EE 330 Energy Systems (Fall 2026) Examination 1

Name KEY A

Instructions: Show all work for full credit. Write answers in indicated places. Attach equation sheet to exam.

1) A 60 Hz, 80-kVA, 2400/230-V_{rms} step-down transformer has **per-unit** equivalent circuit parameters of $R_{series} = 0.02$, $X_{series} = 0.06$, $X_M = 34$, and $R_C = 68$. Using this information, calculate and sketch a fully-labeled approximate transformer equivalent circuit (with units) referred to the **primary** side. Find the turns ratio a. Next, a 72-kVA load with a power factor of 0.866 lagging is connected to the transformer. Assuming that the load is receiving the rated secondary voltage, determine the load current I_L, primary voltage V_P, and voltage regulation VR (use the equivalent circuit). **Extra credit**: Determine the core losses P_{core}, resistive wire losses P_{wire}, and efficiency η of the transformer.

Primary side: $V_{base} = 2400\ V_{rms}$ & $S_{base} = 80$ kVA.

Per (3-63), $Z_{base} = (V_{base})^2 / S_{base} = 2400^2 / 80{,}000 = 72\ \Omega$. Use Z_{base} to compute actual values

$R_{eq,series} = 0.02\, Z_{base} = 0.02\,(72)$ $\Rightarrow$ **$R_{eq,series} = 1.44\ \Omega$**,

$X_{eq,series} = 0.06\, Z_{base} = 0.06\,(72)$ $\Rightarrow$ **$X_{eq,series} = 4.32\ \Omega$**,

$X_M = 34\, Z_{base} = 34\,(72)$ $\Rightarrow$ **$X_M = 2448\ \Omega$**, and

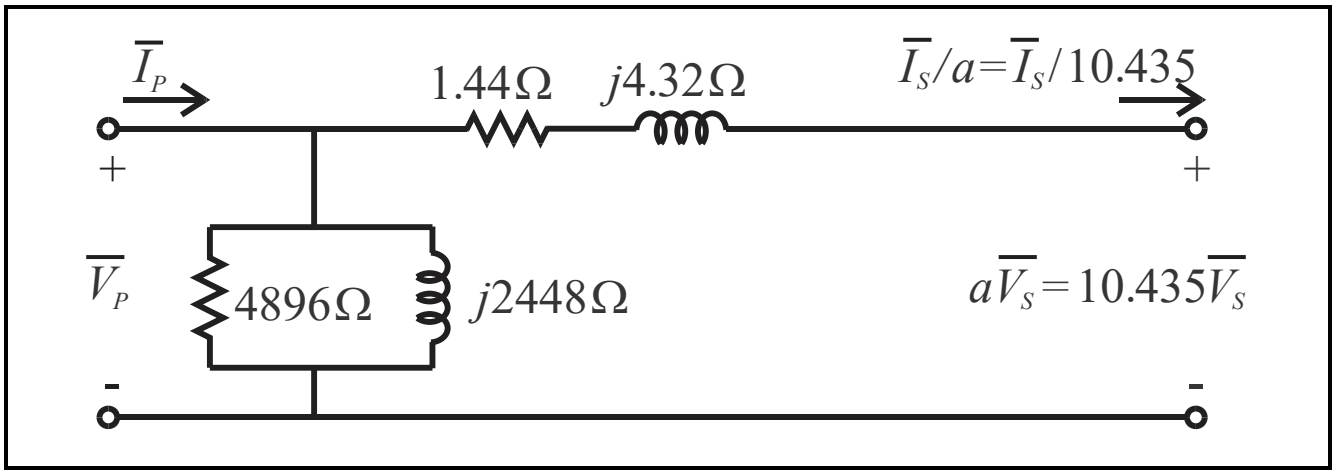
$R_C = 68\, Z_{base} = 68\,(72)$ $\Rightarrow$ **$R_C = 4896\ \Omega$**.

Per (3-5), $a = V_P / V_S = 2400 / 230$ $\Rightarrow$ **$a = 10.43478261$**.

