EE 330 Energy Systems (Fall 2026) Examination 2

Name KEY A

Instructions: Show **all** work for full credit. Write answers in indicated places. Insert equation sheet(s) in exam.

1) A 50 Hz, 6-pole, Y-connected, 3ϕ AC generator has an open circuit line-to-line voltage of 500 V_{rms}. There are 66 turns per coil and the coils are installed in 18 slots around the stator. For each phase, the coils are connected in series. Determine the rotor speed n_m (RPM). Find the open circuit phase voltage $V_{\phi,OC} = E_A$ and magnetic flux ϕ. For an electrical load of 25 kVA with a 0.8 lagging power factor at the rated voltage, the copper losses are 800 W, friction & windage losses are 1.4 kW, and stray/core losses are 600 W. Find the input power P_{in}, applied torque τ_{app}, and efficiency η (%). Use MKS units.

Use (4-23), $f_e = \dfrac{n_m P}{120} \Rightarrow n_m = \dfrac{120 f_e}{P} = \dfrac{120(50)}{6} \Rightarrow$ **n_m = 1000 RPM.**

Per (2-11) for a Y-connection, $V_T = V_{LL} = \sqrt{3}\, V_\phi \Rightarrow V_\phi = V_{LL} / \sqrt{3} = 500 / \sqrt{3}$

$\Rightarrow$ **$V_{\phi,OC}$ = 288.675 V_{rms}.**

Per (4-39), the induced RMS voltage for a single coil is $E_A = \sqrt{2}\,\pi N_C \phi f = V_{\phi,OC}$.

With 18 slots for 3 phases, we have 6 slots/phase (1 coil/2 slots) = 3 coils/phase to generate the total E_A.

For 3 coils, $E_{A,3} = 3\sqrt{2}\,\pi N_C \phi f \Rightarrow \phi = \dfrac{E_{A,3coil}}{3\sqrt{2}\,\pi N_C f} = \dfrac{288.675}{3\sqrt{2}\,\pi(66)50} \Rightarrow$ **ϕ = 0.0065631 Wb.**

The output power is $P_{out} = S\,PF = 25\times10^3\,(0.8) = 20\times10^3 = 20{,}000$ W.

Then, the input power from the prime mover is then

$$P_{in} = P_{out} + \text{losses} = P_{out} + P_{Cu} + P_{F\&W} + P_{stray/core} = 20{,}000 + 800 + 1400 + 600$$

$\Rightarrow$ **P_{in} = 22,800 W = 22.8 kW.**

Also, $P_{in} = \tau_{app}\,\omega_m$. The generator speed is $\omega_m = n_m\,(2\pi/60) = 1000\,(\pi/30) = 104.72$ rad/s.

The applied torque is then $\tau_{app} = P_{in} / \omega_m = 22{,}800 / 104.72 \Rightarrow$ **τ_{app} = 217.724 N-m.**

The efficiency (4-51) is $\eta = P_{out} / P_{in} \times 100\% = 20{,}000 / 22{,}800 \times 100\% \Rightarrow$ **η = 87.7193%.**

n_m = **1000 RPM**	$V_{\phi,OC}$ = **288.675 V_{rms}**	ϕ = **6.563 mWb**
P_{in} = **22.8 kW**	τ_{app} = **217.724 N-m**	η = **87.72%**

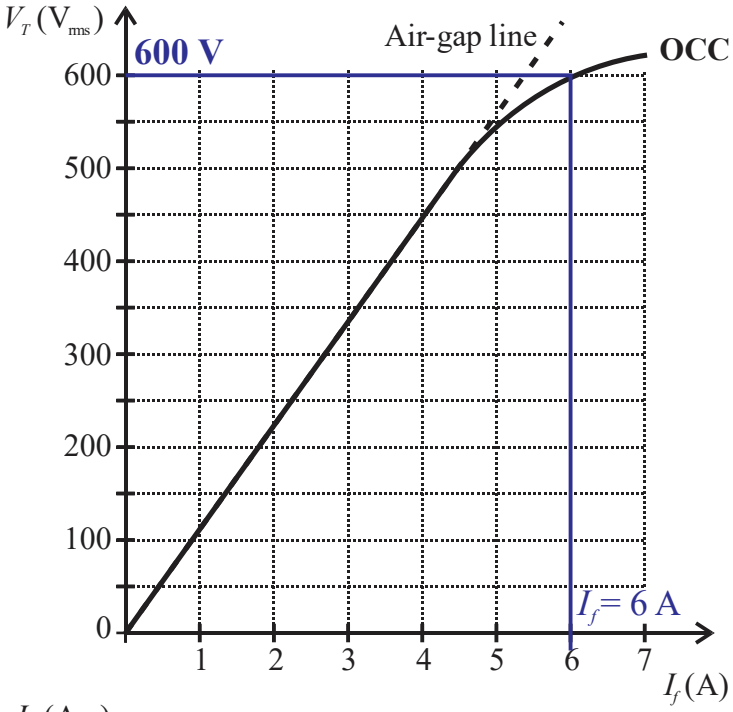
2) A Y-connected 3-phase, synchronous generator has the OCC and SCC shown. The rated field current is 6 A. A 6 V_{dc} voltage applied across two terminals results in a current of 4 A_{dc}. Find the per-phase armature resistance R_A and rated synchronous reactance X_S. Sketch, with labels, the per-phase equivalent armature circuit. At the rated field current, a load draws a line current of 12 A_{rms} with a lagging power factor of 0.82. Find the **phasor** armature $\overline{I}_A$ (assume $\overline{V}_\phi$ at angle 0°) and armature copper losses P_{SCL}.

