Example- A three-phase 6-pole induction motor is Y-connected and rated for a terminal voltage of 220 V_{rms} and 10 hp at 60 Hz. The equivalent circuit parameters are given as

$$R_1 = 0.3\ \Omega,\ X_1 = 0.5\ \Omega,\ R_2 = 0.15\ \Omega,\ X_2 = 0.21\ \Omega, \text{ and}$$

$$X_M = 13.3\ \Omega \text{ with } R_c \text{ neglected/omitted.}$$

The friction, windage, core, & miscellaneous losses are roughly a constant value of 405 W.

Now, let's look at various torque and speed characteristics.

Per (7-1), the synchronous speed is $n_{sync} = 120 f_e / P = 120\ (60) / 6$

$$\Rightarrow \quad \underline{\boldsymbol{n_{sync} = 1200\ \text{RPM}}}.$$

$$\omega_{sync} = n_{sync}\,(\pi/30) = 1200\,(\pi/30) \ \Rightarrow\ \underline{\boldsymbol{\omega_{sync} = 125.6637\ \text{rad/s}}}.$$

For a Y-connected motor, the phase voltage is $V_\phi = 220/\sqrt{3} = 127.02\ V_{rms}$. Let $\angle V_\phi = 0°$. $\bar{V}_\phi = 127\angle 0°\ (V_{rms})$.

Find the Thevenin equivalent values. Per (7-38), the Thevenin voltage is

$$\bar{V}_{TH} = \bar{V}_\phi \frac{jX_M}{R_1 + jX_1 + jX_M} = 127\angle 0° \frac{j13.3}{0.3 + j0.5 + j13.3}$$

$$\Rightarrow \underline{\bar{V}_{TH} = 122.386\angle 1.2454°\ (V_{rms})}.$$

Per (7-38), the Thevenin impedance is

$$\bar{Z}_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + jX_1 + jX_M} = \frac{j13.3(0.3 + j0.5)}{0.3 + j0.5 + j13.3}$$

$$\Rightarrow \underline{\bar{Z}_{TH} = R_{TH} + jX_{TH} = 0.278523 + j0.48794\ (\Omega)}.$$

The slip and speed at the pullout torque are

$$s_{\max} = \frac{R_2}{\sqrt{R_{TH}{}^2 + (X_{TH} + X_2)^2}} = \frac{0.15}{\sqrt{0.27852^2 + (0.48794 + 0.21)^2}} \quad (7\text{-}51)$$

$$\Rightarrow \quad \underline{\boldsymbol{s_{max} = 0.1996 \sim 20\%}},$$

$n_{m,\max} = (1 - s_{\max})\, n_{sync} = (1 - 0.1996)\, 1200 \quad \Rightarrow \quad \underline{\boldsymbol{n_{m,max} = 960.467 \text{ RPM}}}$, and

$\omega_{m,\max} = (1 - s_{\max})\, \omega_{sync} = (1 - 0.1996)\, 125.6637 \quad \Rightarrow \quad \underline{\boldsymbol{\omega_{m,max} = 100.580 \text{ rad/s}}}$.

Per (7-52), the pullout torque is

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{sync}\,[R_{TH} + \sqrt{R_{TH}{}^2 + (X_{TH} + X_2)^2}\,]}$$

$$= \frac{3(122.386)^2}{2(125.6637)\,[0.27852 + \sqrt{0.27852^2 + (0.48794 + 0.21)^2}\,]}$$

$$\Rightarrow \quad \underline{\boldsymbol{\tau_{max} = 173.586 \text{ N-m} = 128.031 \text{ ft-lbs}}}.$$

At startup, the slip is 1 & speed is 0 $\quad \Rightarrow \quad \underline{\boldsymbol{s_{start} = 1}}$, $\underline{\boldsymbol{n_{m,start} = 0}}$, & $\underline{\boldsymbol{\omega_{m,start} = 0}}$.

