ex. Ideal Transformers Circuit

Say I have a portable generator w/ an output of 120 V_{rms} that I want to use to power some load ($\bar{Z}_L = 6 + j2\,\Omega$) in a barn 200m away.

For twin-lead (each way)

$$R = \frac{\ell}{\sigma A}(2) \leftarrow \text{current goes out + back}$$

$$L = \frac{\mu \ell}{\pi} \cosh^{-1}\left(\frac{d}{2a}\right) \leftarrow \text{counts both wires}$$

For 12AWG, Copper wire & $2a = 2.053\text{ mm}$, $d \simeq 6\text{ mm}$, $\sigma_{cu} = 5.8 \times 10^{7}\text{ S/m}$

$$R = \frac{2(200)}{(5.8\times10^{7})\pi\left(\frac{2.053\times10^{-2}}{2}\right)^2} = 2.08336\,\Omega$$

$$L = \frac{(4\pi\times10^{-7})(200)}{\pi} \cosh^{-1}\left(\frac{6\text{mm}}{2.053\text{mm}}\right) = 1.39\times10^{-4}\text{ H}$$

$$\bar{Z}_L = j\omega L = j\,2\pi(60)(1.39\times10^{-4}) = j0.0523\,\Omega$$

$$\underline{\bar{Z}_{wire} = 2.08336 + j0.0523\,\Omega} \leftarrow \text{nearly negligible}$$

$\Downarrow$ Round to

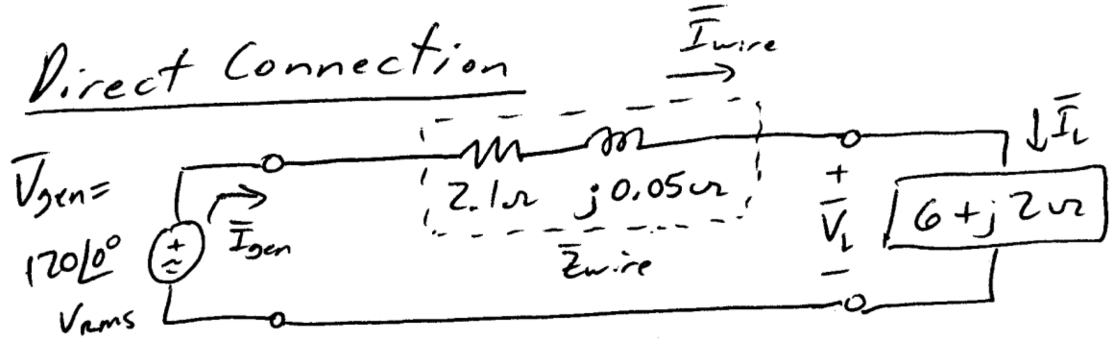
$$\underline{\bar{Z}_{wire} = 2.1 + j0.05\,\Omega}$$

ex. cont.

How much of a difference in terms of power + efficiency will using step-up and step-down transformers make?

Assume 1:4 (step-up) & 4:1 (step-down) transformers are available. ↖ voltage limits for wiring

Direct Connection



$$\bar{I}_{gen} = \bar{I}_{wire} = \bar{I}_L = \frac{120\angle 0°}{(2.1+j0.05)+(6+j2\Omega)}$$

$$\bar{I}_{gen} = 14.362\angle -14.0256° \ A_{rms}$$

$$\bar{S}_{gen} = \bar{V}_{gen}\bar{I}_{gen}^* = (120\angle 0°)(14.362\angle +14.03°)$$

$$= 1672 + j417.7 \ VA$$

$$P_{gen} = 1672 W \qquad Q_{gen} = 417.7 \ VA$$

$$pf_{gen} = \cos 14.03° = 0.97 \text{ lagging}$$

ex. cont.

