

- 3-2** A 20-kVA, 8000/277-V distribution transformer has the following resistances and reactances:
- | | |
|-----------------------------|----------------------------|
| $R_p = 32 \Omega$ | $R_s = 0.05 \Omega$ |
| $X_p = 45 \Omega$ | $X_s = 0.06 \Omega$ |
| $R_c = 250 \text{ k}\Omega$ | $X_M = 30 \text{ k}\Omega$ |

The excitation branch impedances are given referred to the high-voltage side of the transformer.

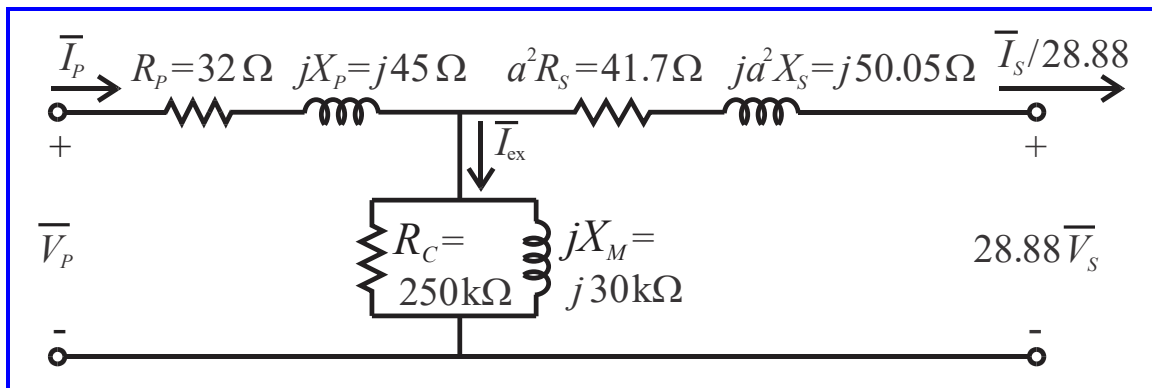
- Find the equivalent circuit of this transformer referred to the high-voltage side.
 - Find the per-unit equivalent circuit of this transformer.
 - Assume that this transformer is supplying rated load at 277 V and 0.8 PF lagging. What is this transformer's input voltage? What is its voltage regulation?
 - What is the transformer's efficiency under the conditions of part (c)?
- For part d), also find the copper and core losses.

a) The turns ratio is (3-5) $a = V_p / V_s = 8000 / 277 = 28.880866$.

Use (3-17) $Z_L' = a^2 Z_L$ to refer R_s and X_s to the high voltage side-

$$R_s' = 28.88^2 (0.05) = 41.7052 \Omega \text{ and } X_s' = 28.88^2 (0.06) = 50.0463 \Omega$$

Per Figure 3-17a, the primary-side phasor equivalent circuit is



- b) Per (3-63), the base impedance is $Z_{base} = (V_{base})^2 / S_{base}$. On the primary side-
- $$Z_{base,P} = 8000^2 / 20,000 = 3200 \Omega.$$

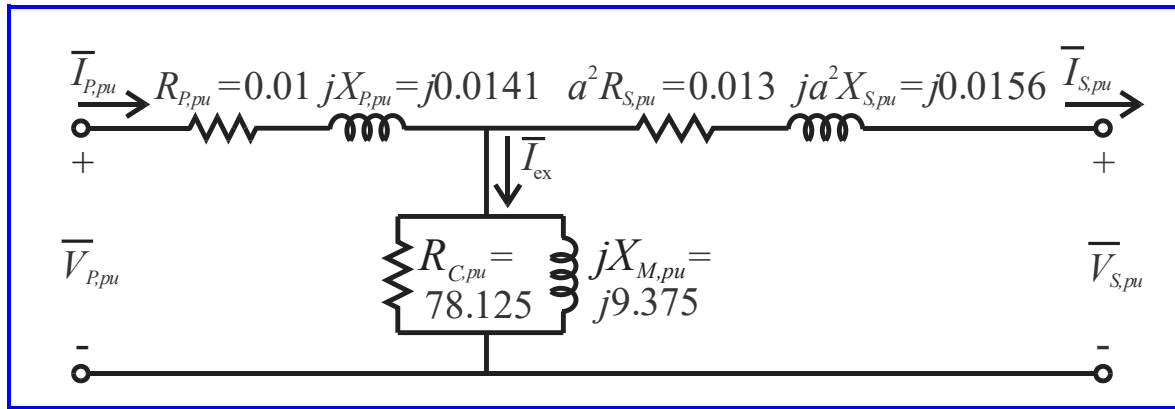
The primary per-unit impedances are then-

$$R_{C,pu} = 250 / 3.2 = 78.125, \quad X_{M,pu} = 30 / 3.2 = 9.375,$$

$$R_{P,pu} = 32 / 3200 = 0.01, \quad X_{P,pu} = 45 / 3200 = 0.01406,$$

$$a^2 R_{S,pu} = 41.7052 / 3200 = 0.01303, \text{ and } a^2 X_{S,pu} = 50.0463 / 3200 = 0.01564.$$

