Chapter 6 Induction Motors

→ No externally supplied dc field current to rotor, only rely on <u>induced</u> rotor currents

→ virtually never used as generators by utilities, some applications in alt. energy (windmills, ...)

6.1 Induction Motor Construction

Two types
- cage rotor  shorting ring(s)
- wound rotor – full set of 3φ windings (like stator)
  - * Usually Y-connected thru brushes/slip rings
  - * more control/more complicated/more maintenance
  - * cost $$

→ cage rotor induction motors far more popular

6.2 Basic Induction Motor Concepts

→ Basics covered in Chap. 6

Key concept $n_{\text{induction motor}} < n_{synch} = \frac{120 f_e}{P}$

→ else τ goes to zero + it slows down

Concept of Rotor Slip

→ care about relative speed/velocities of rotor wrt magnetic field (stator)

$n_{sync} = \frac{120 f_e}{P}$

n_m = mech. speed of shaft

$$n_{slip} = n_{sync} - n_m \equiv \text{slip speed}$$

$$\text{slip} \equiv s = \frac{n_{slip}}{n_{sync}} \times 100\% = \frac{n_{sync} - n_m}{n_{sync}} \times 100\%$$

$$0 \le s \le 1$$

synchronous (at $s = 0$); stationary (motor not turning) (at $s = 1$)

$$n_m = (1-s)\, n_{sync}$$

$$\omega_m = (1-s)\, \omega_{sync}$$

6.2 cont.

Electrical Frequency on the Rotor

→ electrical freq. of e_{ind} + i_{rotor} NOT necessarily the same as f_e due to relative motion of rotor wrt $\bar{B}_{stator}$

→ If $\omega_{rotor} = 0$, $f_{rotor} = f_e$ ← like a stationary transformer

→ If $\omega_{rotor} = \omega_{sync}$, $f_{rotor} = 0$ why? B_{net} doesn't vary w/ time

⇓ linear relationship

$$f_{rotor} \equiv f_r = s f_e = \left(\frac{n_{sync} - n_m}{n_{sync}}\right) f_e$$

$$f_{rotor} = \frac{P}{120}\left(n_{sync} - n_m\right)$$

ex. A 210 V_{rms}, 1/3 hp, 4-pole, 60 Hz, Y-connected induction motor runs at 1790 RPM under load. Find n_{sync}, slip speed, slip, + f_r

$$n_{sync} = \frac{120(60)}{4} = 1800 \text{ RPM}$$

$$n_{slip} = n_{sync} - n_m = 1800 - 1790 = 10 \text{ RPM}$$

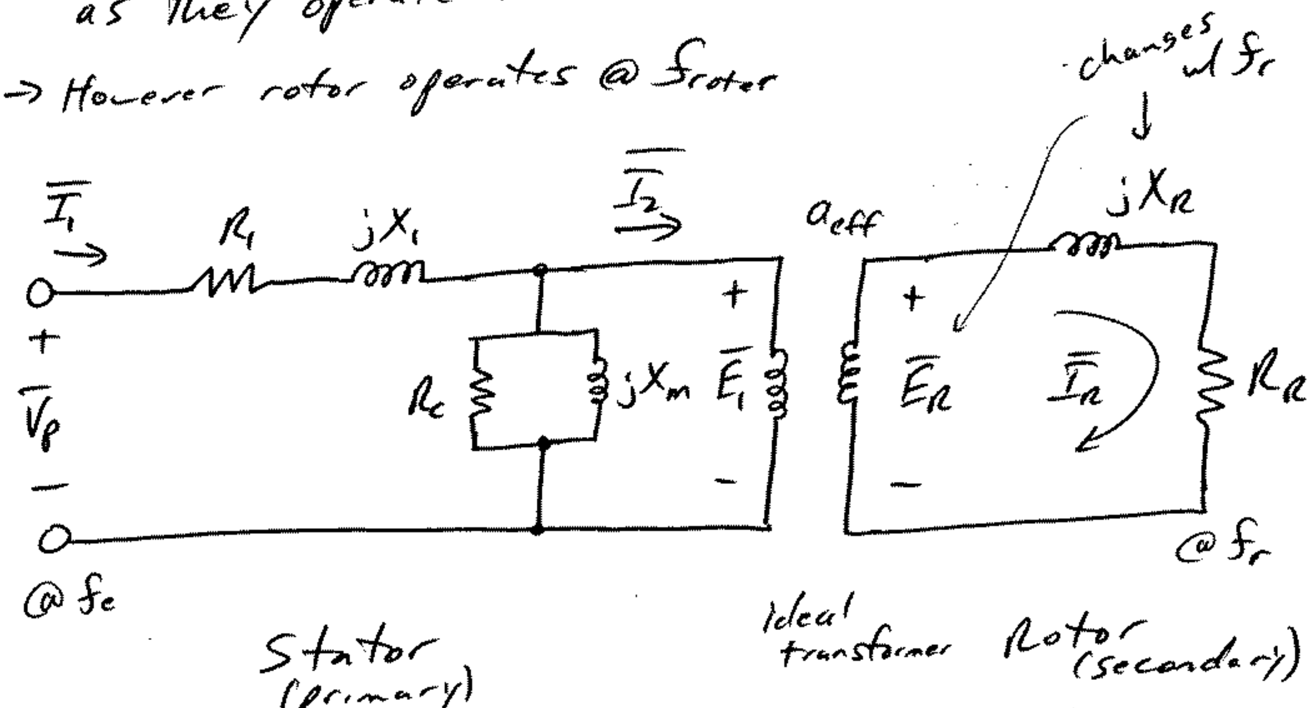
$$s = \frac{n_{slip}}{n_{sync}} \times 100\% = \frac{10}{1800} \times 100\% = 0.556\%$$

$$f_r = s f_e = (0.00555\overline{5})(60) = 0.333 \text{ Hz}$$ ← slow!

6.3 The Equivalent Circuit of an Induction Motor

→ very similar to transformer circuit model as they operate in a similar fashion

→ However rotor operates @ f_{rotor}



$R_1 \equiv$ stator resistance (wire resistance)

$X_1 \equiv$ stator leakage reactance (due to flux leakage)

$R_c \equiv$ equivalent core resistance (losses to eddy currents + hysteresis)

$X_m \equiv$ magnetizing reactance (will be smaller than for a transformer due to the air gap pushing equiv. reluctance up)

$a_{eff} \equiv$ effective turns ratio

$$= \frac{E_1}{E_R} = \underbrace{\frac{N_\phi}{N_R}}_{\text{per phase}} \Big\} \text{ wound rotor only}$$

6.3 cont.

Rotor Circuit Model

$e_{ind} = (\bar{v}_{rel} \times \bar{B}) \cdot \bar{l} \Rightarrow v_{rel} \uparrow \; e_{ind} \uparrow$

If rotor motionless (locked rotor test) v_{rel} = max value & $f_r = f_e$

$\hookrightarrow E_R = E_{RO}$

If rotor @ ω_{sync} $v_{rel} = 0$ & $f_r = 0$

$\Downarrow$ linear relation

$$\underline{E_R = s\,E_{RO}} \quad \text{w/} \quad \underline{f_r = s\,f_e}$$

$\hookrightarrow$ rotor voltage & frequency functions of slip!

$$jX_R = j\,\omega_r L_R = j\,2\pi f_r L_R$$

X_R — rotor reactance

L_R — rotor inductance

$\Downarrow$

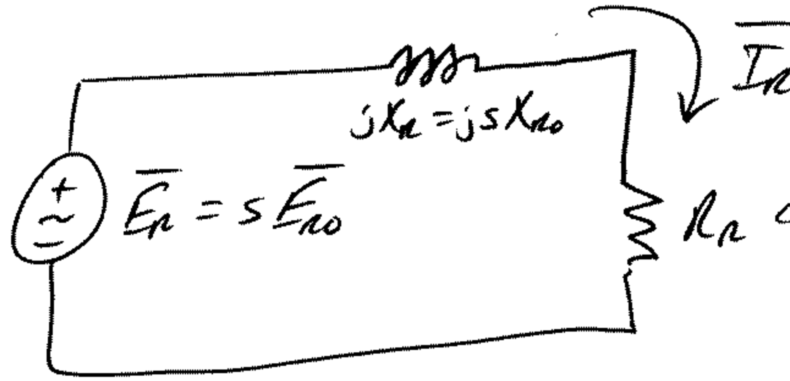
$$\underline{X_R = s\,X_{RO}}$$

X_{RO} — Blocked/locked rotor test reactance

6.3 cont.