Per (7-48), the starting torque is

$$\tau_{ind,s=1} = \tau_{start} = \frac{3V_{TH}{}^2 (R_2 / s)}{\omega_{sync}\,[(R_{TH} + R_2 / s)^2 + (X_{TH} + X_2)^2]}$$

$$= \frac{3(122.386)^2 (0.15)}{125.6637\,[(0.27852 + 0.15)^2 + (0.48794 + 0.21)^2]}$$

$$\Rightarrow \quad \underline{\boldsymbol{\tau_{start} = 79.966 \text{ N-m} = 58.98 \text{ ft-lbs}}}.$$

Note $\tau_{\max} / \tau_{start} = 173.586/79.966 \quad \Rightarrow \quad \underline{\boldsymbol{\tau_{max} / \tau_{start} = 2.17 \text{ or } 217\%}}$.

I'll use MathCad to confirm these values and plot the torque versus speed curve as well as the converted power versus speed curve.

Confirm earlier calculations-

Values given:

$VLL := 220\ Vrms$ $f := 60\ Hz$ $P := 6$ $slip := 0.02$

$R1 := 0.3\ \Omega$ $X1 := 0.5\ \Omega$ $Pfwc := 405\ W$

$R2 := 0.15\ \Omega$ $X2 := 0.21\ \Omega$ $Xm := 13.3\ \Omega$

$nsync := \frac{120 \cdot f}{P}$ $nsync = 1200$ RPM $\omega sync := nsync \cdot \frac{\pi}{30}$ $\omega sync = 125.664$ rad/s

$V\phi := \frac{VLL}{\sqrt{3}}$ $V\phi = 127.017$ Vrms

(7-40) $ZTH := \frac{(R1 + j \cdot X1) \cdot j \cdot Xm}{[(R1 + j \cdot X1) + j \cdot Xm]}$ $ZTH = 0.27852 + 0.48794i\ \Omega$

$RTH := Re(ZTH)$ $XTH := Im(ZTH)$

(7-38) $VTH := V\phi \cdot \frac{j \cdot Xm}{[(R1 + j \cdot X1) + j \cdot Xm]}$ $|VTH| = 122.3861\ V_{rms}$

$\arg(VTH) \cdot \frac{180}{\pi} = 1.245$ deg

Pullout

(7-51) $s_max := \frac{R2}{\sqrt{RTH^2 + (XTH + X2)^2}}$ $s_max = 0.1996$

$nm_max := (1 - s_max) \cdot nsync$ $nm_max = 960.4666$ RPM

$\omega m_max := (1 - s_max) \cdot \omega sync$ $\omega m_max = 100.5798$ rad/s

(7-52) $\tau max := \frac{3 \cdot (|VTH|)^2}{2 \cdot \omega sync \cdot \left[RTH + \sqrt{RTH^2 + (XTH + X2)^2}\right]}$ $\tau max = 173.5861$ N-m

Start-up

$s_start := 1$

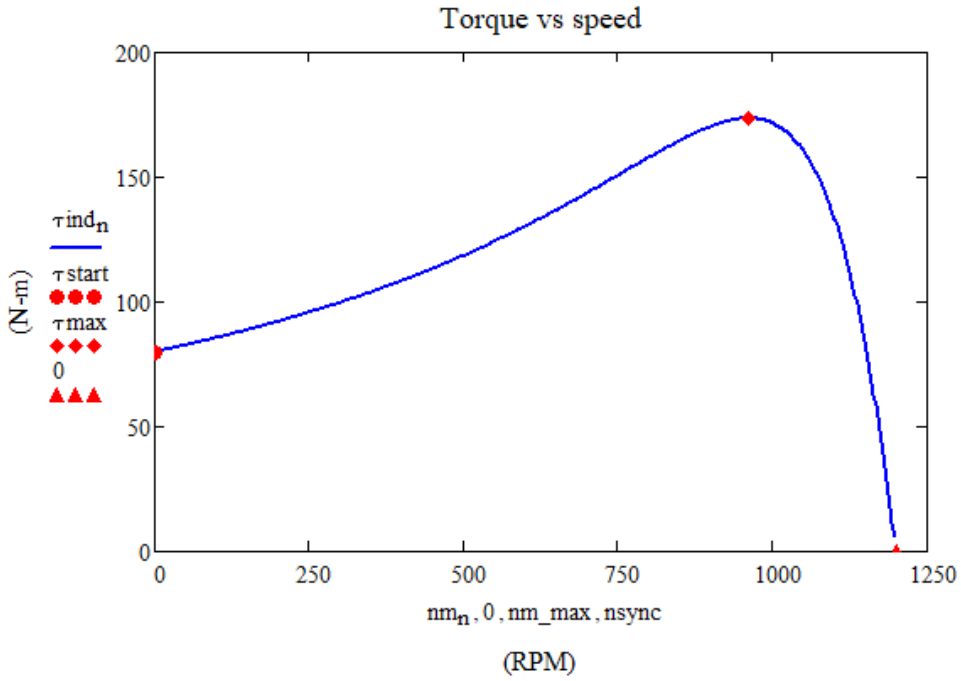
$nm_start := (1 - s_start) \cdot nsync$ $nm_start = 0$ RPM

