}_{bc} = \sqrt{3}\, V_{\phi P}\angle -90°$ **but** $\overline{V}_{b'c'} = \frac{V_{\phi P}}{a}\angle -120° \Rightarrow$ **30° lag**, and

$\overline{V}_{ca} = \sqrt{3}\, V_{\phi P}\angle 150°$ **but** $\overline{V}_{c'a'} = \frac{V_{\phi P}}{a}\angle 120° \Rightarrow$ **30° lag**.