Equiv. Rotor Circuit

@ $f_r = s f_e$



R_R ~ essentially a dc value since f_R small

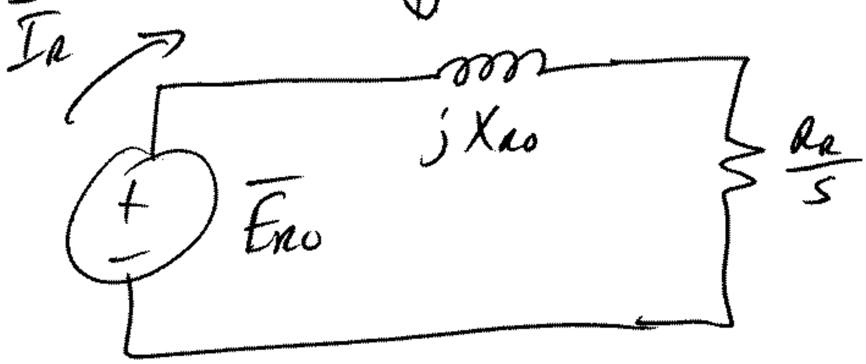
$$\overline{I}_R = \frac{\overline{E}_R}{R_R + jX_R} = \frac{\overline{E}_R}{R_R + jsX_{R0}}$$

$$\overline{I}_R = \frac{\overline{E}_{R0}}{R_R/s + jX_{R0}}$$

Define equivalent rotor impedance $\overline{Z}_{R,eq} = \dfrac{\overline{E}_{R0}}{\overline{I}_R}$

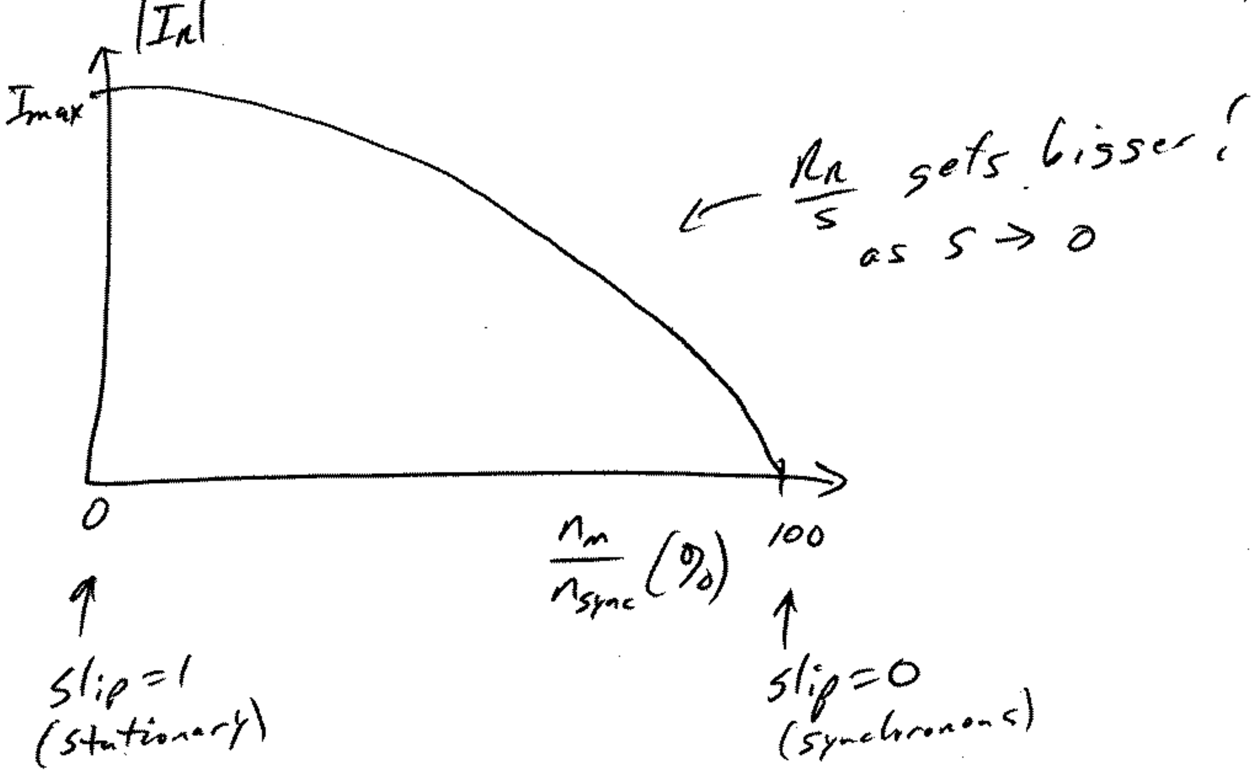
$$\underline{\overline{Z}_{R,eq} = \frac{R_R}{s} + jX_{R0} \quad (\Omega)}$$

$\Downarrow$ New equiv. ckt



@ $f = f_e$!

6.3 cont.



Final Equiv. Circuit for Induction Motor

Refer rotor circuit over to primary (stator) side

$$\left.\begin{aligned} \bar{E}_1 &= \bar{E}_R' = a_{eff}\,\bar{E}_{R0} \\ \bar{I}_2 &= \frac{\bar{I}_R}{a_{eff}} \end{aligned}\right\} \text{from chap 2}$$

$$\bar{Z}_2 = a_{eff}^2\left(\frac{R_R}{s} + jX_{R0}\right) = \frac{a_{eff}^2 R_R}{s} + j\,a_{eff}^2 X_{R0}$$

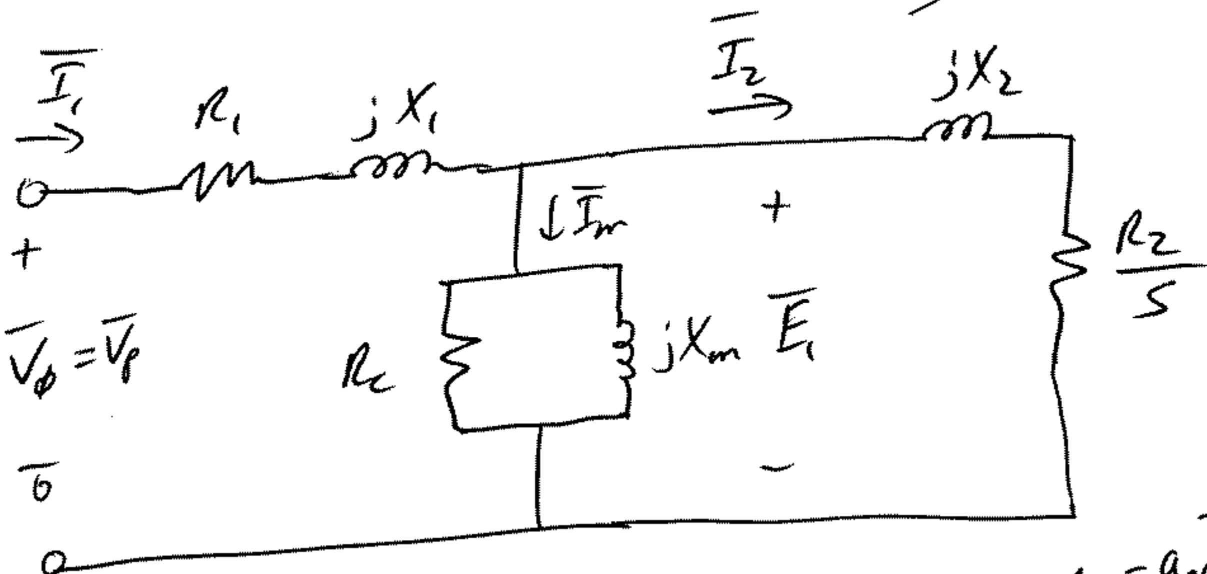
$$= \frac{R_2}{s} + jX_2$$

R_2 + X_2 referred resistance + reactance of rotor!

6.3 cont.

Very possible that we won't know R_R, X_{R0}, + a_{eff} but instead will know R_2 + X_2 for caged rotor machines.

