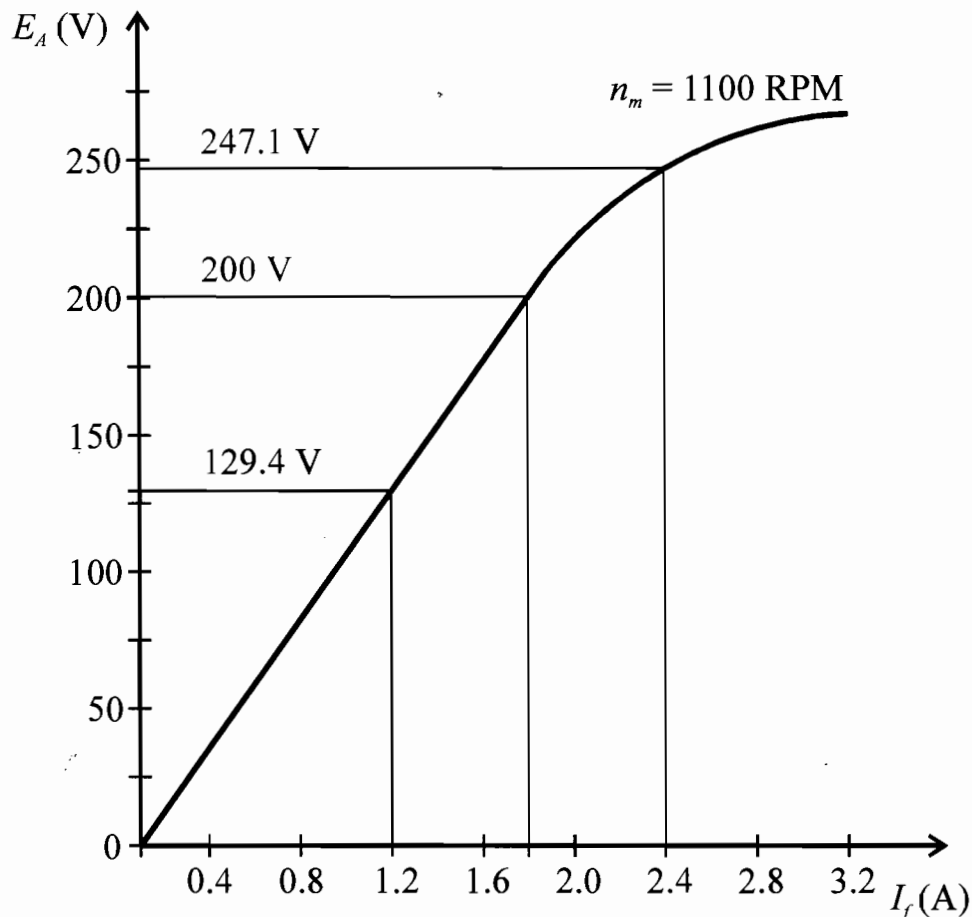
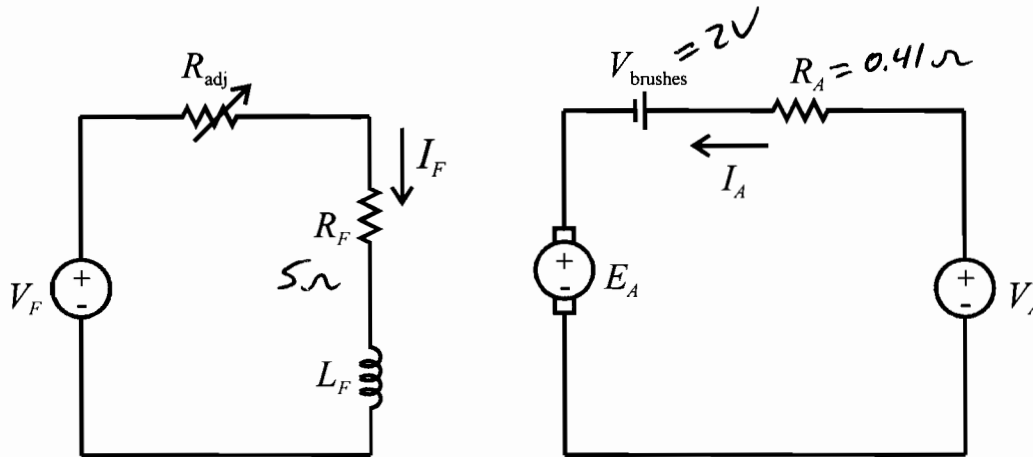