$S_{load} = V_{load}\, I_{load} = V_S\, I_{load} = 72{,}000 = 230\, I_{load}$ $\Rightarrow$ **$I_{load} = I_L = 313.0435\ A_{rms}$.**

$PF_{load} = \cos(\theta_{load}) = \cos(\theta_{VL} - \theta_{IL}) = \cos(0.866) \Rightarrow$ **$\theta_{load} = 30.002911°$.**

Primary-side equivalent circuit



$a =$ **10.435**

1) continued

Assume $\theta_{VL} = 0$. Then, $\overline{V}_L = \overline{V}_S = 230\angle 0^\circ \ (\mathrm{V_{rms}})$.

Further, θ_{IL} = -30.002911° and $\overline{I}_L = \overline{I}_S = 313.0435\angle -30.0029^\circ \ (\mathrm{A_{rms}})$.

Using the turns ratio as shown in the primary-side equivalent circuit,

$a\overline{V}_S = 10.4347861(230\angle 0^\circ) = 2400\angle 0^\circ \ (\mathrm{V_{rms}})$ and

$\overline{I}_S / a = (313.435\angle -30.0029^\circ) / 10.4347861 = 30\angle -30.0029^\circ \ (\mathrm{A_{rms}})$.

Use KVL on the primary-side equivalent circuit to get

$$\overline{V}_P = (30\angle -30.0029^\circ)(1.44 + j4.32) + 2400\angle 0^\circ \ (\mathrm{V_{rms}})$$

$$\Rightarrow \underline{\overline{V}_P = 2503.8577\angle 2.0744^\circ \ (\mathrm{V_{rms}})}.$$

Per (3-68), the voltage regulation is

$$\mathrm{VR} = \frac{V_P / a - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\% = \frac{V_P - aV_{S,\mathrm{fl}}}{aV_{S,\mathrm{fl}}} \times 100\% = \frac{2503.858 - 2400}{2400} \times 100\%$$

$$\Rightarrow \underline{\mathbf{VR = 4.3274\%}}.$$

Extra Credit:

$P_{\mathrm{core}} = V_P^2 / R_C = \ = 2503.858^2 / 4896$ $\Rightarrow \underline{\boldsymbol{P_{\mathrm{core}} = 1280.495\ \mathrm{W}}}$.

$P_{\mathrm{wire}} = (I_S / a)^2 R_{eq,series} = \ = 30^2 (1.44)$ $\Rightarrow \underline{\boldsymbol{P_{\mathrm{wire}} = 1296\ \mathrm{W}}}$.

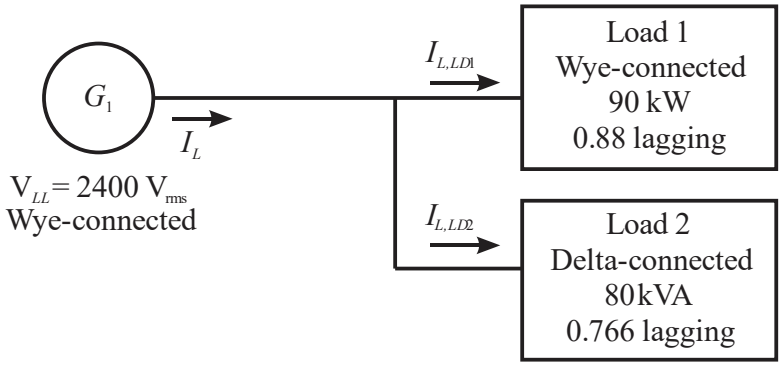
Per (3-72), the efficiency is

$$\eta = \frac{P_{\mathrm{out}}}{P_{\mathrm{out}} + P_{\mathrm{losses}}} \times 100\% = \frac{72{,}000(0.866)}{72{,}000(0.866) + 1280.495 + 1296} \times 100\% \ \Rightarrow \ \underline{\boldsymbol{\eta = 96.03\%}}.$$

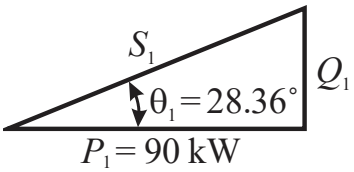
$I_L = \underline{\mathbf{313.0435\ A_{rms}}}$ $V_P = \underline{\mathbf{2503.858\ V_{rms}}}$ $VR = \underline{\mathbf{4.327\%}}$

