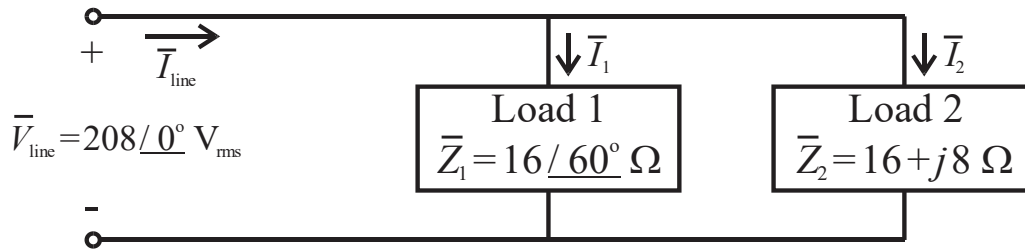


A 60 Hz, 208 V_{rms} single-phase source (assume 0° phase angle) initially supplies two loads ($\bar{Z}_1 = 16\angle 60^\circ \Omega$ & $\bar{Z}_2 = 16 + j8 \Omega$) connected in parallel. a) Find the phasor current (rms) and real, reactive, & apparent powers as well as the power factor for load 1. b) Repeat for load 2. c) Find the phasor line current (rms) as well as the real, reactive, & apparent powers supplied by the source. Also, find the power factor 'seen' by the source. d) We desire to achieve a PF ≥ 0.9 by connecting a capacitive load in parallel. What range of capacitance values will work?



a) By Ohm's Law, $\bar{I}_1 = 208\angle 0^\circ / 16\angle 60^\circ \Rightarrow \bar{I}_1 = 13\angle -60^\circ (\text{A}_{\text{rms}})$.

Per (1-61), the complex power for load 1 is

$$\bar{S}_1 = \bar{V}_{\text{line}} \bar{I}_1^* = 208\angle 0^\circ (13\angle +60^\circ) = 2704\angle 60^\circ (\text{VA}) = 1352 + j2341.733 (\text{VA}).$$

This yields $\Rightarrow \underline{P_1 = 1352 \text{ W}}, \underline{Q_1 = 2341.733 \text{ var}}, \text{ and } \underline{S_1 = 2341.73 \text{ VA}}.$

Per (1-62), the power factor for load 1 is $\text{PF}_1 = \cos(\theta_1) = \cos(60^\circ) \Rightarrow \underline{\text{PF}_1 = 0.5 \text{ lagging}}.$

b) By Ohm's Law, $\bar{I}_2 = 208\angle 0^\circ / (16 + j8) \Rightarrow \bar{I}_2 = 11.62755\angle -26.565^\circ (\text{A}_{\text{rms}})$.

The complex power for load 2 is

$$\bar{S}_2 = \bar{V}_{\text{line}} \bar{I}_2^* = 208\angle 0^\circ (11.63\angle 26.565^\circ) = 2418.53\angle 26.565^\circ (\text{VA}) = 2163.2 + j1081.6 (\text{VA}).$$

This yields $\Rightarrow \underline{P_2 = 2163.2 \text{ W}}, \underline{Q_2 = 1081.6 \text{ var}}, \text{ and } \underline{S_2 = 2418.53 \text{ VA}}.$

The power factor for load 1 is $\text{PF}_2 = \cos(\theta_2) = \cos(26.565^\circ) \Rightarrow \underline{\text{PF}_2 = 0.8944 \text{ lagging}}.$

c) By KCL, $\bar{I}_{\text{line}} = \bar{I}_1 + \bar{I}_2 = 13\angle -60^\circ + 11.628\angle -26.565^\circ \Rightarrow \bar{I}_{\text{line}} = 23.59\angle -44.24^\circ (\text{A}_{\text{rms}})$.

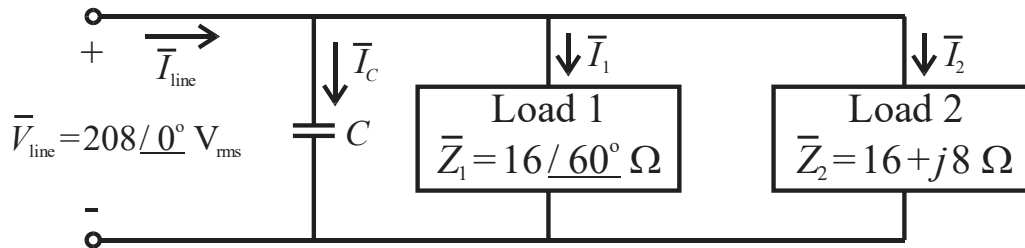
By conservation of complex power:

$$\begin{aligned} \bar{S}_{\text{line}} &= \bar{S}_1 + \bar{S}_2 = 2704\angle 60^\circ + 2418.53\angle 26.565^\circ \\ &= 4906.7\angle 44.24^\circ (\text{VA}) = 3515.2 + j3423.3 (\text{VA}) \end{aligned}$$

This yields $\Rightarrow \underline{P_{\text{line}} = 3515.2 \text{ W}}, \underline{Q_{\text{line}} = 3423.3 \text{ var}}, \text{ and } \underline{S_{\text{line}} = 4906.7 \text{ VA}}.$

The power factor 'seen' is $\text{PF}_{\text{line}} = \cos(\theta_{\text{line}}) = \cos(44.24^\circ) \Rightarrow \underline{\text{PF}_{\text{line}} = 0.7164 \text{ lagging}}.$

d) With power factor correction, the circuit becomes:



For a PF = 0.9, the desired overall phase angle is $\text{PF} = 0.9 = \cos(\theta_{\text{des}}) \Rightarrow \theta_{\text{des}} = \pm 25.842^\circ$.

From the class notes, $C = \frac{P_1 [\tan(\theta_1) - \tan(\theta_2)]}{\omega V^2}$ where P_1 is the power consumed by the load(s), θ_1 is phase angle of the original load(s), θ_2 is the desired overall phase angle, $\omega = 2\pi f$ is the radian frequency, and V is the rms line voltage.

The capacitance needed for undercorrection (PF = 0.9 lagging, $\theta_{\text{des}} = 25.842^\circ$) is

$$C_{\text{low}} = \frac{3515.2 [\tan(44.24^\circ) - \tan(25.842^\circ)]}{2\pi(60) 208^2} \Rightarrow \underline{C_{\text{low}} = 1.055 \times 10^{-4} \text{ F} = 105.5 \mu\text{F}}.$$

The capacitance needed for overcorrection (PF = 0.9 leading, $\theta_{\text{des}} = -25.842^\circ$) is

$$C_{\text{high}} = \frac{3515.2 [\tan(44.24^\circ) - \tan(-25.842^\circ)]}{2\pi(60) 208^2} \Rightarrow \underline{C_{\text{high}} = 3.143 \times 10^{-4} \text{ F} = 314.3 \mu\text{F}}.$$

The range of capacitance values that will work is then $\Rightarrow \underline{105.5 \mu\text{F} \leq C \leq 314.3 \mu\text{F}}.$