Chapter 4 AC Machinery Fundamentals

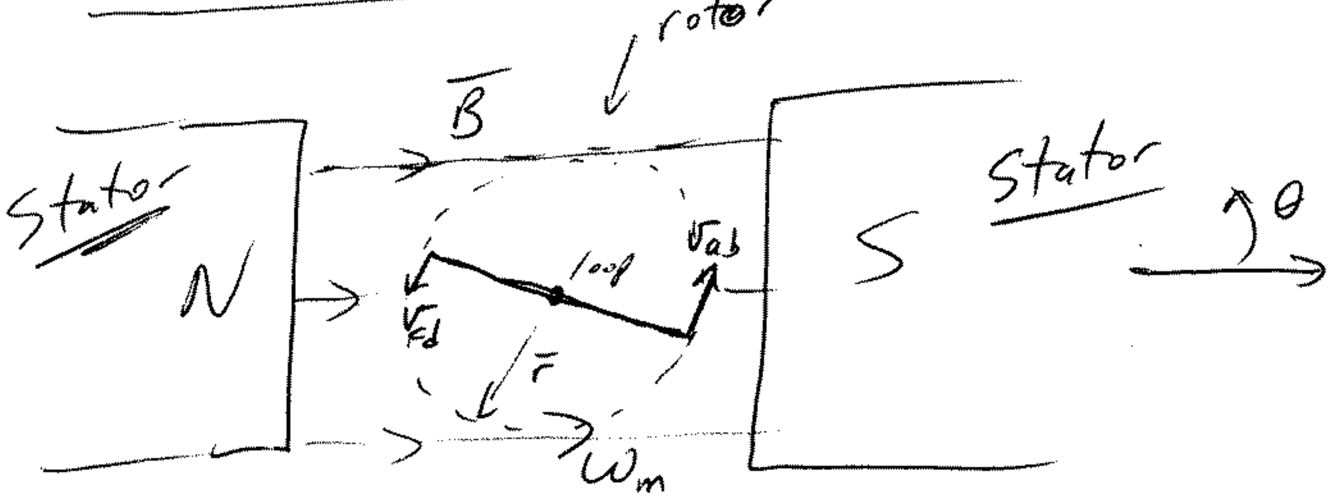
→ directly convert AC electrical energy/power into mechanical energy/power

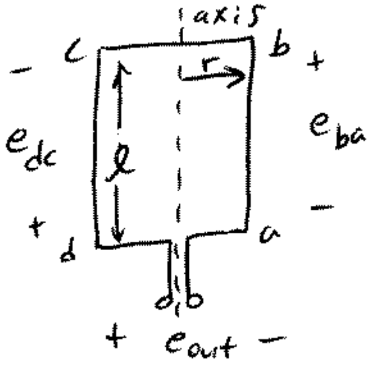
Two Main Types:

1) Synchronous Machines – Most big generators –
   * magnetic field supplied by separate dc source

2) Induction Machines – mainstay of electrical motors
   * magnetic field generated by applied ac source through transformer action/magnetic induction

4.1 Simple Loop in a Uniform Magnetic Field





← Top view of rotor

4.1 cont.

Voltage Induced in a Simple Rotating Loop

Use $e_{ind} = (\bar{v} \times \bar{B}) \cdot \bar{l}$ to find induced voltage on all 4 sides of rotor

1) Side ab

$$\bar{V}_{ab} \times \bar{B} = V_{ab} B \sin\theta \, \hat{a}_{\text{into page}}$$

$\bar{l}$ in direction of wire

$$\underline{e_{ab} = V_{ab} B l \sin\theta = v B l \sin\theta}$$

2) Side bc

$$\bar{V}_{axis-b} = -\bar{V}_{axis-c}$$

side bc straddles axis

$$e_{bc} = e_{cb} = 0$$

cancel out

3) Side cd

$$\bar{V}_{cd} \times \bar{B} = V_{cd} B \sin\theta \, \hat{a}_{\text{out of page}}$$

$$e_{dc} = V_{cd} B l \sin\theta$$

$$e_{dc} = v B l \sin\theta$$

4) Side da → ditto 2) $e_{da} = e_{ad} = 0$

4.1 cont.