Extra credit: $P_{\mathrm{core}} = \underline{\mathbf{1280.495\ W}}$ $P_{\mathrm{wire}} = \underline{\mathbf{1296\ W}}$ $\eta = \underline{\mathbf{96.03\%}}$

2) A one-line diagram of a 60 Hz, 3-ϕ system is shown below. For load 1, find the reactive power Q_1, phase voltage $V_{\phi 1}$, phase current $I_{\phi 1}$, and line current $I_{L,LD1}$. For load 2, find the real power P_2, reactive power Q_2, phase current $I_{\phi 2}$, and line current $I_{L,LD2}$. Then, find the apparent power S, real power P, reactive power Q, power factor PF, phase current I_ϕ, and line current I_L supplied by the generator.



Load 1 (Y-connected)



$\theta_1 = \cos^{-1}(0.88) = +28.3576°$

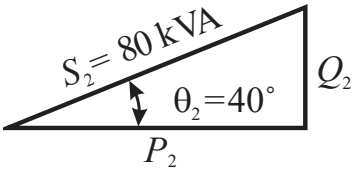
$Q_1 = P_1 \tan(\theta_1) = \ = 90 \tan(28.3576°) \Rightarrow$ **$Q_1 = 48.576854$ kvar**

Per (2-11), $V_{LL} = \sqrt{3}\, V_\phi \Rightarrow V_{\phi 1} = 2400 / \sqrt{3}$ $\Rightarrow$ **$V_{\phi 1} = 1385.64065$ V_{rms}**

Per (2-29), $P = \sqrt{3}\, V_{LL}\, I_L \cos(\theta) \Rightarrow I_{L,LD1} = \dfrac{90 \times 10^3}{\sqrt{3}\,(2400)\,0.88}$ $\Rightarrow$ **$I_{L,LD1} = 24.60299$ A_{rms}**

Per (2-10), $I_L = I_\phi$ for Y-connection. $\Rightarrow$ **$I_{\phi 1} = 24.60299$ A_{rms}**

Load 2 (Δ-connected)



$\theta_2 = \cos^{-1}(0.766) = +40.004°$

$P_2 = S_2 \cos(\theta_2) = \ = 80\,(0.766) \Rightarrow$ **$P_2 = 61.28$ kW**

$Q_2 = S_2 \sin(\theta_2) = \ = 80 \tan(40.004°) \Rightarrow$ **$Q_2 = 51.427246$ kvar**

Per (2-31), $S = \sqrt{3}\, V_{LL} I_L \Rightarrow I_{L,LD2} = \dfrac{80{,}000}{\sqrt{3}\,(2400)}$ $\Rightarrow$ **$I_{L,LD2} = 19.245009$ A_{rms}**

Per (2-29), $P = \sqrt{3}\, V_{LL}\, I_L \cos(\theta) \Rightarrow I_{L,LD1} = \dfrac{90 \times 10^3}{\sqrt{3}\,(2400)\,0.88}$ $\Rightarrow$ **$I_{L,LD1} = 24.60299$ A_{rms}**

Per (2-15), $I_L = \sqrt{3}\, I_\phi \Rightarrow I_{\phi 2} = 19.245009 / \sqrt{3}$ for Y-connection. $\Rightarrow$ **$I_{\phi 2} = 11.1111$ A_{rms}**

$Q_1 =$ **48.577 kvar** $V_{\phi 1} =$ **1385.64 V_{rms}** $I_{\phi 1} =$ **24.603 A_{rms}** $I_{L,LD1} =$ **24.603 A_{rms}**

$P_2 =$ **61.28 kW** $Q_2 =$ **51.427 kvar** $I_{\phi 2} =$ **11.111 A_{rms}** $I_{L,LD2} =$ **19.245 A_{rms}**

2) continued

Y-connected generator

By conservation of complex power,

$$\overline{S}_{\text{gen}} = \overline{S}_1 + \overline{S}_2 = (90 + j48.577) + (61.28 + j51.427)$$

$$\Rightarrow \overline{S}_{\text{gen}} = P + jQ = 151.28 + j100.0041 \text{ (kVA)}$$

$$\Rightarrow \overline{S}_{\text{gen}} = S\angle\theta = 181.34624\angle 33.4668^\circ \text{ (kVA)}$$