referred resistance + reactance



where $R_2 = a_{eff}^2 R_R$

$X_2 = a_{eff}^2 X_{R0}$

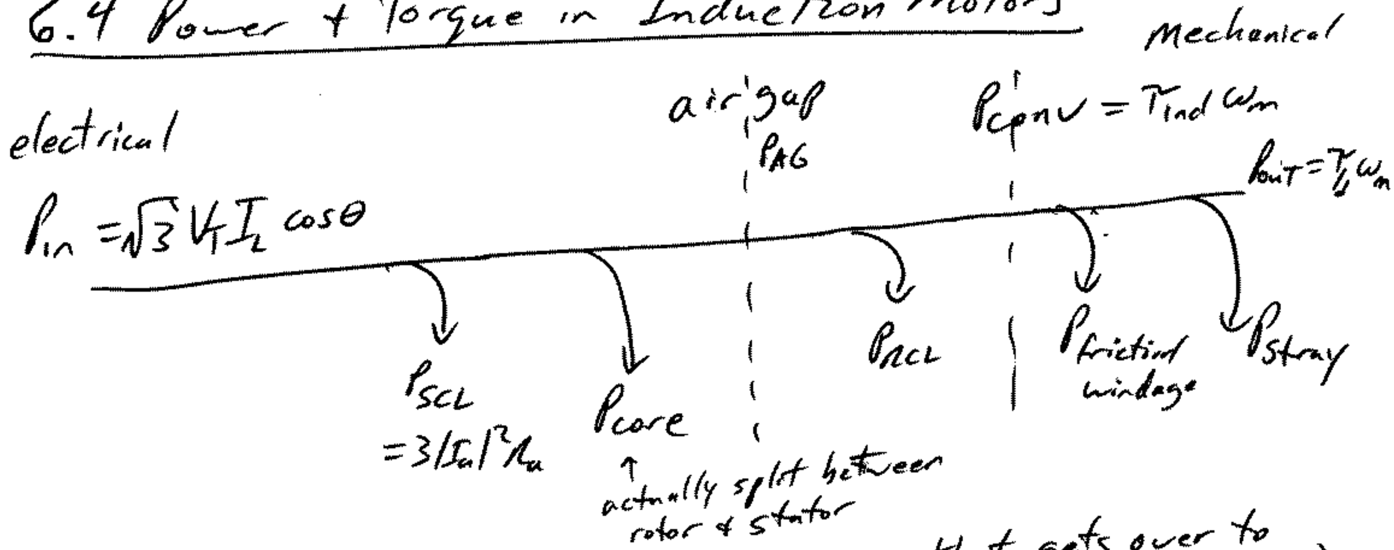
→ everything but s is @ f_e

Note: R_c often omitted or lumped in w/ R_2

why? $X_m << R_c$ due to $\mathcal{R}_{air\text{-}gap}$ being large

so $jX_m \,||\, R_c \simeq \underline{jX_m}$

6.4 Power + Torque in Induction Motors



$P_{AG} = P_{in} - P_{SCL} - P_{core}$ ← power that gets over to rotor (across the air gap)

$P_{conv} = P_{in} - P_{SCL} - P_{core} - P_{RCL} = \tau_{ind}\,\omega_m = \tau_{dev}\,\omega_m$

↑ developed torque

$P_{out} = P_{in} - P_{\substack{all\\losses}} = \tau_{load}\,\omega_m$

$n_m \uparrow \quad \underbrace{P_{friction}\uparrow \quad P_{windage}\uparrow \quad P_{stray}\uparrow}_{P_{rotational}}$ but $P_{core}\downarrow$ $f_{rel.}$ smaller

In terms of the equivalent circuit:

$P_{SCL} = 3\,I_1^2 R_1$

$P_{core} = 3\,\dfrac{E_1^2}{R_C} = 3E_1^2 G_C$

$P_{AG} = P_{in} - P_{SCL} - P_{core} = 3\,I_2^2 \dfrac{R_2}{s}$

$P_{RCL} = 3I_2^2 R_2 = 3I_R^2 R_R$

since $I_2 = \dfrac{I_R}{a_{eff}}$

$+\ R_2 = a_{eff}^2 R_R$

Also,

$\underline{P_{RCL} = s\,P_{AG}}$

$s \to 0 \quad P_{RCL} \to 0$ makes sense

6.4 cont.

Converted power or developed mechanical power

$$P_{conv} = P_{AG} - P_{RCL} = 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2$$

$$P_{conv} = 3 I_2^2 R_2 \left(\frac{1-s}{s} \right)$$

$$= (1-s) P_{AG}$$ $\Leftarrow s=1$ (stationary) $\Rightarrow P_{conv} = 0$!

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{AG}}{\omega_{sync}}$$

Since the air gap power can be expressed as

$P_{AG} = P_{conv} + P_{RCL}$, it would be nice to split the R_2/s equiv. ckt resistance into two parts – 1 for P_{conv} & 1 for P_{RCL}

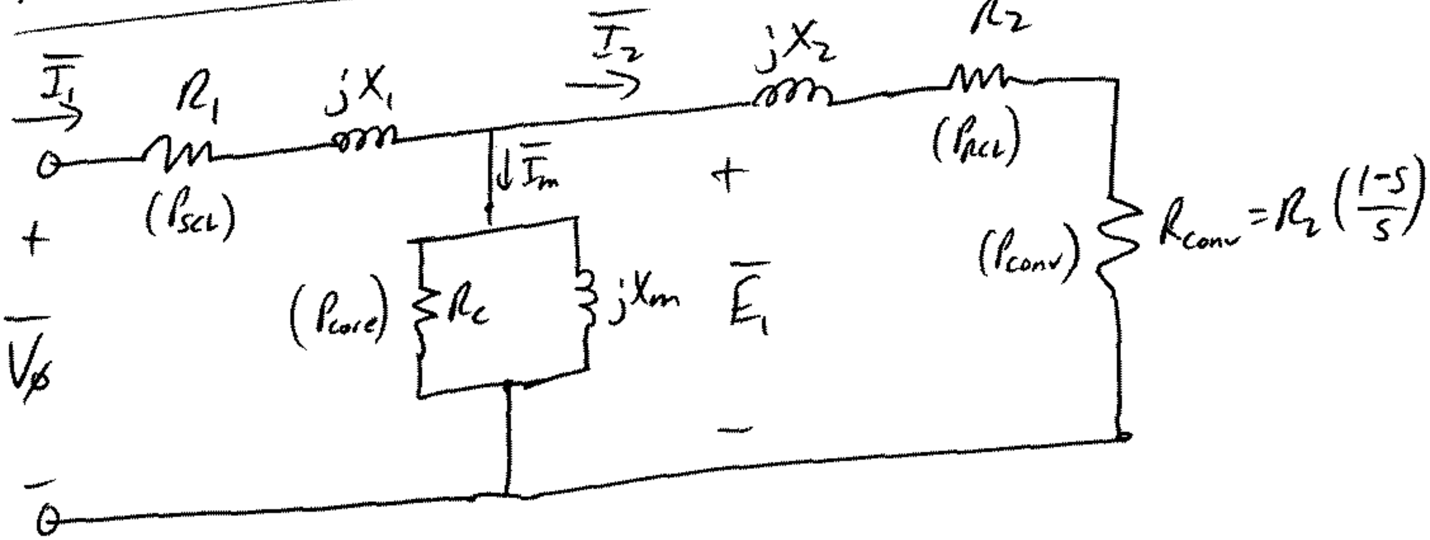
Define $R_{conv} = \frac{R_2}{s} - R_2 = R_2\left(\frac{1-s}{s}\right)$

$$P_{conv} = 3 I_2^2 R_{conv} = 3 I_2^2 R_2 \left(\frac{1-s}{s}\right)$$

& $P_{RCL} = 3 I_2^2 R_2$

6.4 cont.

Modified Equiv. Ckt. Model



6.4 cont.

ex. A 3-ϕ, Y-connected, 220 V_{rms}, 10hp, 60Hz, 6-pole induction motor has

$R_1 = 0.3\,\Omega$, $X_1 = 0.5\,\Omega$

$R_2 = 0.15\,\Omega$, $X_2 = 0.21\,\Omega$

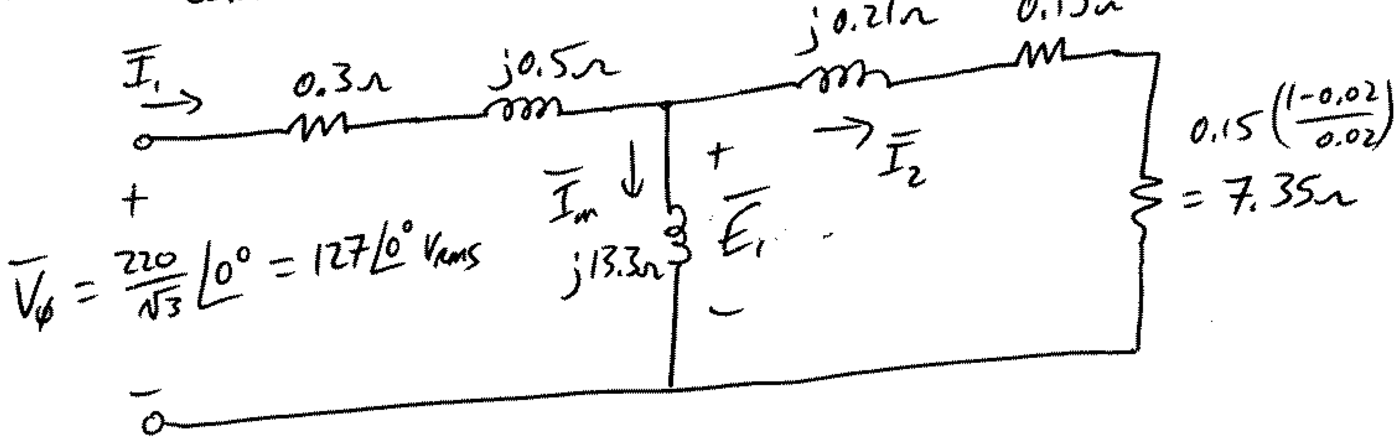
and $X_m = 13.3\,\Omega$ w/ R_c omitted

The total friction, windage, + core losses are 405W (roughly constant). For a slip of 2%, analyze the motor.