From the OCC, $V_{T,OC} = 600\ V_{rms}$.

For a Y-connection per (2-11),

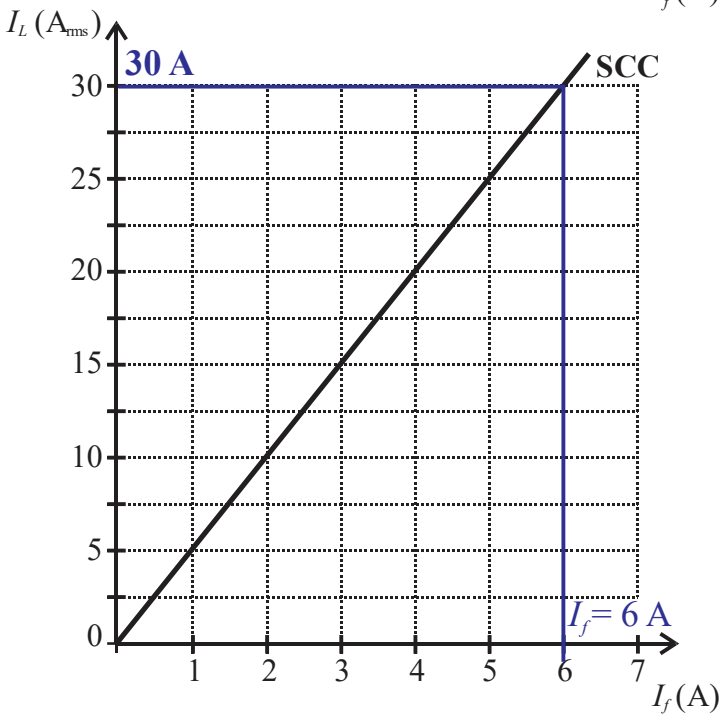
$$V_{\phi,OC} = \frac{V_{LL,OC}}{\sqrt{3}} = \frac{600}{\sqrt{3}}$$

$\Rightarrow$ **$V_{\phi,OC} = E_{A,OC} = 346.41\ V_{rms}$.**



From the SCC, **$I_{L,SC} = I_{\phi,SC} = 30\ A_{rms}$**

for a Y-connection per (2-10).



By Ohm's Law, $R_T = V_{dc} / I_{dc} = V_{dc} / I_{dc} = 6 / 4 = 1.5\ \Omega$.

For a Y-connection, $R_T = 2\ R_A = 1.5\ \Omega$ $\Rightarrow$ **$R_A = 0.75\ \Omega$**.

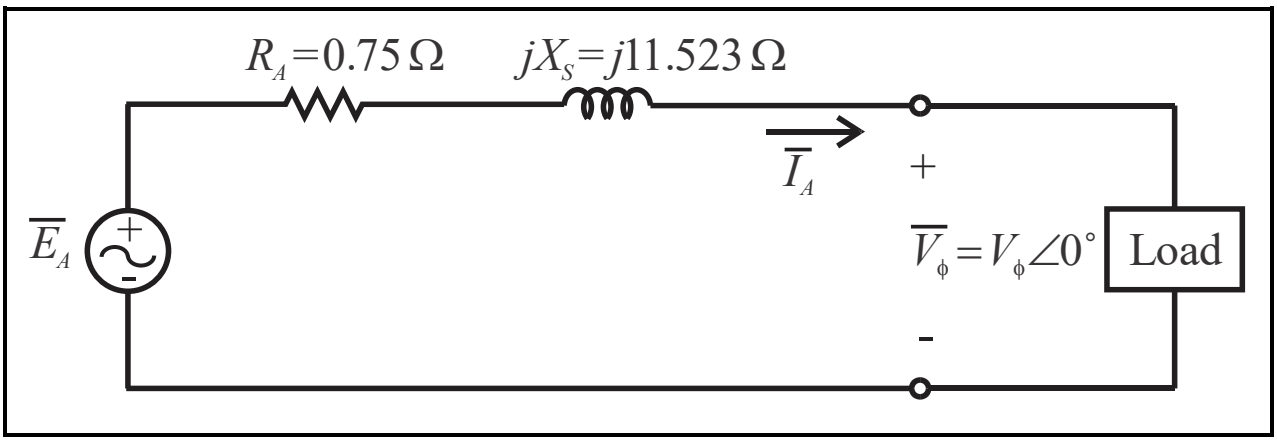
2) cont.