A 220 V, 10 hp separately-excited DC motor has the equivalent circuit and magnetization curve shown. It has a rated field current of 1.8 A, rated armature current of 44 A, and rated speed of 1100 RPM. The field voltage is fixed at 24 V and the field coils have a fixed resistance of $5\ \Omega$ in series with an adjustable resistor (0 to $300\ \Omega$). For the armature, the armature resistance is $0.41\ \Omega$, the voltage drop across the brushes is a fixed value of 2 V, and the armature voltage can be varied from 0 to 260 V. The core ($\propto n_m^{1.5}$) and mechanical ($\propto n_m^3$) losses are 450 W and 400 W respectively at full load.



a) Determine the various losses, torques, and overall efficiency of the motor at full-load.

Field Circuit

$$P_{\text{Field}} = V_F I_{F, \text{rated}} = 24(1.8) = \underline{43.2 \text{ W}} \leftarrow \text{supplied}$$

$$R_{\text{adj}} = ? \Rightarrow I_{F, \text{rated}} = 1.8 \text{ A} = \frac{V_F}{R_{\text{adj}} + R_F} = \frac{24}{R_{\text{adj}} + 5} \Rightarrow \underline{R_{\text{adj}} = 8.3 \Omega}$$

$$P_{R_F} = I_{F, \text{rated}}^2 R_F = 1.8^2(5) = \underline{16.2 \text{ W}}$$

$$P_{R_{\text{adj}}} = I_{F, \text{rated}}^2 R_{\text{adj}} = 1.8^2(8.3) = \underline{27 \text{ W}}$$

Armature

$$P_{\text{Armature}} = V_{A, \text{rated}} I_{A, \text{rated}} = 220(44) = \underline{9680 \text{ W}}$$

$$P_{R_A} = I_{A, \text{rated}}^2 R_A = 44^2(0.41) = \underline{794 \text{ W}}$$

$$P_{\text{Brushes}} = V_{\text{brushes}} I_{A, \text{rated}} = 2(44) = \underline{88 \text{ W}}$$

$$P_{\text{conv}} = E_{A, \text{rated}} I_{A, \text{rated}} = 200(44) = \underline{8800 \text{ W}}$$

^ From mag. curve

$$\text{Check } P_{\text{conv}} \stackrel{?}{=} P_{\text{Armature}} - P_{R_A} - P_{\text{Brushes}} = 9680 - 794 - 88 = \underline{8798 \text{ W}}$$

w/in round-off error

$$P_{\text{out}} = 10 \text{ hp} \left(\frac{746 \text{ W}}{1 \text{ hp}} \right) = \underline{7460 \text{ W}}$$

$$7460 = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{stray}} = 8800 - 400 - 450 - P_{\text{stray}}$$

$$\hookrightarrow \underline{P_{\text{stray}} = 490 \text{ W}}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{7460}{9680 + 43.2} \times 100\% = \underline{76.7\%}$$

↑ armature ↑ field

- b) Over what range of speeds will the motor vary when the adjustable resistor is changed to give field current of 1.2 A and 2.4 A? Assume the armature voltage is held constant at 220 V as is the load on the motor.

From the magnetization curve, the induced emf is 129.4V when $I_f = 1.2\text{ A}$ and $n_m = 1100\text{ RPM}$.

