Chapter 4 AC Machinery Fundamentals

→ directly convert AC electrical energy/power into mechanical energy/power

Two Main Types:

1) Synchronous Machines - most big generators - magnetic field supplied by separate dc source

2) Induction Machines - mainstay of electrical motors
   * magnetic field generated by applied ac source through transformer action/magnetic induction

4.1 Simple Loop in a Uniform Magnetic Field

![Diagram of a simple loop in a uniform magnetic field with labels for stator, rotor, B, loop, and magnetic field.]
Voltage Induced in a Simple Rotating Loop

Use \( E_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{L} \) to find induced voltage on all 4 sides of rotor.

1) Side ab
\[
\mathbf{V}_{ab} \times \mathbf{B} = V_{ab} B \sin \theta \hat{a}_{\text{into pole}}
\]
\( \mathbf{L} \) in direction of wire

\[
E_{ab} = V_{ab} B L \sin \theta = V B L \sin \theta
\]

2) Side bc
\[
\mathbf{V}_{\text{axis-b}} = - \mathbf{V}_{\text{axis-c}} \text{ straddles axis}
\]

\[
E_{bc} = E_{cb} = 0 \text{ cancel out}
\]

3) Side cd
\[
\mathbf{V}_{cd} \times \mathbf{B} = V_{cd} B \sin \theta \hat{a}_{\text{out of pole}}
\]

\[
E_{dc} = V_{cd} B L \sin \theta
\]

\[
E_{ed} = V B L \sin \theta
\]

4) Side d\(a \) \( \Rightarrow \) ditto 2) \( E_{da} = E_{ad} = 0 \)
\[ E_{\text{Tot}} = E_{ab} + E_{bc} + E_{cd} + E_{da} \]

\[ E_{\text{tot}} = 2vB\ell \sin \theta \]

where \( v = \omega_m t \) and \( \theta = \omega t \)

\[ E_{\text{tot}} = 2\omega_m B\ell \sin \omega_m t \quad \text{Note} \quad 2\ell = \text{Area of loop} \]

\[ E_{\text{tot}} = A\beta \omega_m \sin(\omega_m t) \]

Max flux through loop when \( \theta = \omega_m t = 90^\circ \)

\[ \Phi_{\text{max}} = A\beta \]

So

\[ E_{\text{tot}} = \Phi_{\text{max}} \omega_m \sin(\omega_m t) \]

**3 Factors:**

1. \( \Phi_{\text{max}} \) supplied
2. \( \omega_m \) rotation speed
3. Machine geometry size, materials, ...
Torque Induced in a Current-Carrying Loop

Now assume loop carries a current $i$

Each segment experiences a force

\[ \mathbf{F} = i \left( \mathbf{I} \times \mathbf{B} \right) \]

- Current length magnitude vector in direction of current

**Sides ab + cd**

\[ \mathbf{F}_{ab} = i \left( \mathbf{I} \times \mathbf{B} \right) \]
\[ = iI\mathbf{B} \text{ (down)} \]

\[ \mathbf{T}_{ab} = \mathbf{r} \times \mathbf{F}_{ab} \]
\[ = r i I \mathbf{B} \sin \theta \text{ (cw)} \]

\[ \mathbf{F}_{cd} = i \left( \mathbf{I} \times \mathbf{B} \right) \]
\[ = i I \mathbf{B} \text{ (up)} \]

\[ \mathbf{T}_{cd} = \mathbf{r} \times \mathbf{F}_{cd} \]
\[ = r i I \mathbf{B} \sin \theta \text{ (cw)} \]

\[ \text{Note: } \theta_{AB} = \theta_{CD} = \theta \]

H.S. geometry

**Ends bc + da**

\[ \text{No torque, force parallel to axis of rotation.} \]
Total Torque is then

\[ T_{\text{tot}} = T_{\text{induced}} = 2r iA B \sin \theta \]

\[ 2r l = A \equiv \text{Area} \]

\[ = iB A \sin \theta \]

\[ \theta = \omega t \]

\[ = iB A \sin (\omega t) \]

Note: Torque will vary sinusoidally (hard on machines)

Alternately (take-off / variation on magnetic dipole moments)

\[ T_{\text{ind}} = k \left( B_{\text{loop}} \times B_s \right) \]

\[ \Rightarrow T_{\text{ind}} = k \left( B_{\text{loop}} B_s \sin \theta \right) \]

\[ k = \frac{A G}{\mu} \]

Factors:
1) Stator flux density \( i \)
2) Loop flux density which is proportional to \( i \)
3) \( k \) between loop + stator flux
4) Machine geometry + materials (\( \mu \))
4.2 Rotating Magnetic Field

Problem w/ simple loop ⇒ torque varies as loop rotating

Solution

W/ three windings carrying 3φ power it is possible to produce a rotating magnetic field w/ constant magnitude ⇒ constant torque!

Note that $\overline{Ba}$, $\overline{Bb}$, $\overline{Cc}$ are all 120° wrt one another.

Show example

Note: $\overline{Ba}$ → physical surface normal
$\overline{Bb}$ → 11 @ $\theta = 0°$
$\overline{Cc}$ → 11 @ $\theta = 240°$
Three-phase power & rotating magnetic field example

\[
\begin{align*}
n &:= 0..360 \\
\omega t_n &:= n \cdot \frac{2 \cdot \pi}{360}
\end{align*}
\]

\[
\begin{align*}
B_{a_n} &:= 1 \cdot \sin(\omega t_n) \\
B_{b_n} &:= 1 \cdot \sin\left(\omega t_n - \frac{\pi}{3}\right) \cdot e^{j \frac{\pi}{3}} \\
B_{c_n} &:= 1 \cdot \sin\left(\omega t_n - \frac{2 \cdot \pi}{3}\right) \cdot e^{j \frac{2 \pi}{3}}
\end{align*}
\]

\[
B_{\text{tot}_n} := B_{a_n} + B_{b_n} + B_{c_n}
\]

Note magnitude is constant
What about the angle of $B_{\text{tot}}$?

$$B_{\text{tot}}\arg \frac{n}{\pi} := \arg(B_{\text{tot}}) \cdot \frac{180}{\pi}$$

Note angle changes linearly

The linear variation of the angle of $B_{\text{tot}}$ wrt time tells us that the direction of the magnetic flux density vector is rotating wrt time at a rate of one rotation per period of the three-phase power.
4.2 cont.

For this configuration, we get one rotation of $\overline{B_{tor}}$ per electrical period.

$\Rightarrow f_e = f_m$

$\omega_e = \omega_m$

$60\text{Hz} \rightarrow 3600 \text{rpm} = n_m$

2-poles

If we use multiple windings for each of the 3-phases, spaced evenly, we can increase the rotation frequency of $\overline{B_{tor}}$

E.g. $c_1'$ $a_2$

creates

4-poles

$\theta_e = 2\theta_m$

$f_e = 2f_m$

$\omega_e = 2\omega_m$

$\omega_e = 60\text{Hz} \rightarrow 1800 \text{rpm} = n_m$
In general, if we create $p$ poles, we get $\frac{p}{2}$ repetitions of the 3Φ windings and

$$\Theta_e = \frac{p}{2} \Theta_m$$

$$f_e = \frac{p}{2} f_m$$

$$\omega_e = \frac{p}{2} \omega_m$$

$$f_e = \frac{n_m \cdot p}{120} \Rightarrow n_m = f_e \frac{120}{p}$$

Reversing Direction of Magnetic Field Rotation

Why? Reverse Motor direction!

⇒ Swap any 2 of 3 coils' currents

i.e. trade $bb'$ with $cc'$

or