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60)}{6} = 1200 \text{ RPM}$$

$$\omega_{sync} = n_{sync} \frac{\pi}{30} = 125.663706 \text{ rad/s}$$

shaft speed
$$\begin{cases} n_m = (1-s)\,n_{sync} = (1-0.02)\,1200 = 1176 \text{ RPM} \\ \omega_m = 123.1504 \text{ rad/s} \end{cases}$$



6.4 cont.

ex. cont.

To find $\overline{I}_1$ (stator current), determine input impedance

$$\overline{Z}_{TOT} = (0.3+j0.5) + (j13.3)//(7.35+0.15+j0.21)$$
$$= (0.3+j0.5) + (5.5563+j3.229)$$
$$= 5.8563 + j3.7913\,\Omega = \underline{6.976379\angle 32.91845^\circ\,\Omega}$$

$$\overline{I}_1 = \frac{\overline{V}_\phi}{\overline{Z}_{TOT}} = \frac{220/\sqrt{3}}{6.976\angle 32.92^\circ} = \underline{\underline{18.2067\angle -32.9184^\circ\ A_{rms}}}$$

$$pf = \cos\theta = \cos(32.9184467^\circ) = \underline{\underline{0.8394\ \text{lagging}}}$$

$$P_{in} = \sqrt{3}\,V_L I_L \cos\theta = \sqrt{3}(220)(18.2067)\cos 32.918^\circ$$
$$\underline{\underline{P_{in} = 5823.8\ W}}$$

$$P_{SCL} = 3|\overline{I}_1|^2 R_1 = 3(18.2067)^2\, 0.3 = \underline{298.336\ W}$$

$$P_{AG} = P_{in} - P_{SCL} = 5823.8 - 298.336 = \underline{5525.5\ W}$$

$$P_{conv} = (1-s)P_{AG} = (1-0.02)5525.5 = \underline{\underline{5415\ W}}$$

$$P_{out} = P_{conv} - P_{rot} = 5415 - 405 = \underline{\underline{5010\ W}}$$
$$= (5010)\left(\frac{1\,hp}{746\,W}\right) = \underline{6.716\ hp}$$

6.4 cont.

ex. cont.

$$\bar{I}_2 = \bar{I}_1 \frac{j13.3}{j13.3 + (7.5 + j0.21)} = 15.671 \angle{-3.882^\circ} \text{ Arms}$$

$$\bar{I}_m = \bar{I}_1 - \bar{I}_2 = \frac{\bar{E}_1}{j13.3} = 8.84 \angle{-92.78^\circ} \text{ Arms}$$

$$\bar{E}_1 = \bar{V}_\phi \frac{j13.3 // (7.5 + j0.21)}{\bar{Z}_{TOT}} = 117.6 \angle{-2.78^\circ} \text{ Vrms}$$

$$P_{RCL} = 3 |\bar{I}_2|^2 R_2 = 3(15.67)^2 (0.15)$$

$$P_{RCL} = 110.5 \text{ W}$$

$$P_{RCL} = s P_{AG} = (0.02)(5525.5) = 110.5 \text{ W}$$

6.4 cont.

ex. cont.

$$\eta = \frac{P_{OUT}}{P_{in}} \times 100\% = \frac{5010}{5823.8} \times 100\% = \underline{\underline{86\%}}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{5525.5}{125.6637} = \underline{\underline{43.97\ Nm}}$$

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{5010}{123.1504} = \underline{\underline{40.682\ Nm}}$$

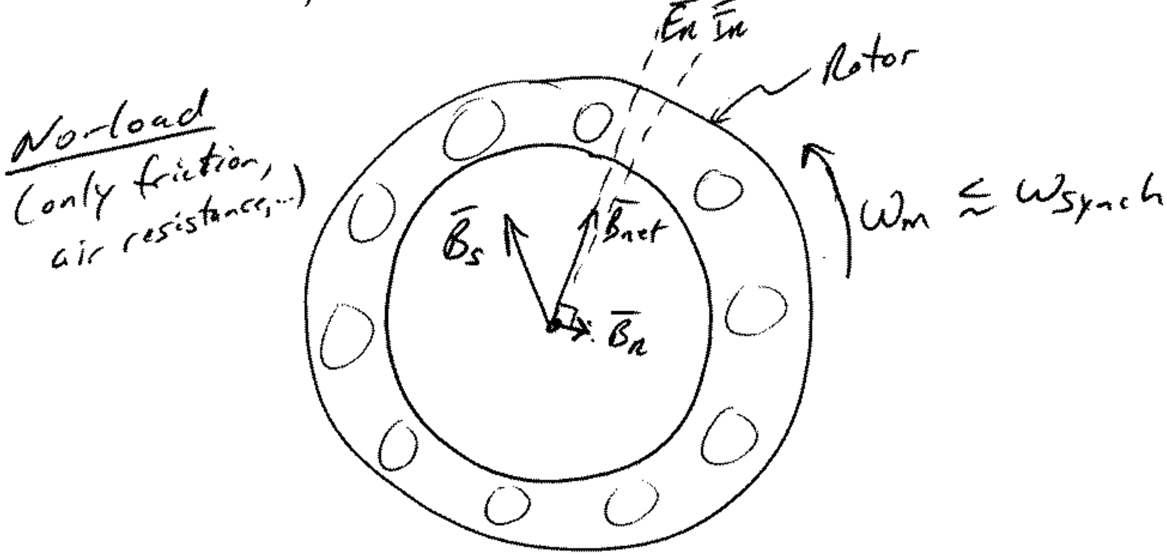
or in English units

$$\tau_{ind} = (43.97)\left(0.7372\ \frac{\text{lb-ft}}{1\text{Nm}}\right) = \underline{\underline{32.4\ \text{lb-ft}}}$$

$$\tau_{load} = 40.682(0.7372) = \underline{\underline{30\ \text{lb-ft}}}$$

6.5 Induction Motor Torque-Speed Characteristics

→ very important topic for all motors



* $\bar{B}_{net}$ due to $\bar{I}_m$ & $|\bar{I}_m| \propto |\bar{E}_1|$
* s very small since $\omega_m \lesssim \omega_{synch}$
  - ↳ f_r very small & v_{rel} very small
  - ↳ $\bar{E}_R$ very small & $\bar{I}_R$ very small
    - ↳ sX_{R0} very small

⇓

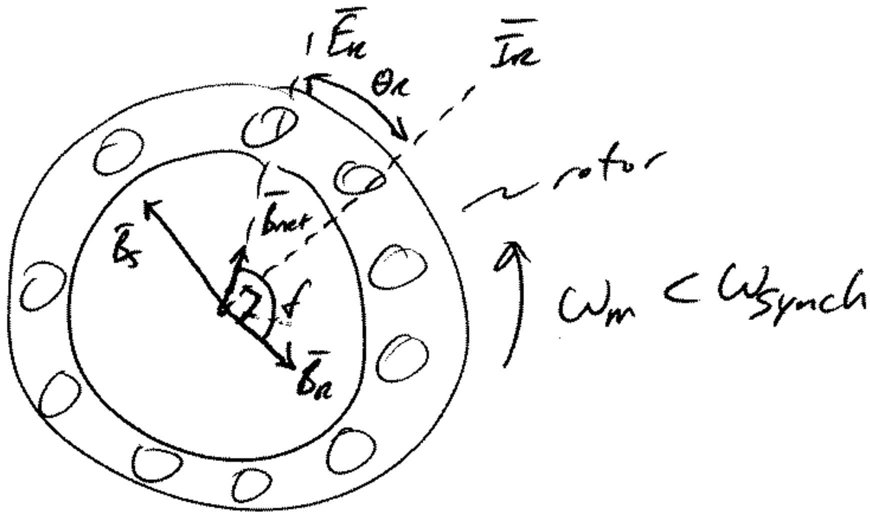
$\bar{E}_R$ & $\bar{I}_R$ approximately in phase!

* $\bar{B}_R \simeq 90°$ behind $\bar{B}_{net}$ & <u>small</u> $\delta \simeq 90°$

$\bar{\tau}_{ind} = k\bar{B}_R \times \bar{B}_{net} \rightarrow \tau_{ind} = kB_R B_{net} \sin\delta$

6.5 cont.

Big Load



* s larger → f_r bigger & v_{rel} bigger
  ↳ $\overline{E}_R$ bigger & $\overline{I}_R$ bigger

  Now sX_{R0} is bigger ⇒ $\overline{I}_R$ lags $\overline{E}_R$

* $\overline{B}_R$ now more than 90° behind $\overline{B}_{net}$ ($\delta > 90°$) and larger

  $|\overline{B}_R| \uparrow$ ⇒ more torque (bigger effect)

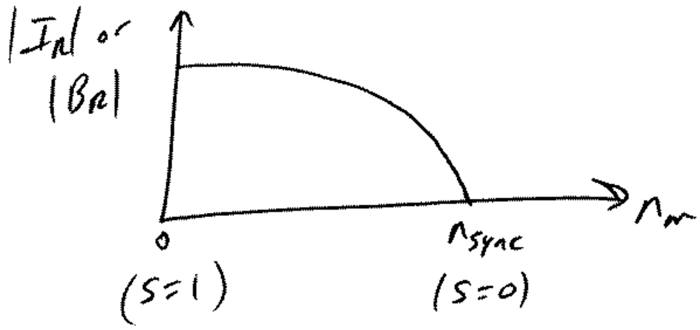
  but $\sin\delta \downarrow$ ⇒ less torque

At some point, the decrease in $\sin\delta$ is equal to increase due to $|\overline{B}_R|$ and the increase in τ_{ind} stops ⇒ any further increase in load will cause motor to stop!

