3–18 A 20-kVA, 20,000/480-V 60-Hz distribution transformer is tested with the following results:

Open-circuit test (measured from secondary side)	Short-circuit test (measured from primary side)
V_{OC} = 480 V	V_{SC} = 1130 V
I_{OC} = 1.51 A	I_{SC} = 1.00 A
P_{OC} = 271 W	P_{SC} = 260 W

a. Find the per-unit equivalent circuit for this transformer at 60 Hz.

b. What would the rating of this transformer be if it were operated on a 50-Hz power system?

c. Sketch the equivalent circuit of this transformer referred to the primary side *if it is operating at 50 Hz.*

a) **Open circuit test (secondary side)**

Per (3-50), $Y_E = |Y_E| = \dfrac{I_{oc}}{V_{oc}} = \dfrac{1.51}{480} = 0.003145833 \text{ S}.$

Per (3-51), $\text{PF}_{oc} = \dfrac{P_{oc}}{V_{oc} I_{oc}} = \dfrac{271}{480(1.51)} = 0.3739 \text{ lagging}.$

Per (3-52), $\theta = \theta_{ZE} = \cos^{-1}\left(\dfrac{P_{oc}}{V_{oc} I_{oc}}\right) = \cos^{-1}(0.3739) = 68.0439°.$

Per (3-53), put the magnitude and phase together to get

$$\overline{Y}_E = Y_E \angle\text{-}\theta_{ZE} = 0.0031458 \angle\text{-}68.044° \text{ S} = 0.001176215 - j0.002917668 \text{ S}.$$

Per (3-49), $\overline{Y}_E = \dfrac{1}{R_C} - j\dfrac{1}{X_M} = 0.001176215 - j0.002917668$ S. Calculate:

$$R_C = \frac{1}{0.001176215} = 850.1847 \ \Omega \quad \& \quad X_M = \frac{1}{0.002917668} = 342.7395 \ \Omega.$$

Per (3-63), the base impedance on the secondary side is

$$Z_{base} = V_{base}^2 / S_{base} = 480^2/20{,}000 = 11.52 \ \Omega.$$

Therefore, the per-unit values are

$$R_{C,pu} = \frac{850.1847}{11.52} = 73.80 \quad \& \quad X_{M,pu} = \frac{342.7395}{11.52} = 29.75.$$

Short circuit test (primary side)

Per (3-54), $Z_{SE} = |Z_{SE}| = \dfrac{V_{sc}}{I_{sc}} = \dfrac{1130}{1.00} = 1130\ \Omega.$

Per (3-55), $\text{PF}_{sc} = \dfrac{P_{sc}}{V_{sc} I_{sc}} = \dfrac{260}{1130(1.0)} = 0.2301$ lagging.

Per (3-52), $\theta = \theta_{SE} = \cos^{-1}\left(\dfrac{P_{sc}}{V_{sc} I_{sc}}\right) = \cos^{-1}(0.2300885) = 76.697718°.$

Per (3-57), put the magnitude and phase together to get

$$\bar{Z}_{SE} = Z_{SE} \angle \theta_{SE} = 1130 \angle 76.7° \ \Omega = 260.00 + j1099.68\ \Omega.$$

Per (3-58), $\bar{Z}_{SE} = 260.00 + j1099.68\ \Omega = R_{eq,P} + jX_{eq,P}$.

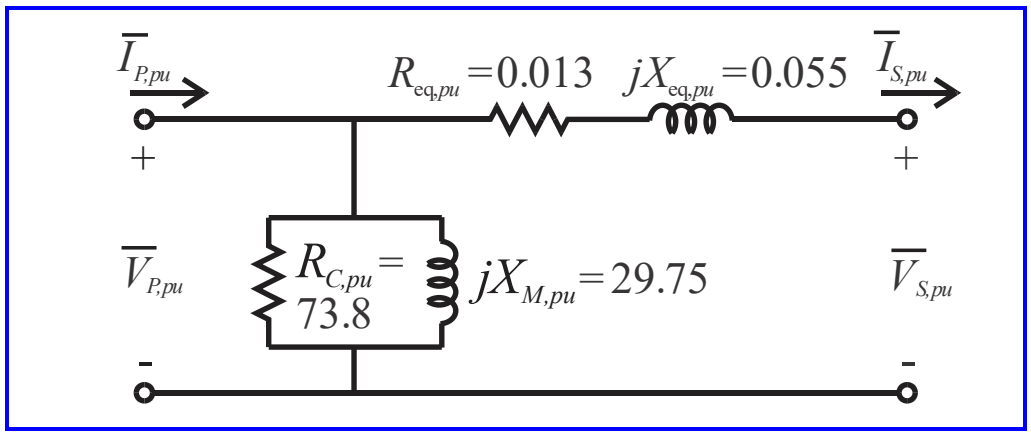
Per (3-63), the base impedance on the primary side is

$$Z_{\text{base}} = V_{\text{base}}^2 / S_{\text{base}} = 20{,}000^2 / 20{,}000 = 20{,}000\ \Omega.$$

Therefore, the per-unit values are

$$R_{eq,pu} = \frac{260}{20{,}000} = 0.013 \quad \& \quad X_{eq,pu} = \frac{1099.68}{20{,}000} = 0.054984 = 0.055.$$

Approximate Per-Unit Equivalent Circuit (per Figure 3-25)



b) Per (3-105), $\phi_{max} = V_{max} / \omega N_P$ must hold constant. Assuming N_P is constant, we get

$20{,}000/2\pi(60)N_P = V_{max,50}/2\pi(50)N_P \Rightarrow V_{max,50} = 16{,}666.7\ V_{rms}$ (primary) and

$480/2\pi(60)N_P = V_{max,50}/2\pi(50)N_P \Rightarrow V_{max,50} = 400\ V_{rms}$ (secondary) at 50 Hz.

Since the voltages were derated by 5/6, the apparent power will be as well

20,000 VA (5/6) $\Rightarrow S_{rated,50} = 16{,}666.7$ VA. In summary, the transformer ratings become

16,666.7 / 400 V_{rms} and 16,666.7 VA at 50 Hz.

c) From earlier, the base impedance on the primary side is $Z_{base} = 20{,}000\ \Omega$. So, the actual primary side parameters at 60 Hz are

$R_C = R_{C,pu} Z_{base} = 73.5\ (20000) = 1{,}470{,}000\ \Omega$,

$X_M = X_{M,pu} Z_{base} = 29.75\ (20000) = 595{,}000\ \Omega$,

$R_{eq} = R_{eq,pu} Z_{base} = 0.013\ (20000) = 260\ \Omega$, and

$X_{eq} = X_{eq,pu} Z_{base} = 0.055\ (20000) = 1100\ \Omega$.

Now, resistances (e.g., R_C & R_{eq}) are NOT frequency dependent. However, inductive reactances $X = \omega L$. Therefore,

$X_{M,50} = 595{,}000(50/60) = 495{,}833.3\ \Omega$ and $X_{eq,50} = 1100(50/60) = 916.67\ \Omega$.

Approximate Primary Equivalent Circuit at 50 Hz

