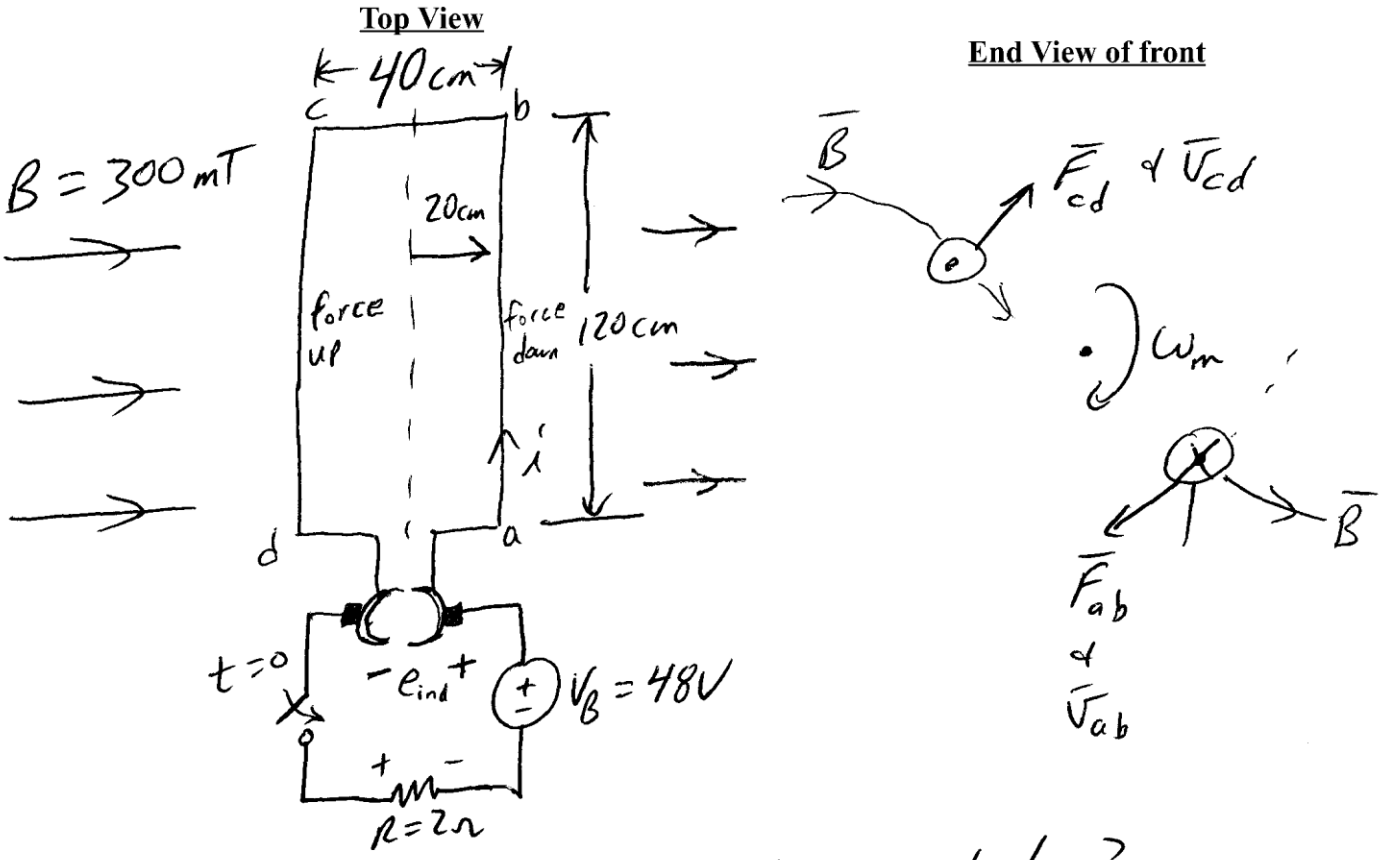
Example- Simple DC Loop



a) Which way will the loop rotate?

⇒ Current flow CCW when V_B applied

By $\bar{F} = i(\bar{l} \times \bar{B})$, the forces are as shown

and the loop rotates CW

b) Find polarity of e_{ind}

Use $e_{ind} = (\bar{v} \times \bar{B}) \cdot \bar{l}$ use RHR

$e_{ab} = v_{ab} B l$ w/ + toward a

$e_{cd} = v_{cd} B l$ w/ + toward c

} e_{ind} polarity shown

ex. cont.

c) Find torque & current @ $t = 0^+$ (assume loop @ rest)

By KVL $\quad V_B = e_{ind} + iR$

($e_{ind} = 0$ (v & $\omega_m = 0$))

$$i = \frac{V_B - e_{ind}}{R} = \frac{48}{2} = \underline{24\,A = i(0^+)}$$

$$\tau_{ind}(0^+) = 2rilB = 2(0.20)\,24\,(1.20)(0.3)$$

$$\underline{\tau_{ind}(0^+) = 3.456\ N\cdot m} \quad (CW)$$

d) Find torque, current, e_{ind}, ω_m, P_{out}, P_{in} at steady-state w/ $\tau_{fric\ windage} = 0.5\ N\cdot m$

At steady-state, $\underline{\tau_{ind_{ss}} = 0.5\ N\cdot m}$ (else v & ω_m wouldn't be constant)

$$i_{ss} = \frac{\tau_{ind_{ss}}}{2rlB} = \frac{0.5}{2(0.2)(1.2)0.3} = \underline{3.47\overline{2}\,A}$$

By KVL, $\quad V_{B_{ss}} = e_{ind_{ss}} + i_{ss}R$

$$48 = e_{ind_{ss}} + (3.47\overline{2})(2)$$

$$\hookrightarrow \underline{e_{ind_{ss}} = 41.05\overline{5}\ V}$$

ex. cont

$$e_{ind_{ss}} = 2rlB\omega_{m,ss}$$

$$\hookrightarrow \omega_{m,ss} = \frac{41.055}{2(0.2)(1.2)(0.3)} = \underline{285.108 \text{ rad/s}}$$

$$n_{m,ss} = \omega_{m,ss}\left(\frac{30}{\pi}\right) = \underline{2722.58 \text{ RPM}}$$

$$P_{out} = \tau_{ss}\omega_{m_{ss}} = 0.5(285.108) = \underline{142.554 \text{ W}}$$

$$P_{in} = V_B i = (48)(3.472) = 166.6\overline{6} \text{ W}$$

$$P_{in} > P_{out} \quad (\text{Motor})$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{142.554}{166.6} \times 100\%$$

$$\underline{\eta = 85.5\%}$$