

example- Transformer VR & Efficiency using Secondary Equiv. Circuit

A transformer is rated $2300 \text{ V}_{\text{rms}} : 208 \text{ V}_{\text{rms}}$ and 45 kVA at 60 Hz . It has per-unit equivalent circuit parameters- $R_{\text{eq}} = 0.02$, $X_{\text{eq}} = 0.04$, $R_C = 90$, & $X_M = 20$. Determine the secondary equivalent circuit. Then, find the voltage regulation (VR) and efficiency for loads of: a) 40 kVA w/ 0.9 leading PF, b) 40 kVA w/ a unity PF, and c) 38 kVA w/ 0.65 lagging PF.

Per (3-5), calculate turns ratio $a = V_P / V_S = 2300/208 \Rightarrow \underline{a = 11.0577}$.

Choose: $S_{\text{base}} = 45 \text{ kVA}$ and $V_{\text{base}} = 208 \text{ V}_{\text{rms}}$ (secondary side).

Then: $I_{\text{base}} = S_{\text{base}} / V_{\text{base}} = 45,000 / 208 \Rightarrow \underline{I_{\text{base}} = 216.346 \text{ A}_{\text{rms}}}$.

Per (3-61), $Z_{\text{base}} = V_{\text{base}} / I_{\text{base}} = 208 / 216.346 \Rightarrow \underline{Z_{\text{base}} = 0.9614222 \Omega}$.

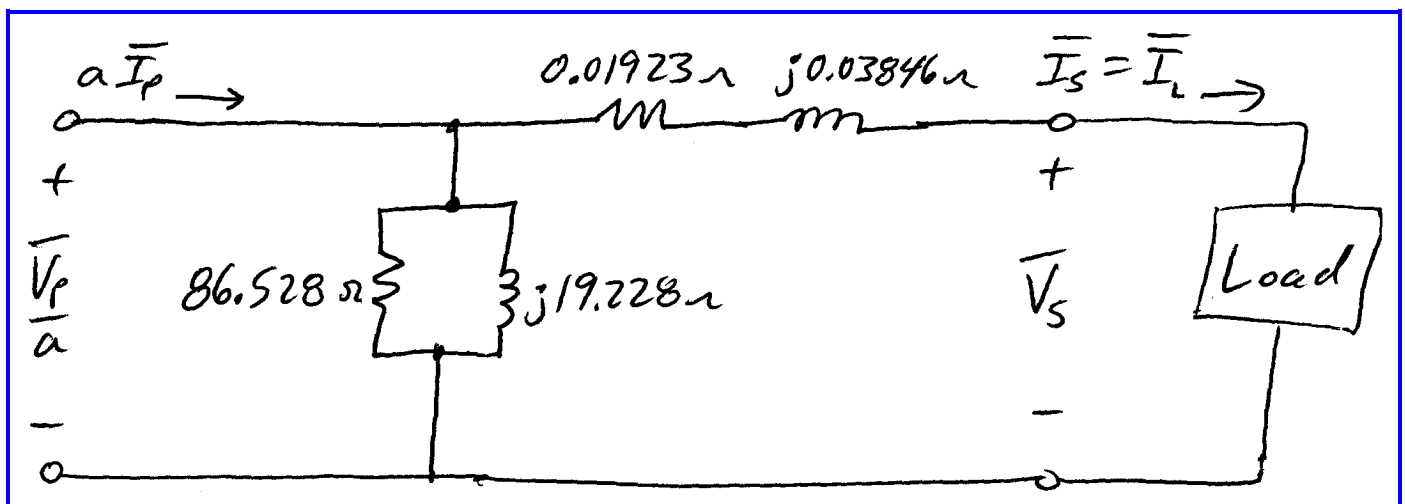
The secondary equivalent circuit values are:

$$R_{C,S} = R_{C,pu} (Z_{\text{base}}) = 90 (0.9614222) \Rightarrow \underline{R_{C,S} = 86.528 \Omega},$$

$$X_{M,S} = X_{M,pu} (Z_{\text{base}}) = 20 (0.9614222) \Rightarrow \underline{X_{M,S} = 19.2284 \Omega},$$

$$R_{eq,S} = R_{eq,pu} (Z_{\text{base}}) = 0.02 (0.9614222) \Rightarrow \underline{R_{eq,S} = 0.019228 \Omega},$$

$$\text{and } X_{eq,S} = X_{eq,pu} (Z_{\text{base}}) = 0.04 (0.9614222) \Rightarrow \underline{X_{eq,S,pu} = 0.038457 \Omega}.$$

Secondary Equivalent Circuit

a) Given $S_a = 40$ kVA and $PF_a = 0.9$ leading. Let $\bar{V}_S = 208\angle 0^\circ$ (V_{rms}).

Then, $I_S = I_L = S_L / V_S = 40,000 / 208 = 192.3077$ A_{rms}.

Using the power factor, $\theta_a = \theta_V - \theta_I = \cos^{-1}(0.9) = -25.842^\circ \Rightarrow \theta_I = 25.842^\circ$.