$\omega m_start := (1 - s_start) \cdot \omega sync$ $\omega m_start = 0$ rad/s

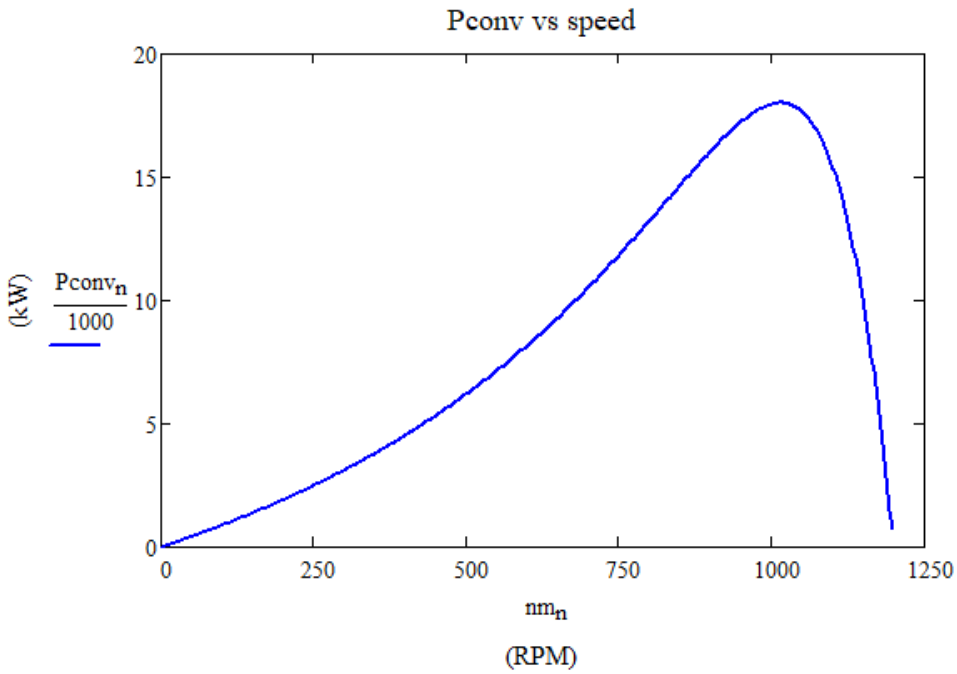
(7-48) $\tau start := \frac{3 \cdot (|VTH|)^2 \cdot \frac{R2}{s_start}}{\omega sync \cdot \left[\left(RTH + \frac{R2}{s_start}\right)^2 + (XTH + X2)^2\right]}$ $\tau start = 79.966$ N-m

$n := 0..399 \qquad nm_n := 3n \qquad s_n := \frac{nsync - nm_n}{nsync}$

(7-48) $$\tau ind_n := \frac{3 \cdot \left(|VTH| \right)^2 \cdot \frac{R2}{s_n}}{\omega sync \cdot \left[\left(RTH + \frac{R2}{s_n} \right)^2 + (XTH + X2)^2 \right]}$$



(7-35) $$Pconv_n := \tau ind_n \cdot nm_n \cdot \frac{\pi}{30}$$



How does doubling R_2 change the torque?

