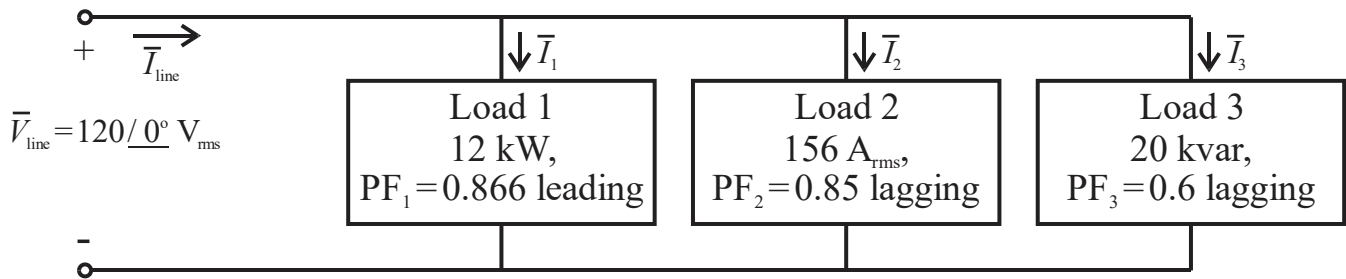


A 60 Hz, 120 V_{rms} single-phase source (assume 0° phase angle) is connected to three loads connected in parallel. Load 1 consumes 12 kW of power and has a power factor $pf_1 = 0.866$ leading. Load 2 draws 156 A_{rms} of current and has $pf_2 = 0.85$ lagging. Load 3 uses 20 kvar of reactive power and has a power factor $pf_3 = 0.6$ lagging. Determine: a) the complex power consumed, phasor current (rms) drawn by, and impedance of load 1, b) repeat for load 2, c) repeat for load 3, d) the total complex power and phasor current (rms) supplied by the source, and e) the overall power factor 'seen' by the source.



- a) Per (1-62), power factor for load 1 is $PF_1 = 0.866 = \cos(\theta_1) \Rightarrow \theta_1 = -30.0029^\circ$ (leading).
 $\Rightarrow \theta_1 = -30.0029^\circ = \theta_V - \theta_{I_1} = 0 - \theta_{I_1} \Rightarrow \theta_{I_1} = 30.0029^\circ$.

Per (1-51), the power is $P_1 = V_1 I_1 \cos(\theta_1)$.

$$\Rightarrow I_1 = P_1 / V_1 \cos(\theta_1) = 12,000 / 120(0.866) \Rightarrow I_1 = 115.47344 \text{ A}_{rms}.$$

Putting current magnitude and phase together $\Rightarrow \bar{I}_1 = 115.47 \angle 30^\circ (\text{A}_{rms})$.

Per (1-61), the complex power for load 1 is $\bar{S}_1 = \bar{V}_{line} \bar{I}_1^* = 120 \angle 0^\circ (115.47 \angle -30^\circ)$

$$\Rightarrow \bar{S}_1 = 13,856.8 \angle -30^\circ (\text{VA}) = 12,000 - j6,929 (\text{VA}).$$

By Ohm's Law, $\bar{Z}_1 = \bar{V}_{line} / \bar{I}_1 = 120 \angle 0^\circ / (115.47 \angle 30^\circ)$

$$\Rightarrow \bar{Z}_1 = 0.900 - j0.520 \Omega = 1.0392 \angle -30.00^\circ \Omega$$

- b) Per (1-62), power factor for load 2 is $PF_2 = 0.85 = \cos(\theta_2) \Rightarrow \theta_2 = 31.78833^\circ$ (lagging).

$$\Rightarrow \theta_2 = 31.78833^\circ = \theta_V - \theta_{I_2} = 0 - \theta_{I_2} \Rightarrow \theta_{I_2} = -31.78833^\circ.$$

Putting 156 A_{rms} current magnitude and phase together $\Rightarrow \bar{I}_2 = 156 \angle -31.788^\circ (\text{A}_{rms})$.

Per (1-61), the complex power for load 2 is $\bar{S}_2 = \bar{V}_{line} \bar{I}_2^* = 120 \angle 0^\circ (156 \angle 31.788^\circ)$

$$\Rightarrow \bar{S}_2 = 18,720 \angle 31.788^\circ (\text{VA}) = 15,913 + j9,861 (\text{VA}).$$

By Ohm's Law, $\bar{Z}_2 = \bar{V}_{line} / \bar{I}_2 = 120 \angle 0^\circ / (156 \angle -31.788^\circ)$

$$\Rightarrow \bar{Z}_2 = 0.654 + j0.405 \Omega = 0.7692 \angle 31.788^\circ \Omega$$

- c) Per (1-62), the power factor for load 3 is $\text{PF}_3 = 0.6 = \cos(\theta_3) \Rightarrow \theta_3 = 53.1301^\circ$.
 $\Rightarrow \theta_3 = 53.13^\circ = \theta_V - \theta_{I_3} = 0 - \theta_{I_3} \Rightarrow \theta_{I_3} = -53.13^\circ$.

Per (1-52), the reactive power is $Q_3 = V_3 I_3 \sin(\theta_3)$.

$$\Rightarrow I_3 = Q_3 / V_3 \sin(\theta_3) = 20,000 / 120 \sin(53.13) \Rightarrow I_3 = 208.33334 \text{ A}_{\text{rms}}.$$

Putting current magnitude and phase together $\Rightarrow \underline{\bar{I}_3 = 208.33 \angle -53.13^\circ (\text{A}_{\text{rms}})}.$

Per (1-61), the complex power for load 3 is $\bar{S}_3 = \bar{V}_{\text{line}} \bar{I}_3^* = 120 \angle 0^\circ (208.33 \angle 53.13^\circ)$

$$\Rightarrow \underline{\bar{S}_3 = 25,000 \angle 53.13^\circ (\text{VA}) = 15,000 + j20,000 (\text{VA})}.$$

By Ohm's Law, $\bar{Z}_3 = \bar{V}_{\text{line}} / \bar{I}_3 = 120 \angle 0^\circ / (208.33 \angle -53.13^\circ)$

$$\Rightarrow \underline{\bar{Z}_3 = 0.3456 + j0.4608 \Omega = 0.576 \angle 53.13^\circ \Omega}$$

- d) By conservation of complex power:

$$\bar{S}_{\text{line}} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 13,856.8 \angle -30^\circ + 18,720 \angle 31.788^\circ + 25,000 \angle 53.13^\circ$$

$$\Rightarrow \underline{\bar{S}_{\text{line}} = 48,655.24 \angle 28.12^\circ (\text{VA}) = 42,911.99 + j22,932.36 (\text{VA})}.$$

By KCL, $\bar{I}_{\text{line}} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 115.47 \angle 30^\circ + 156 \angle -31.788^\circ + 208.33 \angle -53.13^\circ$

$$\Rightarrow \underline{\bar{I}_{\text{line}} = 405.46 \angle -28.12^\circ (\text{A}_{\text{rms}})}.$$

- e) Per (1-62), the overall power factor is $\text{PF}_{\text{line}} = \cos(\theta_{S_{\text{line}}}) = \cos(28.12^\circ)$

$$\Rightarrow \underline{\text{PF}_{\text{line}} = 0.882 \text{ lagging}}.$$