This yields $\bar{I}_S = 192.3077\angle 25.842^\circ$ (A_{rms}).

$$\frac{\bar{V}_P}{a} = \bar{I}_S(0.01923 + j0.03846) + \bar{V}_S$$

$$\begin{aligned} \text{By KVL, } &= 192.3\angle 25.842^\circ (0.01923 + j0.03846) + 208\angle 0^\circ. \\ &= 208.2685\angle 2.275^\circ \text{ (V}_{\text{rms}}) \end{aligned}$$

Per (3-68), the voltage regulation is

$$VR_a = \frac{\frac{V_P}{a} - V_S}{V_S} \times 100\% = \frac{208.2685 - 208}{208} \times 100\% \Rightarrow \underline{VR_a = 0.129\%}.$$

The output power is $P_{\text{out},a} = S_a (PF_a) = 40,000(0.9) \Rightarrow \underline{P_{\text{out},a} = 36,000 \text{ W}}.$

$$\text{The core losses are } P_{\text{core},a} = \frac{\left(\frac{V_P}{a}\right)^2}{R_{C,S}} = \frac{208.2685^2}{86.528} \Rightarrow \underline{P_{\text{core},a} = 501.29 \text{ W}}.$$

$$\begin{aligned} \text{The wire/copper losses are } P_{\text{Cu}} &= I_S^2 R_{eq,S} = 192.31^2 (0.01923) \\ &\Rightarrow \underline{P_{\text{Cu},a} = 711.1 \text{ W}}. \end{aligned}$$

Per (3-72), the efficiency is

$$\eta_a = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}} \times 100\% = \frac{36,000 \times 100\%}{36,000 + (711.11 + 501.29)} \Rightarrow \underline{\eta_a = 96.74\%}.$$

b) Given $S_b = 45 \text{ kVA}$ and $\text{PF}_b = 1$. Let $\bar{V}_S = 208\angle 0^\circ \text{ (V}_{\text{rms}})$.

Then, $I_S = I_L = S_L / V_S = 45,000 / 208 = 216.346 \text{ A}_{\text{rms}}$.

Using the power factor, $\theta_b = \theta_V - \theta_I = \cos^{-1}(1) = 0^\circ \Rightarrow \theta_I = 0^\circ$.

So, $\bar{I}_S = 216.346\angle 0^\circ \text{ (A}_{\text{rms}})$.

$$\begin{aligned} \text{By KVL, } \frac{\bar{V}_P}{a} &= 216.346\angle 0^\circ (0.01923 + j0.03846) + 208\angle 0^\circ \\ &= 212.323\angle 2.246^\circ \text{ (V}_{\text{rms}}) \end{aligned}$$

Per (3-68), the voltage regulation is

$$VR_b = \frac{\frac{V_P}{a} - V_S}{V_S} \times 100\% = \frac{212.323 - 208}{208} \times 100\% \Rightarrow \underline{VR_b = 2.08\%}.$$

The output power is $P_{\text{out},b} = S_b (\text{PF}_b) = 45,000(1) \Rightarrow \underline{P_{\text{out},b} = 45,000 \text{ W}}$.

$$\text{The core losses are } P_{\text{core},b} = \frac{\left(\frac{V_P}{a}\right)^2}{R_{C,S}} = \frac{212.323^2}{86.528} \Rightarrow \underline{P_{\text{core},b} = 521 \text{ W}}.$$

The wire/copper losses are $P_{\text{Cu}} = I_S^2 R_{eq,S} = 216.35^2 (0.01923) \Rightarrow \underline{P_{\text{Cu},b} = 900 \text{ W}}$.

Per (3-72), the efficiency is

$$\eta_b = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}} \times 100\% = \frac{45,000 \times 100\%}{45,000 + (521 + 900)} \Rightarrow \underline{\eta_b = 96.94\%}.$$

c) Given $S_c = 38$ kVA and $PF_c = 0.65$ lagging. Let $\bar{V}_s = 208\angle 0^\circ$ (V_{rms}).

Then, $I_s = I_L = S_L / V_s = 38,000 / 208 = 182.6923$ A_{rms}.

Using power factor, $\theta_c = \theta_V - \theta_I = \cos^{-1}(0.9) = 49.4584^\circ \Rightarrow \theta_I = -49.4584^\circ$.

So, $\bar{I}_s = 182.6923\angle -49.4584^\circ$ (A_{rms}).

$$\begin{aligned} \text{By KVL, } \frac{\bar{V}_p}{a} &= 182.6923\angle -49.4584^\circ (0.01923 + j0.03846) + 208\angle 0^\circ \\ &= 215.63085\angle 0.504^\circ \text{ (V}_{rms}\text{)} \end{aligned}$$

Per (3-68), the voltage regulation is

$$VR_c = \frac{\frac{V_p}{a} - V_s}{V_s} \times 100\% = \frac{215.63 - 208}{208} \times 100\% \Rightarrow \underline{VR_c = 3.67\%}.$$

The output power is $P_{out,c} = S_c (PF_c) = 38,000(0.65) \Rightarrow \underline{P_{out,c} = 24,700 \text{ W}}.$

$$\text{The core losses are } P_{core,c} = \frac{\left(\frac{V_p}{a}\right)^2}{R_{C,S}} = \frac{215.63^2}{86.528} \Rightarrow \underline{P_{core,c} = 537.36 \text{ W}}.$$

$$\begin{aligned} \text{The copper losses are } P_{Cu} &= I_s^2 R_{eq,S} = 182.6923^2 (0.01923) \\ &\Rightarrow \underline{P_{Cu,c} = 641.78 \text{ W}}. \end{aligned}$$

Per (3-72), the efficiency is

$$\eta_c = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\% = \frac{24,700 \times 100\%}{24,700 + (641.78 + 537.36)} \Rightarrow \underline{\eta_c = 95.4\%}.$$