PF = $\cos(\theta_S) = \cos(33.4668^\circ) = P/S = 151.28/181.3462$ $\Rightarrow$ **PF = 0.8342 lagging**

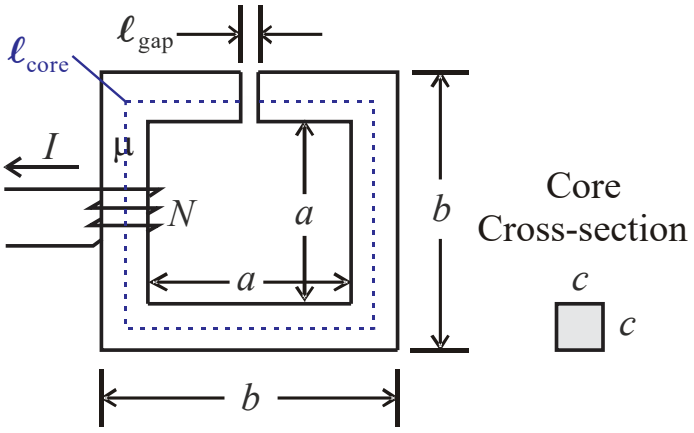
Per (2-31), $S = \sqrt{3}\, V_{LL} I_L \Rightarrow I_L = \dfrac{181{,}346.24}{\sqrt{3}\,(2400)}$ $\Rightarrow$ $\boldsymbol{I_L = 43.625\ A_{rms}}$

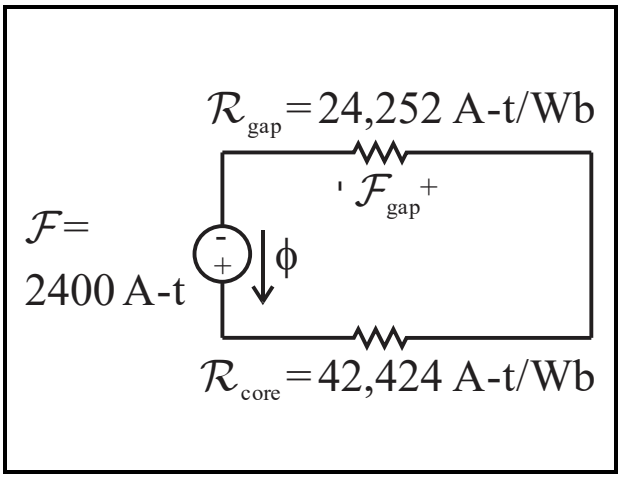
Per (2-10), $I_L = I_\phi$ for Y-connection. $\Rightarrow$ $\boldsymbol{I_{\phi 1} = 43.625\ A_{rms}}$

S = **181.346 kVA** P = **151.28 kW** Q = **100.004 kvar**

PF = **0.8342 lagging** I_ϕ = **43.625 A_{rms}** I_L = **43.625 A_{rms}**

3) For the magnetic core shown, sketch & label the equivalent magnetic circuit w/ the magnetomotive force (*mmf*) source and core & air gap reluctances shown inside box. Find the equivalent reluctance "seen" by the coil and magnitude of the magnetic flux circulating through the core (indicate its direction on sketch). Determine the drop in *mmf* across the air gap and magnitude of the magnetic field in the air gap. Given: $N = 160$, $I = 15$ A, $a = 100$ cm, $b = 150$ cm, $c = 25$ cm, $\mu = 1500\mu_0$, and $\ell_{gap} = 2$ mm. Assume the magnetic field is evenly distributed across the cross-section of the core, the coil is tightly wound, and that fringing increases the effective cross-sectional area of the air gap by 5%.





Core & gap areas: $A_{core} = c^2 = 0.25^2 = 0.0625\ \text{m}^2$ & $A_{gap} = 1.05c^2 = 0.065625\ \text{m}^2$

Core mean path length: $\ell_{core} = 4\,(a + c/2 + c/2) - \ell_{gap} = 4\,(1 + 0.25) - 0.002 = 4.998$ m.