Direct cont.

$$\bar{V}_{wire} = \bar{I}_{wire}(\bar{Z}_{wire}) = (14.362\angle{-14.03^\circ})(2.1+j0.05)$$
$$= \underline{30.16875\angle{-12.662^\circ}}\ V_{rms}$$

$$\bar{S}_{wire} = \bar{V}_{wire}\bar{I}_{wire}^{*} = (30.17\angle{-12.66^\circ})(14.362\angle{+14.03^\circ})$$
$$\underline{\bar{S}_{wire} = 433.16 + j10.313\ VA},\ \underline{P_{wire} = 433.2W},\ \underline{Q_{wire} = 10.3\,VAR}$$

$$\bar{V}_L = \bar{I}_{wire}\bar{Z}_L = (14.362\angle{-14.03^\circ})(6+j2)$$
$$= \underline{90.833\angle{4.409^\circ}}$$

Very bad (lights dim, $I\uparrow$ for constant power)

$$\bar{S}_L = \bar{V}_L\bar{I}_L^{*} = (90.833\angle{4.409^\circ})(14.362\angle{+14.03^\circ})$$
$$= 1237.6 + j412.5\ VA$$

$$P_L = \underline{1237.6\,W} \quad + \quad \underline{Q_L = 412.5\ VAR}$$

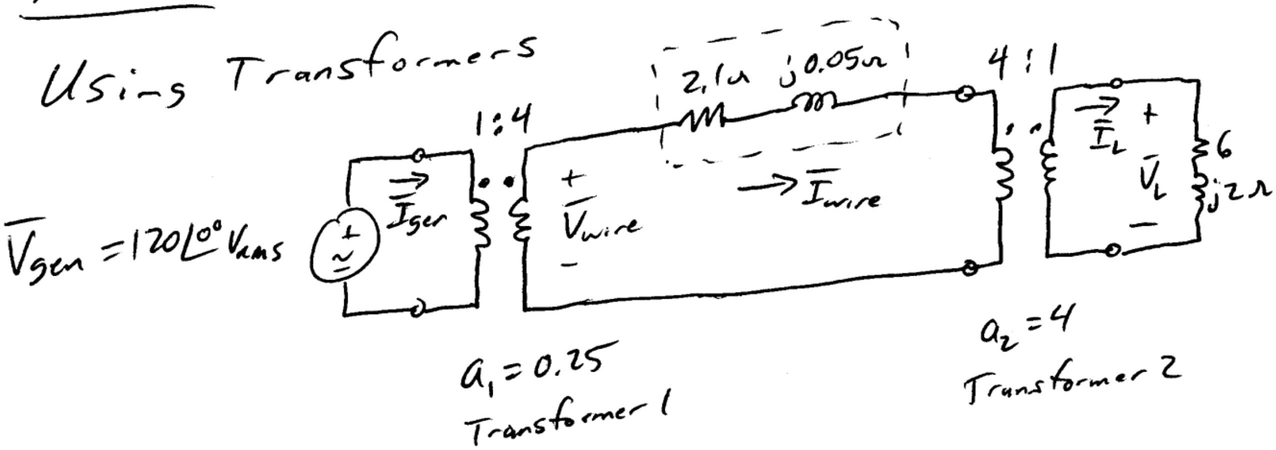
$$\%\ loss = \frac{P_{wire}}{P_{gen}} \times 100\% = \frac{433.2}{1672} \times 100\% = \underline{26\ \%}$$

$$\%\ load = efficiency = \frac{P_L}{P_{gen}} \times 100\% = \underline{74\%}$$

not good

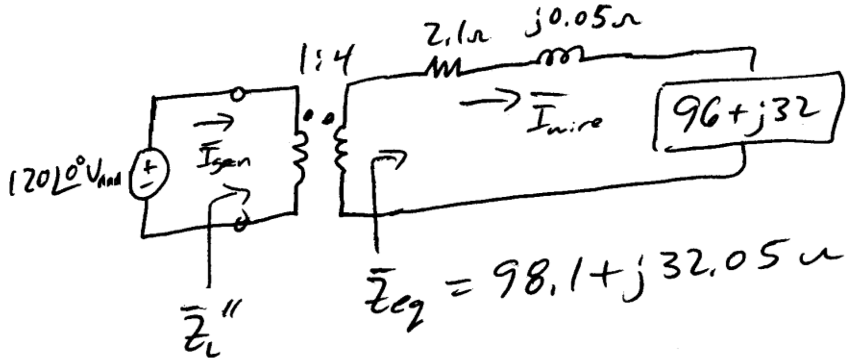
ex. cont.