Pullout w/ $2R_2$

(7-51) $s_max2 := \dfrac{2R2}{\sqrt{RTH^2 + (XTH + X2)^2}}$ $\quad s_max2 = 0.3992$

$nm_max2 := (1 - s_max2) \cdot nsync$ $\quad nm_max2 = 720.9333$ RPM

$\omega m_max2 := (1 - s_max2) \cdot \omega sync$ $\quad \omega m_max2 = 75.496$ rad/s

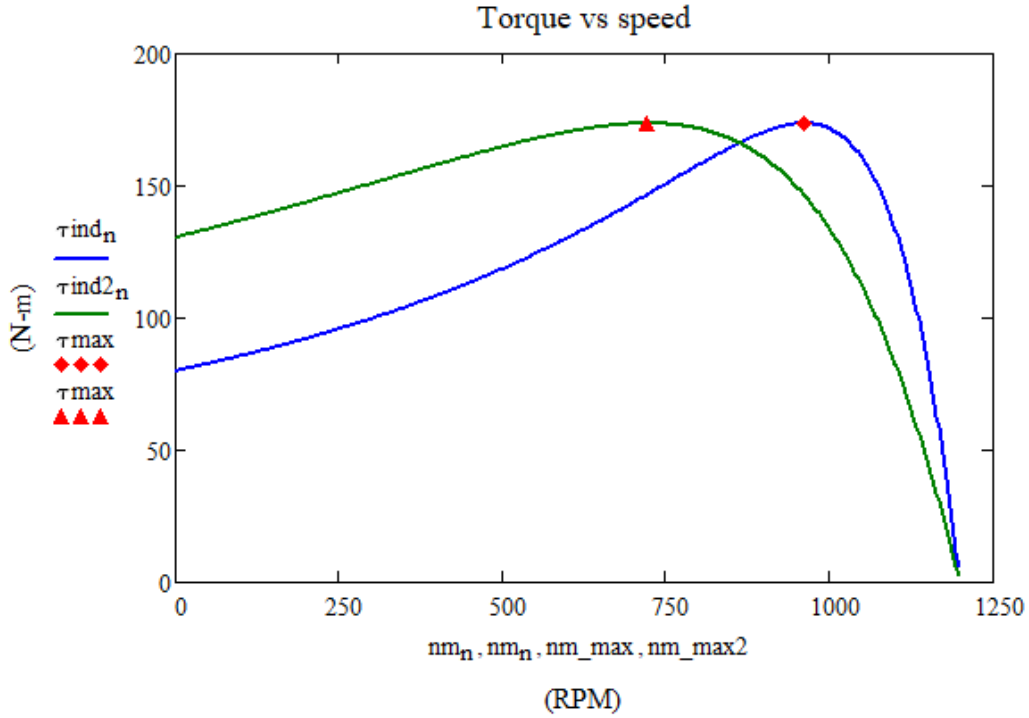
(7-52) $\tau max2 := \dfrac{3 \cdot (|VTH|)^2}{2 \cdot \omega sync \cdot \left[RTH + \sqrt{RTH^2 + (XTH + X2)^2}\right]}$ $\quad \tau max2 = 173.5861$ N-m

Same!

Start-up w/ $2R_2$

(7-48) $\tau start2 := \dfrac{3 \cdot (|VTH|)^2 \cdot \dfrac{2R2}{s_start}}{\omega sync \cdot \left[\left(RTH + \dfrac{2R2}{s_start}\right)^2 + (XTH + X2)^2\right]}$ $\quad \tau start2 = 130.5349$ N-m

(7-48) $\tau ind2_n := \dfrac{3 \cdot (|VTH|)^2 \cdot \dfrac{2R2}{s_n}}{\omega sync \cdot \left[\left(RTH + \dfrac{2R2}{s_n}\right)^2 + (XTH + X2)^2\right]}$



Substantial increase in starting torque. No change in maximum torque. However, the speed at which the maximum torque occurs decreases.