6.5 cont.

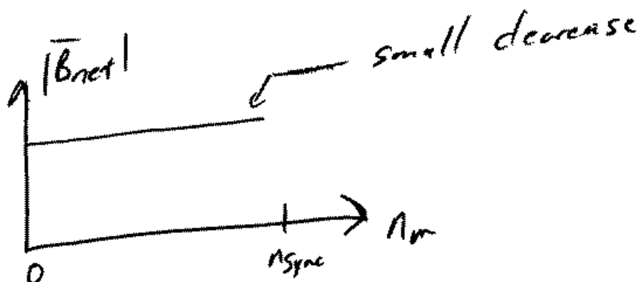
Summary

1) $\bar{B}_R \propto \bar{I}_R$ and $\bar{I}_R \uparrow$ w/ $s \uparrow$



2) $|\bar{B}_{net}| \approx$ constant assuming losses across $R_1 + jX_1$ are small

$\propto |\bar{I}_m| \propto |\bar{E}_1|$

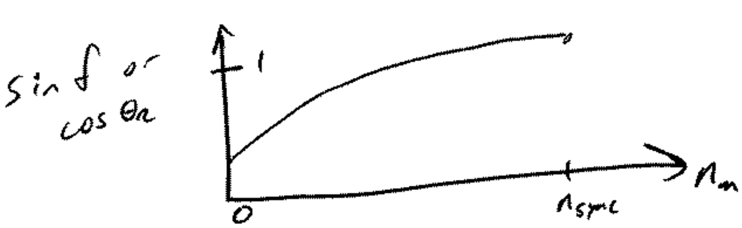


3) $\sin\delta \Rightarrow \delta = \theta_R + 90^\circ$

power factor angle of rotor

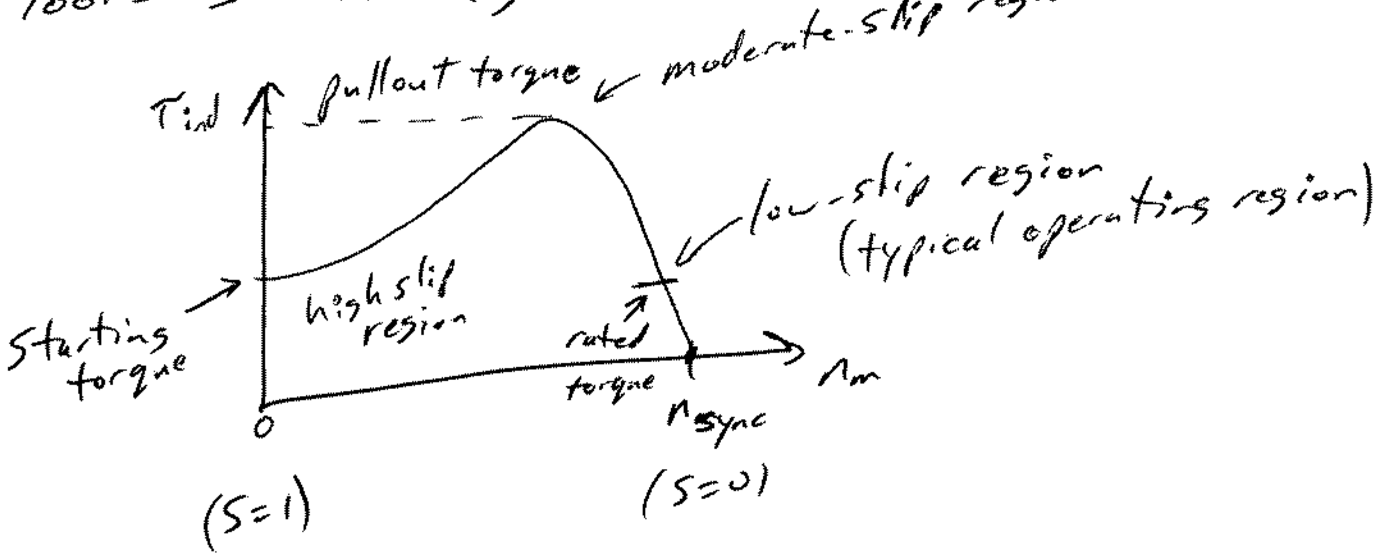
$$\sin(\theta_R + 90^\circ) = \cos\theta_R = pf_R$$

where $\theta_R = \tan^{-1}\left(\frac{X_R}{R_R}\right) = \tan^{-1}\left(\frac{sX_{R0}}{R_R}\right)$



6.5 cont.

The overall Torque vs n_m curve then looks something like –



→ pullout torque usually 2 to 2.5 times rated load torque

→ starting torque ~1.5 to 2 times rated load torque

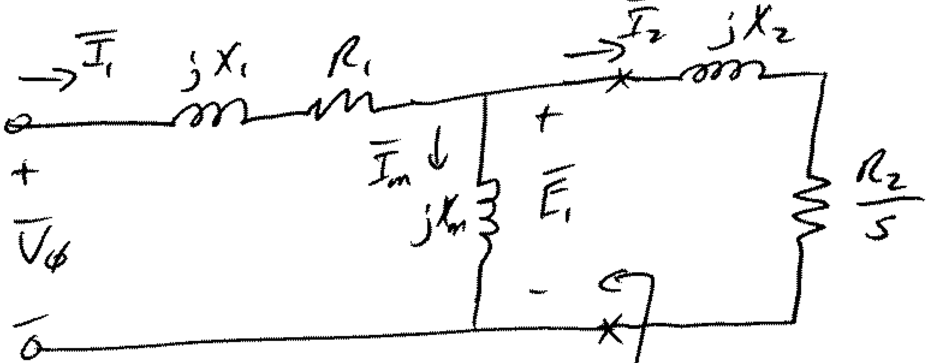
$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{AG}}{\omega_{sync}}$$

↳ Can we put this in terms of equivalent ckt parameters?

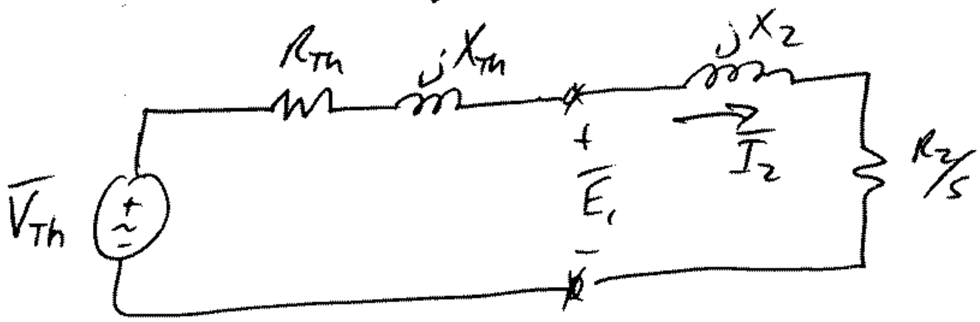
yes!

6.5 cont.

Ignoring/neglecting R_{core}



Find Thevenin equivalent circuit



$$\overline{V}_{Th} = \overline{V}_\phi \left(\frac{jX_m}{R_1 + jX_1 + jX_m} \right)$$

magnitude $$V_{Th} = \frac{V_\phi X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} \simeq \frac{V_\phi X_m}{X_1 + X_m} \quad (R_1 << X_1 + X_m)$$

$$\overline{Z}_{Th} = R_{Th} + jX_{Th} = \frac{jX_m (R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

approx. $$R_{Th} \simeq R_1 \left(\frac{X_m}{X_1 + X_m} \right)^2$$

$$X_{Th} \simeq X_1$$

6.5 cont.

$$\bar{I}_2 = \frac{\bar{V}_{TH}}{R_{TH} + R_2/s + j(X_2 + X_{TH})}$$

$$I_2 = \frac{V_{TH}}{\sqrt{(R_{TH} + R_2/s)^2 + (X_2 + X_{TH})^2}}$$

$$P_{AG} = 3 I_2^2 \, R_2/s$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

$$\tau_{ind} = \frac{3 V_{TH}^2 (R_2/s)}{\omega_{sync}\left[(R_{TH} + R_2/s)^2 + (X_2 + X_{TH})^2\right]}$$

Notes:

* Can stop induction motor by reversing $\bar{B}_{stator}$ (ie switch phases)
* τ_{ind} vs n_m ~linear near n_{synch} (low-slip)
  $\rightarrow R_2/s >> X_R$
* can't exceed pull-out torque (motor stops)
* starting torque > full-load torque (else it won't start!)
* $\tau_{ind} \propto (V_{TH})^2$
* $n_m > n_{sync} \Rightarrow$ generator!