Using Transformers



Use (2-15), to refer load impedance across transformer 2

$$\overline{Z}_L' = a_2^2 \, Z_L = 4^2 (6 + j2) = 96 + j32 \ \Omega$$



The "load "seen" by the generator is then

$$\overline{Z}_L'' = a_1^2 \, \overline{Z}_{eq} = (0.25)^2 (98.1 + j32.05)$$
$$= 6.13125 + j2.003125 \ \Omega \leftarrow \text{close to } \overline{Z}_L!$$

$= 6.45 \angle 18.0926° \ \Omega$

ex. cont.

Transformers cont.

$$\bar{I}_{gen,T} = \frac{120\angle 0^\circ}{6.13 + j2.003} = 18.60415\angle -18.0926^\circ \text{ Arms}$$

$$\bar{S}_{gen} = \bar{V}_{gen}\,\bar{I}_{gen,T}^{*} = (120\angle 0^\circ)(18.60415\angle +18.0926^\circ)$$

$$= \underline{2122.1 + j693.3 \text{ VA}}$$ ← More power out!

$P_{gen,T} = \underline{2122 \text{ W}}$ vs. $P_{gen,D} = \underline{1672 \text{ W}}$

$Q_{gen,T} = 693.3$ VAR vs. $Q_{gen,D} = 417.7$ VAR

By (2-5) $\dfrac{\bar{I}_p}{\bar{I}_s} = \dfrac{1}{a}$, so $\bar{I}_{wire,T} = a_1 \bar{I}_{gen,T}$

$$\bar{I}_{wire,T} = (0.25)(18.60415\angle -18.0926^\circ)$$

$$\underline{\bar{I}_{wire,T} = 4.6510375\angle -18.0926^\circ \text{ Arms}}$$

$$\bar{V}_{wire,T} = \bar{I}_{wire,T}\,(2.1 + j0.05)$$

$$= 9.77\angle -16.73^\circ \text{ Vrms}$$ ← Much lower!

$$\bar{S}_{wire,T} = \bar{V}_{wire,T}\,\bar{I}_{wire,T}^{*}$$

$$= (9.77\angle -16.73^\circ)(4.651\angle +18.093^\circ)$$

$$= 45.4 + j1.1 \text{ VA}$$

Big Redux! $\begin{cases} P_{wire,T} = 45.4 \text{ W} & \text{vs. } P_{wire,D} = 433.2 \text{ W} \\ Q_{wire,T} = 1.1 \text{ VAR} & \text{vs. } Q_{wire,T} = 10.3 \text{ VAR} \end{cases}$

ex. cont.

Transformers cont.

Using (2-5) $\frac{\bar{I}_p}{\bar{I}_s} = 1/a$ again, $\bar{I}_L = a_2 \bar{I}_{wire,T}$

$$\bar{I}_{L,T} = (4)(4.651\angle -18.0926^\circ) = \underline{18.60415 \angle -18.0926^\circ \text{ A}_{rms}}$$

$$= \bar{I}_{gen,T} \;!$$

$$\bar{V}_{L,T} = \bar{I}_{L,T} * (6 + j2) = (18.604\angle -18.09^\circ)(6+j2)$$

$$\underline{\bar{V}_{L,T} = 117.663\angle 0.342^\circ \text{ V}_{rms}}$$ ← only minor voltage drop

$$\bar{S}_{L,T} = \bar{V}_{L,T}\,\bar{I}_{L,T}^{*} = (117.663\angle 0.342^\circ)(18.604\angle +18.09^\circ)$$

$$= 2076.7 + j\,692.2 \text{ VA}$$

$P_{L,T} = 2076.7$ W vs $P_{L,0} = 1237.6$ W

$Q_{L,T} = 692.2$ VAR vs. $Q_{L,0} = 412.5$ VAR

$$\% \text{ loss} = \frac{P_{wire,T}}{P_{gen,T}} * 100\% = \frac{45.4}{2122.1} \times 100\% = \underline{2.1\%}$$

$$\% \text{ load} = \text{efficiency} = \frac{P_{L,T}}{P_{gen,T}} * 100\% = \frac{2076.7}{2122.1} \times 100\%$$

$$= \underline{\underline{97.9\%}}$$