Example- A three-phase 6-pole induction motor is Y-connected and rated for a terminal voltage of 220 V_{rms} and 10 hp at 60 Hz. The equivalent circuit parameters are given as

$$R_1 = 0.3\ \Omega,\ X_1 = 0.5\ \Omega,\ R_2 = 0.15\ \Omega,\ X_2 = 0.21\ \Omega, \text{ and}$$

$$X_m = 13.3\ \Omega \text{ with } R_c \text{ neglected/omitted.}$$

The friction, windage, core, & miscellaneous losses are roughly a constant value of 405 W. For a slip of 2%, analyze the motor.

Per (7-1), the synchronous speed is $n_{sync} = 120 f_e / P = 120\ (60) / 6$

$$\Rightarrow \quad \underline{\boldsymbol{n_{sync} = 1200\ \text{RPM}}}.$$

$$\omega_{sync} = n_{sync}\,(\pi/30) = 1200\,(\pi/30) \ \Rightarrow\ \underline{\boldsymbol{\omega_{sync} = 125.6637\ \text{rad/s}}}.$$

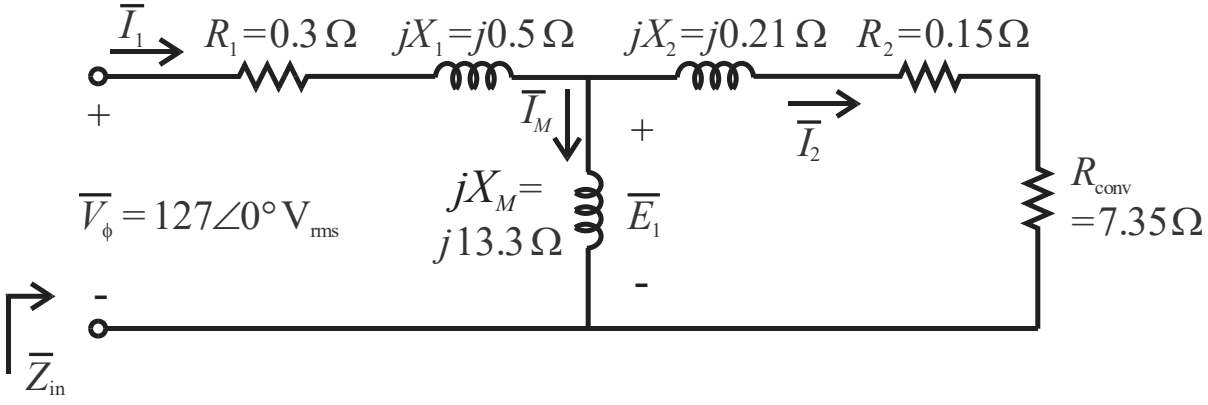
Per (7-6), the shaft speed is $n_m = (1 - s)\, n_{sync} = (1 - 0.02)\ 1200$

$$\Rightarrow \quad \underline{\boldsymbol{n_m = 1176\ \text{RPM}}}.$$

$$\omega_m = n_m\,(\pi/30) = 1176\,(\pi/30) \ \Rightarrow\ \underline{\boldsymbol{\omega_m = 123.1504\ \text{rad/s}}}.$$

Next, draw the phase equivalent circuit. The input phase voltage is needed. For a Y-connection, $V_\phi = 220 / \sqrt{3} = 127.02\ V_{rms}$. Let $\angle V_\phi = 0°$. Also, calculate the converted resistance

$$R_{conv} = R_2\left(\frac{1-s}{s}\right) = 0.15\left(\frac{1-0.02}{0.02}\right) = 7.35\,\Omega$$



First, calculate the input impedance

$$\begin{aligned}\bar{Z}_{\text{in}} &= (R_1 + jX_1) + [jX_M \,//\, (jX_2 + R_2 + R_{\text{conv}})] \\ &= (0.3 + j0.5) + [j13.3 \,//\, (j0.21 + 0.15 + 7.35) \\ &= (0.3 + j0.5) + (5.556286 + j3.29127) \\ &= 5.8563 + j3.7913 = 6.97638\angle 32.91845^\circ\ \Omega\end{aligned}$$

Find the power factor of the motor

$$\text{PF} = \cos\theta = \cos(32.91845^\circ) \quad \Rightarrow \ \underline{\textbf{PF = 0.8394 lagging}}.$$

Find the input current for the stator

$$\bar{I}_1 = \bar{V}_\phi / \bar{Z}_{\text{in}} = 127\angle 0^\circ / 6.9764\angle 32.918^\circ \Rightarrow \underline{\bar{I}_1 = 18.2067\angle -32.918^\circ\ (\text{A}_{\text{rms}})}.$$

Per (2-29),

$$P_{\text{in}} = \sqrt{3}\, V_{LL}\, I_L \cos\theta = \sqrt{3}(220)\, 18.2067\, (0.8394) \quad \Rightarrow \ \underline{\boldsymbol{P_{\text{in}}} \textbf{ = 5823.804 W}}.$$