6.5 cont.

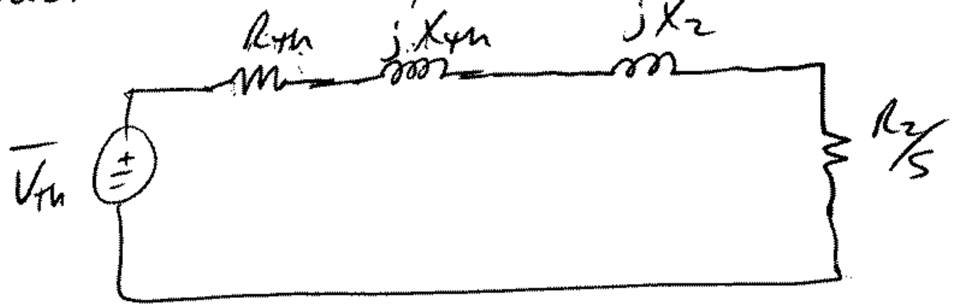
Be nice to know $\tau_{ind,max}$ (AKA pullout torque)

$$\tau_{ind,max} = \frac{(P_{AG})_{max}}{\omega_{sync}}$$

where $P_{AG} = 3 I_2^2 \, R_2/s$

Need to find where P_{AG} is maximized

→ Back in circuits, we found for the ckt



max. power is transferred when

$$R_2/s_{max} = \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}$$

$$s_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$

* directly proportional to $R_2 = a_{eff}^2 R_R$!

6.5 cont.

Putting s_{max} into our τ_{ind} eqn yields -

$$(\tau_{ind})_{max} = \tau_{max} = \frac{3 V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

Note! τ_{max} doesn't depend on $R_2 = a_{eff}^2 R_R$

but s_{max} does depend on $R_2 = a_{eff}^2 R_R$

$\Downarrow$

By controlling R_R, we can put τ_{max} at whatever n_m (speed) we want!

Application - move τ_{max} to near $n_m = 0$ when starting motor by increasing R_R ($s_{max} \simeq 1$ corresponds to $n_m = 0$); remove once motor gets moving

$\Downarrow$

more starting torque!

6.5 cont.

ex. Revisit earlier example

3ϕ, Y-connected, 220 Vrms, 10 hp, 60 Hz, 6-pole

motor w/ $R_1 = 0.3\,\Omega$ $X_1 = 0.5\,\Omega$

$R_2 = 0.15\,\Omega$ $X_2 = 0.21\,\Omega$

and $X_m = 13.3\,\Omega$

where

$n_{sync} = 1200$ RPM & $\omega_{sync} = 125.6637$ rad/s

Find s_{max}, T_{max}, $T_{start\text{-}up}$

$$\overline{V}_{Th} = \overline{V}_\phi \frac{jX_m}{R_1 + j(X_1 + X_m)} = \left(\frac{220}{\sqrt{3}}\angle 0^\circ\right)\frac{j13.3}{0.3 + j(0.5 + 13.3)}$$

$$= 122.386\angle 1.2454^\circ \text{ Vrms} \qquad \left(\frac{220}{\sqrt{3}} = 127 \text{ Vrms}\right)$$

$$\overline{Z}_{Th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} = \frac{(j13.3)(0.3 + j0.5)}{0.3 + j(13.3 + 0.5)}$$

$$= 0.278523 + j0.48794\,\Omega$$

$$s_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}} = \frac{0.15}{\sqrt{0.2785^2 + (0.488 + 0.21)^2}}$$

$$= 0.1996 \simeq 20\% \text{ slip}$$

6.5 cont.

ex. cont.

$$\tau_{max} = \frac{3V_{Th}^2}{2\omega_{sync}\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}\right]}$$

$$= \frac{3(122.386)^2}{2(125.6637)\left[0.2785 + \sqrt{0.2785^2 + (0.488 + 0.21)^2}\right]}$$

$$\tau_{max} = 173.6 \text{ N·m} \simeq 128 \text{ ft-lbs}$$

@ $\omega_{max} = (1 - s_{max})\omega_{sync} = 100.58 \text{ rad/s}$

$n_{max} = (1 - s_{max})n_{sync} = 960.5 \text{ rpm}$

$$\tau_{start}(s=1) = \frac{3V_{Th}^2(R_2/1)}{\omega_{sync}\left[(R_{Th} + R_2/1)^2 + (X_{Th} + X_2)^2\right]}$$

$$= \frac{3(122.386)^2(0.15)}{125.6637\left[(0.2785 + 0.15)^2 + (0.488 + 0.21)^2\right]}$$

$$= 79.97 \text{ N·m} = 59 \text{ ft-lbs}$$

6.6 Variations in Induction Motor Torque-Speed Characteristics

Trade-offs

* Bigger R_R → better start-up torque but worse steady-state efficiency (if R_R can't be varied as w/ wound rotor motors)

* It is possible, w/in limits, to cause R_R to vary w/ f_R based on rotor bar cross section (read section 7.6)

* Leads to various "design classes" for cage rotor induction motors as designated by Nat'l Electrical Manf'turers Assoc. (NEMA)

6.7 Trends in Induction Motor Design

* Better materials e.g. mag. steels, insulation, ...
* Better efficiency
* Better construction (e.g. smaller air gaps)

⋮

6.8 Starting Induction Motors

→ Got to be careful w/ initial current surge; can cause voltage dips

→ Can estimate start-up current from NEMA code letter
Table shown in Fig 6-34

$$S_{start} = (\text{rated hp})(\text{code letter factor})$$

$$I_{L,start} = \frac{S_{start}}{\sqrt{3}\, V_T}$$

e.x. Use earlier 10hp motor example w/ 220 Vrms and assume code letter B

upper bound to be safe

$$S_{start} = (10\,hp)\left(3.55\,\frac{kVA}{hp}\right) = 35.5\,kVA$$

$$I_{L,start} = \frac{35.5 \times 10^3}{\sqrt{3}\,(220)} = \underline{93.16\ A_{rms}}\ !$$

(earlier example had $I_L = 18.2$ Arms @ 2% slip)

6.8 cont.

How can we deal w/ this large current?

→ In-line inductors to damp down current surge (or resistors)

→ Start w/ lower V_T before applying full V_T (reduces torque) using switches and/or an auto transformer

→ Also include various protection features

* Short ckt protection w/ fuses
* Overload protection (some sort of thermal breaker)
* Under voltage protection (relay opens)

6.9 Speed Control of Induction Motors

* Various solid-state drives can accomplish this by:

    1) vary f_{el} + hence synchronous speed

    2) vary slip s by changing R_R and/or V_T

  * read on own

6.10 Solid-State Induction Motor Drives

→ can provide 0 to 120 Hz <u>and</u> 0 to Vrated <u>both</u>!

→ read on own

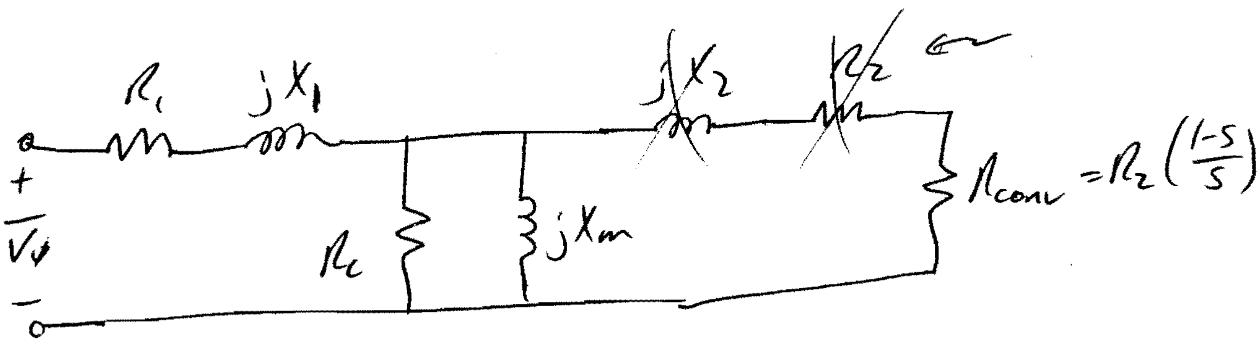
6.11 Determining Circuit Model Parameters

Want to find/measure $R_1, R_2, X_1, X_2,$ + X_m.