Calculate reluctances, using (1-32) $\mathcal{R} = \ell / \mu A$, as:

$\mathcal{R}_{core} = \ell_{core} / \mu A_{core} = 4.998 / [1500(4\pi \cdot 10^{-7})0.0625] \Rightarrow \mathcal{R}_{core} = 42{,}424.34163$ (A-t/Wb),

$\mathcal{R}_{gap} = \ell_{gap} / \mu A_{gap} = 0.002 / [(4\pi \cdot 10^{-7})0.065625] \Rightarrow \mathcal{R}_{core} = 24{,}252.1818$ (A-t/Wb) &

(1-33) $\mathcal{R}_{eq} = \mathcal{R}_{core} + \mathcal{R}_{gap} = 42{,}424.34163 + 24{,}252.1818 \Rightarrow$ **$\mathcal{R}_{eq} = 66{,}676.52$ (A-t/Wb)**.

Calculate coil mmf, using (1-27) $\mathcal{F} = Ni = 160(15) \Rightarrow \mathcal{F} = 2400$ A-t.

Per (1-28), find magnetic flux $\phi = \mathcal{F} / \mathcal{R}_{eq} = 2400 / 66{,}676.52 \Rightarrow$ **$\phi = 35.995$ mWb**.

Calculate gap mmf, $\mathcal{F}_{gap} = \phi\,\mathcal{R}_{gap} = 0.035995\ (42{,}424.34) \Rightarrow$ **$\mathcal{F}_{gap} = 872.9495$ A-t**.

$H_{gap} = \mathcal{F}_{gap} / \ell_{gap} = 872.95 / 0.002 = B_{gap} / \mu_0 = (\phi / A_{gap}) / \mu_0 \Rightarrow$ **$H_{gap} = 436{,}474.76$ A/m**.

equiv. reluctance = **66,676.52 (A-t/Wb)** |magnetic flux| = **$\phi = 35.995$ mWb**

mmf drop across air gap = **$\mathcal{F}_{gap} = 872.95$ A-t** |magnetic field in air gap| = **$H_{gap} = 436{,}474.8$ A/m**

EE 330 Energy Systems (Fall 2026) Examination 1R

Name KEY R

Instructions: Show all work for full credit. Write answers in indicated places. Insert equation sheet(s) in exam. Worth up to 30 points. Will replace worst scoring exam 1 problem.

1) A 60 Hz, 16-kVA, 480/200-V_{rms} step-down single-phase transformer produced by Acme, Corporation has **per-unit** equivalent circuit parameters of $R_{eq,pu}$ = 1.6%, $X_{eq,pu}$ = 8%, $X_{M,pu}$ = 32, and $R_{C,pu}$ = 100. Using this information, calculate the secondary-side base impedance $Z_{base,S}$ and turns ratio a. Then, calculate the secondary-side core resistance $R_{C,S}$, magnetizing reactance $X_{M,S}$, and series equivalent resistance $R_{eq,S}$ & reactance $X_{eq,S}$. Sketch a fully-labeled approximate transformer equivalent circuit (w/ units) referred to **secondary** side.

Secondary side: $V_{base} = 200\ V_{rms}$ & $S_{base} = 16$ kVA.

Per (3-63), $Z_{base} = (V_{base})^2 / S_{base} = 200^2 / 16{,}000 =$ **2.5 Ω**.

Per (3-5), $a = V_P / V_S = 480 / 200$ ⇒ $a = 2.4$.

Use Z_{base} to compute actual values for secondary side equivalent transformer circuit-

$R_{eq,series} = 0.016\, Z_{base} = 0.016\,(2.5)$ ⇒ $R_{eq,series} = 0.04\ \Omega$,