Per (5-23), $Z_S = \sqrt{R_A^2 + X_S^2} = \dfrac{E_{A,OC}}{I_{A,SC}} = \dfrac{V_{\phi,OC}}{I_{A,SC}}$. So,

$$\sqrt{0.75^2 + X_S^2} = \frac{600/\sqrt{3}}{30} \quad \Rightarrow 0.75^2 + X_S^2 = 133.3333 \quad \Rightarrow X_S = \sqrt{133.3333 - 0.75^2}$$

$$\Rightarrow \quad \underline{\boldsymbol{X_S = 11.522623\ \Omega}}.$$

Per-phase equivalent armature circuit $R_A =$ **0.75 Ω** $X_S =$ **11.523 Ω**



We are given $I_L = I_A = 12\ A_{rms}$.

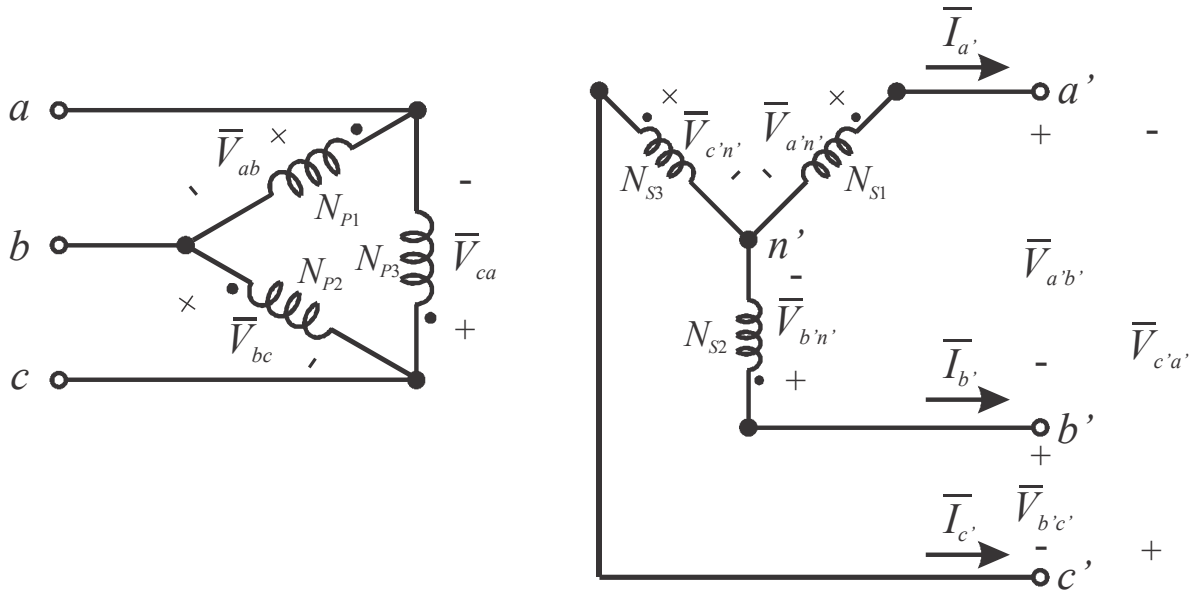
Using the lagging PF = cos $\theta \Rightarrow \theta = \theta_V - \theta_I = \cos^{-1}(0.82) = 34.9152° \Rightarrow \theta_I = -34.9152°$.

Therefore, the armature current is $\underline{\overline{I}_A = 12\angle -34.9152°\ (A_{rms})}$.

Per (4-53), $P_{SCL} = 3\,I_A^2\,R_A = 3\,(12^2)\,0.75 \Rightarrow$ $\underline{\boldsymbol{P_{SCL} = 324\ W}}$.

$$\overline{I}_A = \underline{12\angle -34.9152°\ (A_{rms})} \qquad P_{SCL} = \underline{\textbf{324 W}}$$

3) A 200 kVA, 480/2400 V_{rms} (at terminals), Δ-Y connected (wired as shown) three-phase transformer is running at the rated voltages. Given $\overline{V}_{ab} = V_{LL,P}\angle 0°$, $\overline{V}_{bc} = V_{LL,P}\angle 120°$, and $\overline{V}_{ca} = V_{LL,P}\angle 240°$, find the phasor secondary line-to-line voltages $\overline{V}_{a'b'}$, $\overline{V}_{b'c'}$, & $\overline{V}_{c'a'}$, phasor secondary phase voltages $\overline{V}_{a'n'}$, $\overline{V}_{b'n'}$, & $\overline{V}_{c'n'}$, line-to-line turns ratio a_{LL}, and phase turns ratio a_ϕ. Put phasor angles in degrees.