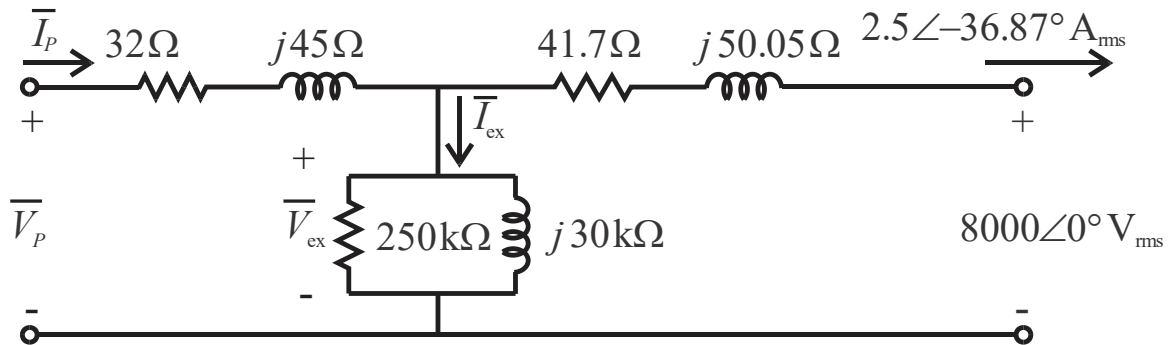
Per Figure 3-17a & Figure 3-25, the primary-side per-unit phasor equivalent circuit is



- c) Assuming a phase angle of 0, $\bar{V}_L = \bar{V}_S = 277\angle 0^\circ$ (V_{rms}). Per the circuit in part a), the referred secondary voltage is $28.88\bar{V}_S = 28.88(277\angle 0^\circ) = 8000\angle 0^\circ$ (V_{rms}).

For a rated load, $I_L = S_{\text{rated}} / V_L = 20,000 / 277 = 72.2022$ (A_{rms}) with a phase angle found using $\theta_{VL} - \theta_{IL} = \cos^{-1}(\text{PF}) \Rightarrow -\theta_{IL} = -\cos^{-1}(0.8) = -36.87^\circ$.

So, $\bar{I}_L = \bar{I}_S = 72.2\angle -36.87^\circ$ (A_{rms}). Per the circuit in part a), the referred secondary current is $\bar{I}_S / 28.88 = (72.2\angle -36.87^\circ) / 28.88 = 2.5\angle -36.87^\circ$ (A_{rms}). This gives



Using KVL on RHS of the circuit,

$$\bar{V}_{\text{ex}} = (41.7 + j50.05)(2.5\angle -36.87^\circ) + (8000\angle 0^\circ) = 8158.56626\angle 0.2636^\circ \text{ (V}_{\text{rms}}\text{)}.$$

Using KCL on top middle node of the circuit,

$$\begin{aligned} \bar{I}_P &= \bar{I}_{\text{ex}} + (2.5\angle -36.87^\circ) = \bar{V}_{\text{ex}} \bar{Y}_{\text{ex}} + (2.5\angle -36.87^\circ) \\ &= (8158.56626\angle 0.2636^\circ) \left(\frac{1}{250,000} + \frac{1}{j30,000} \right) + (2.5\angle -36.87^\circ). \\ &= 2.6974\angle -41.06^\circ \text{ (A}_{\text{rms}}\text{)} \end{aligned}$$

Use KVL on LHS of the circuit to get the phasor input voltage,

$$\begin{aligned}\bar{V}_p &= \bar{I}_p(32 + j45) + \bar{V}_{ex} = (2.6974 \angle -41.06^\circ)(32 + j45) + (8158.56626 \angle 0.2636^\circ) \\ &= 8303.61 \angle 0.4993^\circ \text{ (V}_{\text{rms}}\text{)} \\ \Rightarrow \underline{\bar{V}_p} &= \underline{8303.61 \angle 0.4993^\circ \text{ (V}_{\text{rms}}\text{)}}\end{aligned}$$

Per (3-68), the voltage regulation is

$$\begin{aligned}\text{VR} &= \frac{V_p / a - V_{S,\text{fl}}}{V_{S,\text{fl}}} \times 100\% = \frac{V_p - aV_{S,\text{fl}}}{aV_{S,\text{fl}}} \times 100\% = \frac{8303.61 - 8000}{8000} \times 100\% \\ \Rightarrow \underline{\text{VR}} &= \underline{3.8\%}.\end{aligned}$$

d) To get the efficiency, find the output, copper, & core powers-

$$P_{\text{out}} = S_{\text{rated}} \cos(\theta) = S_{\text{rated}} \text{PF} = 20,000 (0.8) = 16,000 \text{ W}$$

$$P_{\text{Cu}} = I_p^2 (32) + 2.5^2 (41.7) = 2.6974^2 (32) + 2.5^2 (41.7) \Rightarrow \underline{P_{\text{Cu}}} = \underline{493.488 \text{ W}}.$$

$$P_{\text{core}} = V_{\text{ex}}^2 / R_{\text{core}} = 8158.56626^2 / 250,000 \Rightarrow \underline{P_{\text{core}}} = \underline{266.2488 \text{ W}}.$$

Per (3-72), the efficiency is

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{losses}}} \times 100\% = \frac{16,000}{16,000 + 493.488 + 266.2488} \times 100\% \Rightarrow \underline{\eta} = \underline{95.47\%}.$$