Per (8-13), $\frac{E_A}{E_{A0}} = \frac{n_m}{n_0}$. To apply this equation, we need the induced emf @ full-load. By KVL,

$$\begin{aligned} E_{A,FL} &= V_A - I_A R_A - V_{\text{brushes}} \\ &= 220 - 44(0.41) - 2 = 199.96\text{ V} \sim 200\text{ V} \end{aligned}$$

(Note: $I_f = 1.8\text{ A}$ for this case.)

$$\text{For } I_f = 1.2\text{ A}, \quad n_m = n_0 \left(\frac{E_A}{E_{A0}} \right) = 1100 \left(\frac{200}{129.4} \right) = \underline{\underline{1700\text{ RPM}}}$$

For an $I_f = 2.4\text{ A}$, the induced emf @ 1100 RPM would be 247.1 V from the magnetization curve. Therefore, the related speed will be

$$n_m = 1100 \left(\frac{200}{247.1} \right) = \underline{\underline{890\text{ RPM}}}$$

- c) At rated field current, determine the no-load speed of the motor when the armature voltage is: a) 50 V, b) 100 V, c) 150 V, d) 200 V, & e) 260 V. Neglect losses.

W/out losses, at no-load, $T_{ind} = K\phi I_A = 0 \Rightarrow I_A = 0$

By KVL, $V_A = E_A + V_{brushes} + I_A R_A$

$$\hookrightarrow E_{A,NL} = V_A - V_{brush} - \overset{0}{I_A} R_A$$

From the magnetization curve, $E_{A0} = 200V$ w/ $I_{f, rated} = 1.8A$ at speed $n_0 = 1100 \text{ RPM}$.

a) $V_A = 50V \Rightarrow E_{A,NL,a} = 50 - 2 = 48V$

Per (8-13) $n_{m,a} = n_0 \frac{E_A}{E_{A0}} = 1100 \left(\frac{48}{200} \right) = \underline{264 \text{ RPM}}$

b) $V_A = 100V \Rightarrow E_{A,NL,b} = 100 - 2 = 98V$

$$n_{m,b} = 1100 \left(\frac{98}{200} \right) = \underline{539 \text{ RPM}}$$

c) $V_A = 150V \Rightarrow E_{A,NL,c} = 150 - 2 = 148V$

$$n_{m,c} = 1100 \left(\frac{148}{200} \right) = \underline{814 \text{ RPM}}$$

d) $V_A = 200V \Rightarrow E_{A,NL,d} = 200 - 2 = 198V$

$$n_{m,d} = 1100 \left(\frac{198}{200} \right) = \underline{1089 \text{ RPM}}$$

e) $V_A = 260V \Rightarrow E_{A,NL,e} = 260 - 2 = 258V$

$$n_{m,e} = 1100 \left(\frac{258}{200} \right) = \underline{\underline{1419 \text{ RPM}}}$$

- d) Estimate the no-load speed (neglect losses) and speed regulation for this motor at rated conditions.

From problem description, $V_A = 220V$, $I_f = 1.8A$

$$\text{Using the results of part c), } E_{A_{NL}} = V_A - V_{\text{brush}} - I_A R_A \\ = 220 - 2 = \underline{218V}$$

and $E_{A_0} = 200V$ at $n_0 = 1100 \text{ RPM}$ w/ $I_f = 1.8A$.

$$n_{NL} = n_0 \frac{E_A}{E_{A_0}} = 1100 \left(\frac{218}{200} \right) = \underline{1199 \text{ RPM}}$$

$$\text{For a full-load, } E_{A_{FL}} = V_A - V_{\text{brush}} - I_A R_A \\ = 220 - 2 - 44(0.41) = 200V$$

$$\hookrightarrow n_{FL} = 1100 \frac{200}{200} = \underline{1100 \text{ RPM}}$$

Per (8-2)

$$SR = \frac{n_{NL} - n_{FL}}{n_{FL}} \times 100 \% \\ = \frac{1199 - 1100}{1100} \times 100 \%$$