$\overline{V}_{ab} = V_{LL,P}\angle 0°$, $\overline{V}_{bc} = V_{LL,P}\angle 120°$, & $\overline{V}_{ca} = V_{LL,P}\angle 240°$ ⇒ NOT standard *abc* sequence.

Per (3-96), the line-to-line turns ratio for a Δ-Y connection is $a_{LL} = \dfrac{\overline{V}_{LP}}{\overline{V}_{LS}} = \dfrac{a_\phi}{\sqrt{3}} = \dfrac{480}{2400}$

$\Rightarrow$ $\underline{\boldsymbol{a_{LL} = 0.2}}$, and

$$a_\phi = \sqrt{3}\, a_{LL} = \sqrt{3}\,\frac{480}{2400} \qquad \Rightarrow \quad \underline{\boldsymbol{a_\phi = 0.34641}}.$$

Per (3-5), the turns ratio is $\dfrac{\overline{V}_P}{\overline{V}_S} = a = a_\phi$ for the phase voltages (i.e., coil pairs).

For coil set #1, $\overline{V}_{S1} = \overline{V}_{a'n'} = \dfrac{\overline{V}_{P1}}{a_\phi} = \dfrac{\overline{V}_{ab}}{a_\phi} = \dfrac{480\angle 0°}{0.34641}$ $\Rightarrow$ $\underline{\overline{V}_{a'n'} = 1385.64\angle 0°\ (V_{rms})}$.

For coil set #2, $\overline{V}_{S2} = \overline{V}_{b'n'} = \dfrac{\overline{V}_{P2}}{a_\phi} = \dfrac{\overline{V}_{bc}}{a_\phi} = \dfrac{480\angle 120°}{0.34641}$ $\Rightarrow$ $\underline{\overline{V}_{b'n'} = 1385.64\angle 120°\ (V_{rms})}$.

For coil set #3, $\overline{V}_{S3} = \overline{V}_{c'n'} = \dfrac{\overline{V}_{P3}}{a_\phi} = \dfrac{\overline{V}_{ca}}{a_\phi} = \dfrac{480\angle 240°}{0.34641}$ $\Rightarrow$ $\underline{\overline{V}_{c'n'} = 1385.64\angle 240°\ (V_{rms})}$.

3) cont.

By KVL on the secondary transformer circuit-

$$\bar{V}_{a'b'} = \bar{V}_{a'n'} - \bar{V}_{b'n'} = 1385.64\angle 0° - 1385.64\angle 120° \quad \Rightarrow \quad \underline{\bar{V}_{a'b'} = 2400\angle -30° \ (\mathrm{V_{rms}})},$$

$$\bar{V}_{b'c'} = \bar{V}_{b'n'} - \bar{V}_{c'n'} = 1385.64\angle 120° - 1385.64\angle 240° \quad \Rightarrow \quad \underline{\bar{V}_{b'c'} = 2400\angle 90° \ (\mathrm{V_{rms}})}, \text{ and}$$

$$\bar{V}_{c'a'} = \bar{V}_{c'n'} - \bar{V}_{a'n'} = 1385.64\angle 240° - 1385.64\angle 0°$$

$$\Rightarrow \underline{\bar{V}_{c'a'} = 2400\angle -150° \ (\mathrm{V_{rms}}) = 2400\angle 210° \ (\mathrm{V_{rms}})}.$$

$a_{LL} = \underline{\mathbf{0.2}}$ $a_{\phi} = \underline{\mathbf{0.3464}}$

$\bar{V}_{a'b'} = \underline{2400\angle -30° \ (\mathrm{V_{rms}})}$ $\bar{V}_{b'c'} = \underline{2400\angle 90° \ (\mathrm{V_{rms}})}$ $\bar{V}_{c'a'} = \underline{2400\angle 210° \ (\mathrm{V_{rms}})}$

$\bar{V}_{a'n'} = \underline{1385.64\angle 0° \ (\mathrm{V_{rms}})}$ $\bar{V}_{b'n'} = \underline{1385.64\angle 120° \ (\mathrm{V_{rms}})}$ $\bar{V}_{c'n'} = \underline{1385.64\angle 240° \ (\mathrm{V_{rms}})}$