$X_{eq,series} = 0.08\, Z_{base} = 0.08\,(2.5)$ ⇒ $X_{eq,series} = 0.2\ \Omega$,

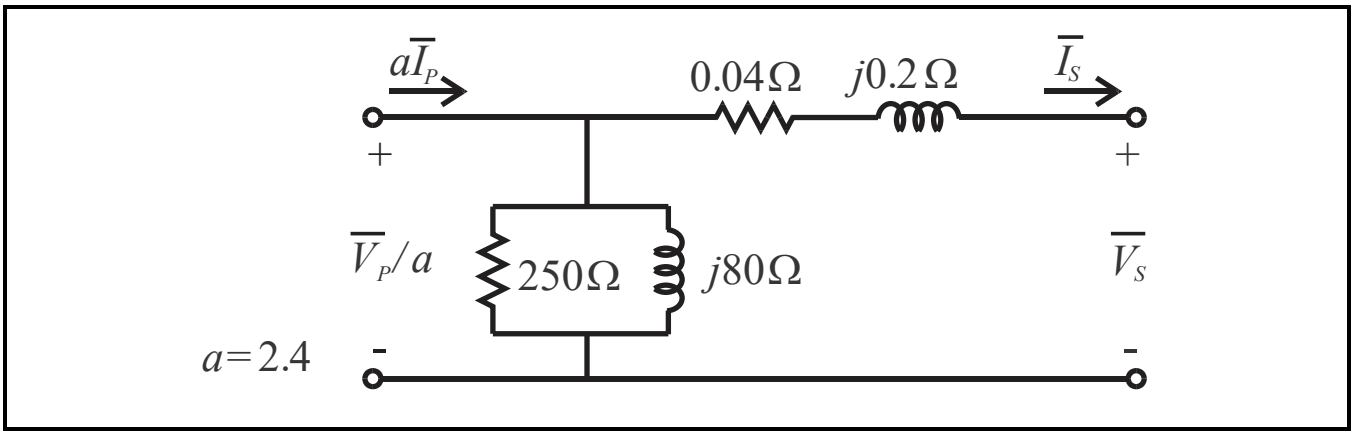
$X_M = 32\, Z_{base} = 32\,(2.5)$ ⇒ $X_M = 80\ \Omega$, and

$R_C = 100\, Z_{base} = 100\,(2.5)$ ⇒ $R_C = 250\ \Omega$.

$Z_{base,S} =$ **2.5 Ω** $a =$ **2.4** $R_{C,S} =$ **250 Ω**

$X_{M,S} =$ **80 Ω** $R_{eq,S} =$ **0.04 Ω** $X_{eq,S} =$ **0.2 Ω**

Secondary-side equivalent circuit (see Figure 3-18b in text)



1) continued Next, a 14-kVA load with a power factor of 0.5 lagging is connected to the transformer. Assuming the load is receiving the rated secondary voltage at a phase angle of 0°, find the RMS phasor load current $\bar{I}_L = \bar{I}_S$ and voltage $\bar{V}_L = \bar{V}_S$, referred RMS phasor primary voltage $\bar{V}_P / a$, and voltage regulation VR.

Rated secondary voltage is 200 V_{rms} w/ phase angle of 0 $\Rightarrow$ $\underline{\bar{V}_L = \bar{V}_S = 200\angle 0°\ (V_{rms})}$.

$S_{load} = V_{load} I_{load} = V_S I_{load} = 14{,}000 = 200\, I_{load} \Rightarrow \underline{I_{load} = I_L = 70\ A_{rms}}$.

$PF_{load} = \cos(\theta_{load}) = \cos(\theta_{VL} - \theta_{IL}) = \cos(0.5) \Rightarrow \underline{\theta_{load} = 60°}$.

Since $\theta_{VL} = 0$, $\theta_{IL} = -60°$ $\Rightarrow$ $\underline{\bar{I}_L = \bar{I}_S = 70\angle -60°\ (A_{rms})}$.

Use KVL on the secondary-side equivalent circuit to get

$$\bar{V}_P / a = \bar{I}_S (R_{eq,S} + jX_{eq,S}) + \bar{V}_S$$
$$= (70\angle -60°)(0.04 + j0.2) + 200\angle 0°\ (V_{rms})$$
$$\Rightarrow \underline{\bar{V}_P / a = 213.573365\angle 1.2275°\ (V_{rms})}.$$

Per (3-68), the voltage regulation is

$$VR = \frac{V_P / a - V_{S,fl}}{V_{S,fl}} \times 100\% = \frac{213.57 - 200}{200} \times 100\% \Rightarrow \underline{\mathbf{VR = 6.78668\%}}.$$

$\bar{I}_L = \bar{I}_S = \underline{\mathbf{70\angle -60°\ (A_{rms})}}$ $\bar{V}_L = \bar{V}_S = \underline{\mathbf{200\angle 0°\ (V_{rms})}}$

$\bar{V}_P / a = \underline{\mathbf{213.57\angle 1.23°\ (V_{rms})}}$ $VR = \underline{\mathbf{6.79\%}}$