$$\underline{SR = 9 \%}$$

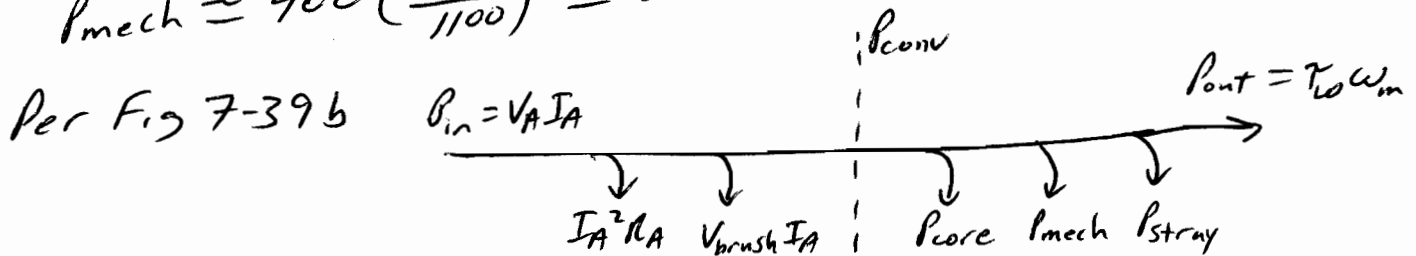
- e) Estimate the no-load speed, armature current, and speed regulation for this motor at rated conditions with losses taken into consideration. You may assume that the stray losses are approximately constant.

From problem description, $P_{core} = 450 \text{ W @ } 1100 \text{ RPM}$
 $P_{mech} = 400 \text{ W @ } 1100 \text{ RPM}$

Using the part d) $n_{NL} = 1199 \text{ RPM}$ speed estimate

$$P_{core} \approx 450 \left(\frac{1199}{1100} \right)^{1.5} \approx 512.1 \text{ W}$$

$$P_{mech} \approx 400 \left(\frac{1199}{1100} \right)^3 \approx 518.0 \text{ W}$$



At no-load, $P_{out} = T_{L0} \omega_m = 0 = P_{in} - I_A^2 R_A - V_{brush} I_A - P_{core} - P_{mech} - P_{stray}$

$$0 = 220 I_A - I_A^2 (0.41) - 2 I_A - 512.1 - 518 - 490 \quad \text{from part a)}$$

$$\hookrightarrow -0.41 I_A^2 + 218 I_A - 1520.1 = 0$$

↓ solve using TI-68 polynomial root solver

$$I_A = \underline{7.06686 \text{ A}} \quad \text{or } 524.64 \text{ A not realistic}$$

↪ bigger than $I_{A,FL} = 44 \text{ A}$

Per KVL, $E_{A,NL} = V_A - V_{brush} - I_A R_A$

$$= 220 - 2 - 7.06686(0.41) = 215.1 \text{ V}$$

1st estimate $n_{NL} = n_0 \frac{E_A}{E_{A0}} = 1100 \frac{215.1}{200} = \underline{1183.06 \text{ RPM}}$

↑ Bit lower than w/out losses considered.

e) cont. Repeat process using new estimate of the no-load speed.

$$P_{\text{core}} \approx 450 \left(\frac{1183}{1100} \right)^{1.5} = 501.9 \text{ W}$$

$$P_{\text{mech}} \approx 400 \left(\frac{1183}{1100} \right)^3 = 497.55 \text{ W}$$

Now $P_{\text{out}} = 0 = P_{\text{in}} - I_A^2 R_A - V_{\text{brush}} I_A - P_{\text{core}} - P_{\text{mech}} - P_{\text{stray}}$

$$0 = 220 I_A - I_A^2 (0.41) - 2 I_A - 501.9 - 497.55 - 490$$

$$-0.41 I_A^2 + 218 I_A - 1489.45 = 0$$

$$\Downarrow \text{TI-68}$$

$$I_A = \underline{6.9225 \text{ A}} \quad \text{or } \cancel{524.6 \text{ A}} \quad \text{Not realistic, } > I_{A, \text{FL}}$$

Per KVL, $E_{A, \text{NL}} = 220 - 2 - 6.9225(0.41) = 215.16 \text{ V}$

2nd estimate $n_{\text{NL}} = 1100 \frac{215.16}{200} = \underline{1183.4 \text{ RPM}}$

only 0.03% difference
STOP iterative process

$$S R_{\text{w/losses}} = \frac{1183.4 - 1100}{1100} \times 100\%$$

$$\underline{S R_{\text{w/losses}} = 7.58\%}$$

$\Rightarrow$ as expected, motor runs slower when losses are considered.