No-Load Test

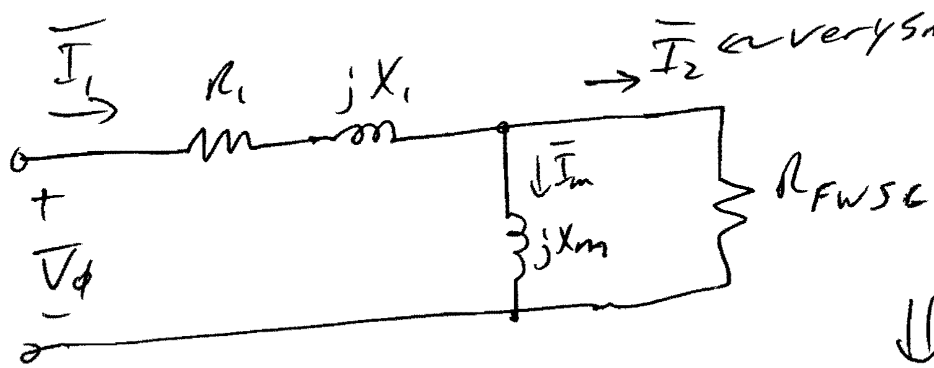
* Find rotational losses (e.g., friction, windage)
* slip very small ← @ No-load, this accounts for mech. losses

↳ $R_{conv} = R_2\left(\frac{1-s}{s}\right) \gg R_2$ and X_2



↓ combine R_c and R_{conv}

No-load



⇓

* much bigger than X_m
* lump together friction, windage, stray + core losses

6.11 cont.

No-load cont.

AKA: P_{stray}

$$P_{in,NL} = P_{SCL} + \underbrace{P_{core} + P_{F+W} + P_{misc}}_{P_{rot}}$$

$$= 3|\bar{I}_1|^2 R_1 + P_{rot}$$

Most of voltage drops across $jX_1 + jX_m$,

so

$$|Z_{eq}| = \frac{V_\phi}{I_{1,NL}} \approx \underline{X_1 + X_m}$$

assumes $R_{FWSC} \parallel jX_m \approx jX_m$

So, what do we measure?

1) All 3 line currents – $I_{L,NL} = \frac{I_A + I_B + I_C}{3}$
2) All 3 line-to-line voltages – $V_{LL,NL} = \frac{V_{AB} + V_{BC} + V_{CA}}{3}$
   (full rated voltage)

   $\uparrow V_{T,NL}$
3) $P_{in,NL}$

Taking averages

still need more info!

6.11 cont.

DC Test for Stator Resistance

Y-conn.: $R_T = 2R_1 \Rightarrow R_1 = R_T/2$

Δ-conn.: $R_T = \frac{2}{3}R_1 \Rightarrow R_1 = \frac{3}{2}R_T$

Result

How?

1) Use ohmmeter, measure R_{TAB}, R_{TBC}, R_{TCA}

$$R_T = \frac{R_{TAB} + R_{TBC} + R_{TCA}}{3}$$

2) More accurate to apply DC test voltage such that $I_{DC} = I_{L,rated}$ (heats up windings, simulating operating conditions

↳ Measure I_{DC} w/ multimeter
Measure V_{DC} w/ multimeter

$$R_T = \frac{V_{DC}}{I_{DC}}$$

for best accuracy repeat test across AB, BC, & CA and average

$$R_T = \frac{\left(\frac{V_{DC}}{I_{DC}}\right)_{AB} + \left(\frac{V_{DC}}{I_{DC}}\right)_{BC} + \left(\frac{V_{DC}}{I_{DC}}\right)_{CA}}{3}$$

6.11 cont.

Locked-Rotor Test ($s=1$) (AKA: Blocked-Rotor Test)

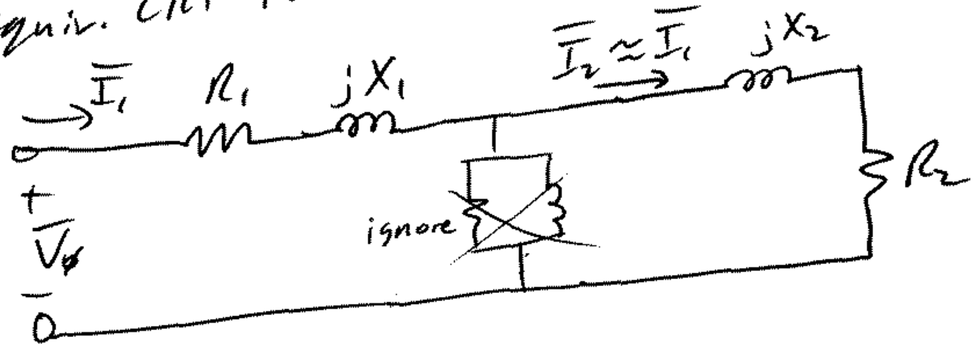
→ analogous to transformer short ckt test

→ Since $s=1$, $R_2/s = R_2$ (smaller #) and

$R_{conv} = 0$

→ $\bar{I}_m$ very small in comparison to $\bar{I}_2$ since $R_2 + jX_2$ are smaller than $R_c \| jX_m$

Equiv. Ckt for Locked-Rotor Test



Issue: What do we do about frequency?

In normal op'n, $f_R \approx 1$ to 3Hz at slips of 2 to 4%. W/ locked rotor $f_e = f_R$, so we can get abnormal results! Some testers will reduce f_e to 25% of normal [e.g. 0.25(60)=15Hz]. Others just use 60Hz + accept inaccuracy.

Locked-Rotor cont.

What do we measure?

1) Measure all 3 line currents – $I_{L,LR} = \dfrac{I_A + I_B + I_C}{3}$
   ($\sim I_{L,rated}$, adjust V_T to get)

2) Measure all 3 line-line voltages – $V_{LL,LR} = \dfrac{V_{AB} + V_{BC} + V_{CA}}{3}$
   (adjust w/ variac) $\uparrow V_{T,LR}$

3) Measure $P_{in,LR}$

Using this data, we get:

$$pf_{LR} = \cos\theta_{LR} = \frac{P_{in}}{\sqrt{3}\, V_{T,LR}\, I_{L,LR}}$$

$$\hookrightarrow \theta_{LR} = \cos^{-1}(pf_{LR})$$ ← also impedance phase $\measuredangle$

$$|\bar{Z}_{LR}| = \frac{V_\phi}{I_1} = \frac{V_{T,LR}}{\sqrt{3}\, I_{L,LR}}$$

$$\bar{Z}_{LR} = |\bar{Z}_{LR}| \angle \theta_{LR} = R_{LR} + jX'_{LR}$$

$$= |\bar{Z}_{LR}| \cos\theta_{LR} + j\,|\bar{Z}_{LR}| \sin\theta_{LR}$$

6.11 cont.

Now, we can start getting some equiv. ckt values.

$$R_{LR} = |Z_{LR}| \cos\theta_{LR} = R_1 + R_2$$

R_1 is known from DC test

$$R_2 = R_{LR} - R_1$$

$$X_{LR}' = X_1' + X_2' \quad \leftarrow \text{at } f_e = f_{Test}$$

$$X_{LR} = \frac{f_{rated}}{f_{test}} X_{LR}' = X_1 + X_2 \quad \leftarrow \text{No way to separate easily}$$

Fig. 6-56

Rule-of-thumb division of X_1 + X_2

Rotor Design	X_1	X_2
Wound	$0.5 X_{LR}$	$0.5 X_{LR}$
Design A	$0.5 X_{LR}$	$0.5 X_{LR}$
B	$0.4 X_{LR}$	$0.6 X_{LR}$
C	$0.3 X_{LR}$	$0.7 X_{LR}$
D	$0.5 X_{LR}$	$0.5 X_{LR}$

6.11 cont.

ex. A 5 hp, 60 Hz, Y-connected, 210 V_{rms}, 4-pole, design class A motor is tested to determine its per-phase equivalent circuit. The following results are measured –

No-Load Test: $210.5\ V_{rms} = V_{T,ave}$, $I_{L,ave} = 4.45\ A_{rms}$, and $250W = P_{in,NL}$

Locked Rotor Test: $V_{T,ave,LR} = 28.66\ V_{rms}$, $I_{L,ave,LR} = 12.8\ A_{rms}$, $P_{in,LR} = 480W$ @ $\underline{f_e = 15\,Hz}$

DC Test: $V_{DC} = 19.8V$ $\quad I_{DC} = 12.83A$

From DC Test: $R_T = \frac{V_{DC}}{I_{DC}} = \frac{19.8}{12.83} = 1.54326\,\Omega = 2R_1$ (Y-connected)

(6-61)

$$\underline{\underline{R_1 = 0.77163\,\Omega}}$$

From No-Load Test: $V_\phi = \frac{210.5}{\sqrt{3}} = 121.532\ V_{rms}$ ← Y-connected

(6-60) $|Z_{eq}| = \frac{V_{\phi NL}}{I_{,NL}} = \frac{121.532}{4.45} = 27.3106\,\Omega \cong X_1 + X_m$

(6-25) $P_{SCL} = 3I_1^2 R_1 = 3(4.45)^2 0.77163 = 45.84\ W$

(6-58) $P_{rot} = P_{in,NL} - P_{SCL,NL} = 250 - 45.84 = 204.16\ W$

6.11 cont.

ex. cont.