Per (7-25), $P_{\text{SCL}} = 3I_1^2\, R_1 = 3(18.2067)^2\, 0.3$ $\Rightarrow \ \underline{\boldsymbol{P_{\text{SCL}}} \textbf{ = 298.336 W}}.$

Per (7-27), $P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} - P_{\text{core}} = 5823.804 - 298.336 - 0 \Rightarrow \underline{\boldsymbol{P_{\text{AG}}} \textbf{ = 5525.5 W}}.$

Per (7-32), $P_{\text{RCL}} = s\, P_{\text{AG}} = 0.02\, (5525.4685)$ $\Rightarrow \underline{\boldsymbol{P_{\text{RCL}}} \textbf{ = 110.509 W}}.$

Per (7-33), $P_{\text{conv}} = (1 - s)\, P_{\text{AG}} = (1 - 0.02)\, 5525.4685$ $\Rightarrow \underline{\boldsymbol{P_{\text{conv}}} \textbf{ = 5414.95 W}}.$

Per (7-34), $P_{\text{out}} = P_{\text{conv}} - P_{\text{F\&W}} - P_{\text{misc}} = 5414.95 - 405$ $\Rightarrow \underline{\boldsymbol{P_{\text{out}}} \textbf{ = 5009.95 W}}$

or, $P_{\text{out}} = 5009.95$ W (1 hp / 746 W) $\Rightarrow \ \underline{\boldsymbol{P_{\text{out}}} \textbf{ = 6.716 hp}}.$

To verify some of these power numbers, calculate

$$\bar{E}_1 = \bar{V}_\phi - \bar{I}_1(R_1 + jX_1) = 127\angle 0^\circ - (18.2067\angle -32.91845^\circ)(0.3 + j0.5)$$

$$\Rightarrow \underline{\bar{E}_1 = 117.578\angle -2.278^\circ\ (\text{V}_{\text{rms}})}.$$

$$\overline{I}_2 = \overline{E}_1 / (jX_2 + R_2 + R_{\text{conv}}) = 117.57775\angle -2.27798^\circ / (j0.21 + 0.15 + 7.35)$$
$$\Rightarrow \underline{\overline{I}_2 = 15.6709\angle -3.882^\circ \, (\text{A}_{\text{rms}})}.$$

$$\overline{I}_M = \overline{I}_1 - \overline{I}_2 = (18.2067\angle -32.918^\circ) - (15.6709\angle -3.882^\circ)$$
$$\Rightarrow \underline{\overline{I}_M = 8.8404\angle -92.278^\circ \, (\text{A}_{\text{rms}})}.$$

Now, we can check/recalculate some of the power values-

Per (7-28), $P_{\text{AG}} = 3I_2^2 (R_2 / s) = 3(15.6709)^2 (0.15/0.02) \Rightarrow \underline{\boldsymbol{P_{\text{AG}} = 5525.5 \text{ W}}}$.

Per (7-32), $P_{\text{RCL}} = 3I_2^2 R_2 = 3(15.6709)^2 0.15 \Rightarrow \underline{\boldsymbol{P_{\text{RCL}} = 110.51 \text{ W}}}$.

Per (7-31), $P_{\text{conv}} = 3I_2^2 R_2 (1 - s) / s = 110.51 (1 - 0.02) / 0.02$
$\Rightarrow \underline{\boldsymbol{P_{\text{conv}} = 5414.975 \text{ W}}}$.

Per (7-34), $P_{\text{out}} = P_{\text{conv}} - P_{\text{F\&W}} - P_{\text{misc}} = 5414.975 - 405 \Rightarrow \underline{\boldsymbol{P_{\text{out}} = 5009.98 \text{ W}}}$

or, $P_{\text{out}} = 5009.98$ W (1 hp / 746 W) $\Rightarrow \underline{\boldsymbol{P_{\text{out}} = 6.716 \text{ hp}}}$ **(< rated 10 hp)**.

[All answers are the same to within round-off error.]

The motor efficiency is

$$\eta = P_{\text{out}} / P_{\text{in}} \times 100\% = 5009.98/5823.804 \times 100\% \Rightarrow \underline{\boldsymbol{\eta = 86.03\%}}.$$

The induced torque is

(7-35) $\tau_{\text{ind}} = P_{\text{conv}} / \omega_m = 5414.95/123.1504 \Rightarrow \underline{\boldsymbol{\tau_{\text{ind}} = 43.97 \text{ (N-m)}}}$,

or (7-36) $\tau_{\text{ind}} = P_{\text{AG}} / \omega_{\text{sync}} = 5525.5/125.6637 \Rightarrow \underline{\boldsymbol{\tau_{\text{ind}} = 43.97 \text{ (N-m)}}}$.

Per Figure 7-13, the load torque is

$$\tau_{\text{load}} = P_{\text{out}} / \omega_m = 5009.95/123.1504 \Rightarrow \underline{\boldsymbol{\tau_{\text{ind}} = 40.68 \text{ (N-m)}}}.$$