From Locked Rotor Test:

$$(6\text{-}63)\quad |Z_{LR}| = \frac{V_\phi}{I_1} = \frac{28.66/\sqrt{3}}{12.8} = 1.2927\ \Omega$$

$$(6\text{-}62)\quad pf = \cos\theta_{LR} = \frac{P_{in,LR}}{\sqrt{3}\,V_{T,LR}\,I_{L,LR}} = \frac{480}{\sqrt{3}\,(28.66)\,12.8} = 0.7554$$

$$\hookrightarrow \theta_{LR} = 40.937^\circ$$

$$\bar{Z}_{LR} = |Z_{LR}|\angle\theta_{LR} = 1.2927\angle 40.937^\circ = \underbrace{0.97656}_{R_{LR}} + j\underbrace{0.84703}_{X'_{LR}}\ \Omega$$

$$(6\text{-}65)\quad R_{LR} = R_1 + R_2 = 0.97656 \Rightarrow R_2 = 0.97656 - 0.77163\ \Omega$$

$$\underline{R_2 = 0.2049\ \Omega}$$

$$(6\text{-}68)\quad X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = \left(\frac{60}{15}\right)(0.84703) = 3.38812\ \Omega$$

$$= X_1 + X_2$$

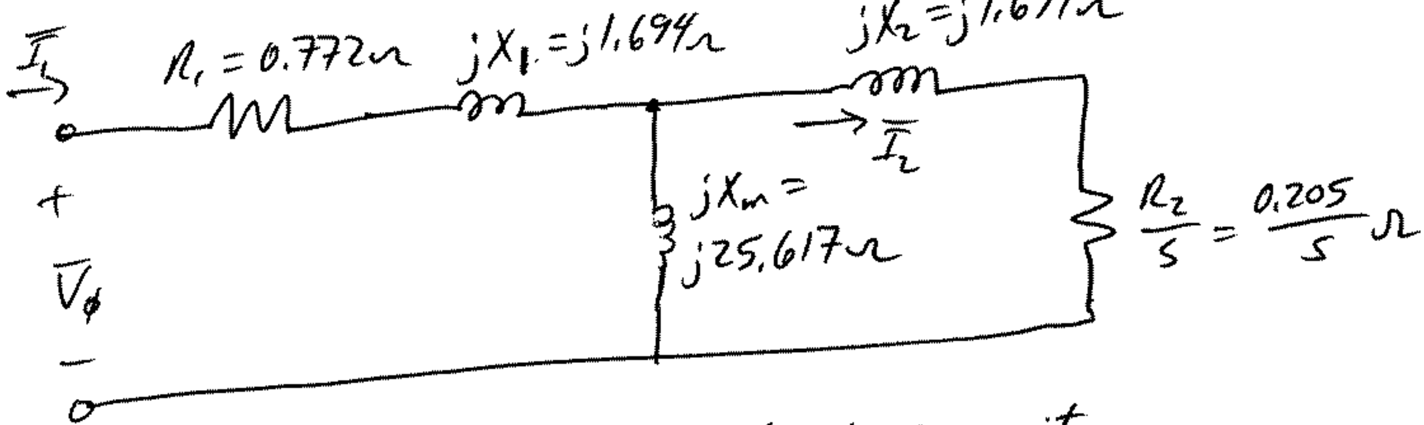
Per Fig. 6-56, $X_1 = 0.5 X_{LR}$ & $X_2 = 0.5 X_{LR}$ for class A

$$\underline{X_1 = X_2 = 1.694\ \Omega}$$

$$X_m = 27.3106 - 1.69406 = \underline{25.61654\ \Omega}$$

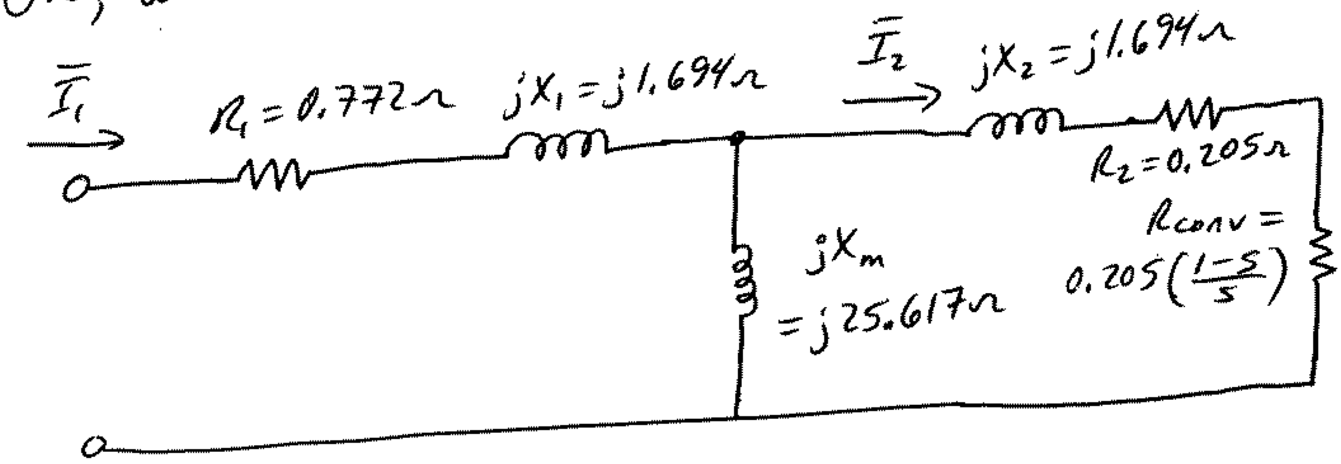
6.11 cont

ex. cont.



Per-phase Equivalent Circuit

OR, with R_2 & R_{conv} separated,



6.12 Induction Generator

* If an induction machine is driven at a mechanical speed $n_m > n_{synch} \Rightarrow$ Generator

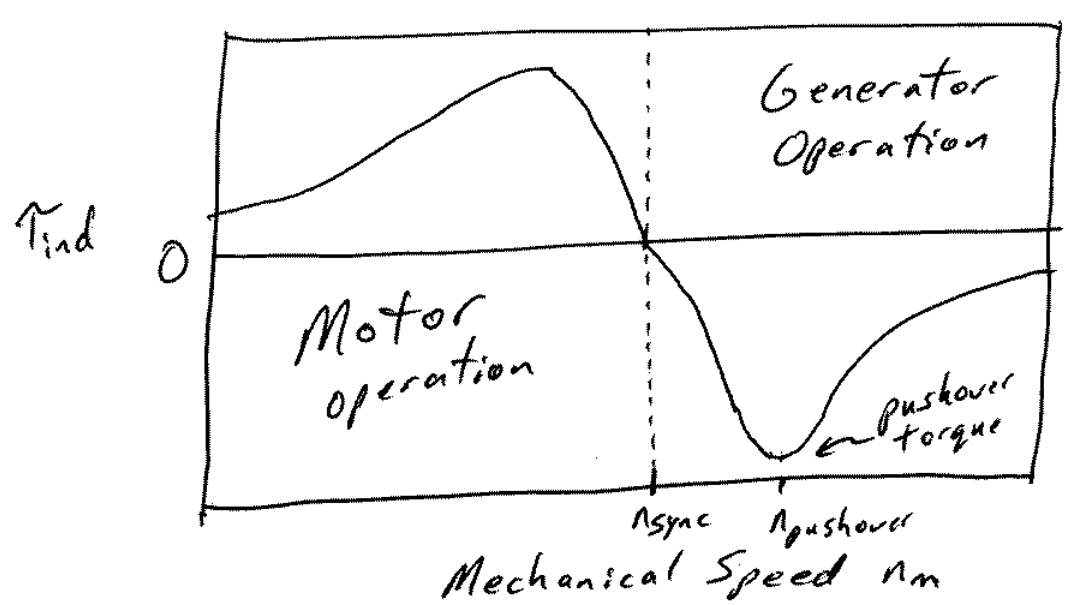
* As shown in typical torque-speed characteristic, there is a maximum possible induced torque for an induction generator ($\tau_{pushover}$). If n_m exceed $n_{pushover}$, the generator will overspeed. why?

Below $n_{pushover}$, $\Sigma\tau = \tau_{prime\ mover} + \tau_{induced} = 0$

$\hookrightarrow \tau = J\alpha \Rightarrow \alpha = 0$ (No acceleration)

Above $n_{pushover}$, $\Sigma\tau = \tau_{prime\ move} + \tau_{induced} > 0$

$\Rightarrow \alpha > 0$ (generator accelerates until something gives)



6.12 cont.

Other problems w/ induction generators:

* Can NOT produce reactive power
* Can NOT control output voltage

⇓

Need external source

Advantage – very simple construction/light

– used (some) w/ windmills
(rectify AC output to DC, convert back to 60Hz AC)

– also used to recover/scavenge energy/power (competes w/ permanent magnet generators).

6.13 Induction Motor Ratings

Output Power (e.g. ½ hp)

Voltage (e.g. 208 V_{rms}) ← what will saturate mag. core

Current (e.g. 10 A_{rms}) ← what will overheat windings

power factor

Speed

Nominal efficiency

NEMA Design Class

Start Code

⇒ Not all of these quantities will be given/available on all motors