$e_{TOT} = e_{ab} + e_{bc} + e_{dc} + e_{da}$

$$\underline{\underline{e_{TOT} = 2vBl\sin\theta}}$$

where $v = r\omega_m$ & $\theta = \omega_m t$

$e_{TOT} = 2r\omega_m Bl\sin\omega_m t$ Note $2rl$ = Area of loop

$$\underline{\underline{e_{TOT} = AB\omega_m \sin(\omega_m t)}}$$

Max flux thru loop when $\theta = \omega_m t = 90°$

$\phi_{max} = AB$

So

$$\underline{e_{TOT} = \phi_{max}\omega_m \sin(\omega_m t)}$$

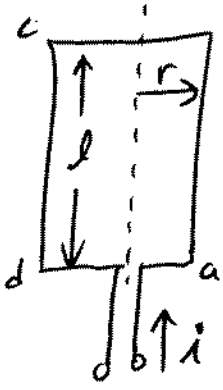
↳ Three factors:

GENERATOR

1) ϕ_{max} supplied
2) ω_m rotation speed
3) machine geometry size, materials, ...

Torque Induced in a Current-Carrying Loop

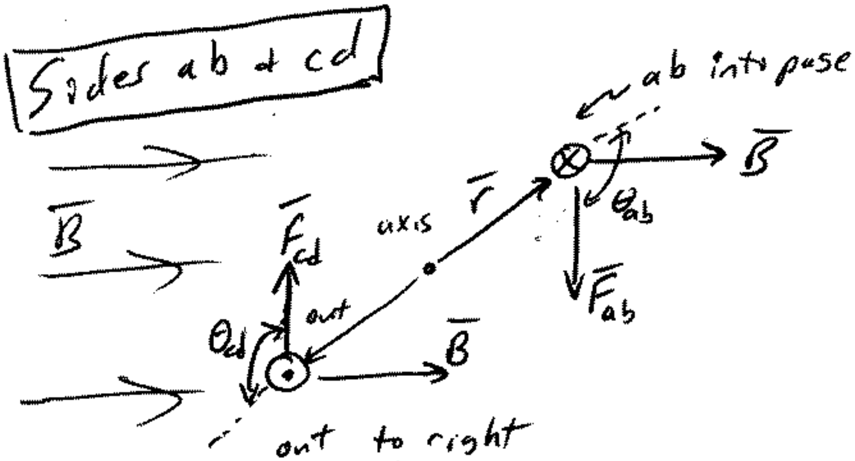
Now assume loop carries a current i



Each segment experiences a force

$$\overline{F} = \underset{\substack{\uparrow \\ \text{current} \\ \underline{\text{magnitude}}}}{i}\,(\underset{\substack{\uparrow \\ \text{length} \\ \text{vector} \\ \text{in direction} \\ \text{of current}}}{\overline{l}} \times \overline{B})$$

Sides ab + cd



$$\overline{F}_{ab} = i(\overset{\substack{\text{into page} \\ \downarrow}}{\overline{l}} \times \overset{\substack{\text{to right} \\ \swarrow}}{\overline{B}})$$

$$= i l B \text{ (down)}$$

$$\tau_{ab} = \overline{r} \times \overline{F}_{ab}$$

$$= r i l B \sin\theta_{AB} \text{ (cw)}$$

$$\overline{F}_{cd} = i(\overline{l} \times \overline{B})$$

$$= i l B \text{ (up)}$$

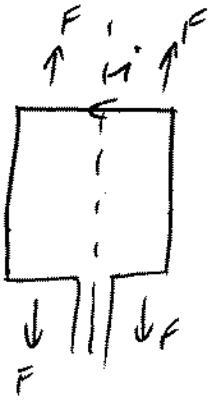
$$\tau_{cd} = \overline{r} \times \overline{F}_{cd}$$

$$= r i l B \sin\theta_{CD} \text{ (cw)}$$

Note: $\theta_{AB} = \theta_{CD} = \theta$

H.S. geometry

Ends bc + da



~ No torque, force parallel to axis of rotation.

4.1 cont.

Total Torque is then

$$\tau_{TOT} = \tau_{induced} = 2ri\ell B\sin\theta \qquad 2r\ell = A \equiv \text{Area}$$

$$= iBA\sin\theta \qquad \theta = \omega_m t$$

$$= iBA\sin(\omega_m t)$$

Note! Torque will vary sinusoidally (hard on machines)

Alternately (take-off/variation on magnetic dipole moments)

$$\bar{\tau}_{ind} = K\,\bar{B}_{loop} \times \bar{B}_s \quad \Rightarrow \quad \tau_{ind} = K B_{loop} B_S \sin\theta$$

stator mag. flux density

flux density produced by loop $B_{loop} = \frac{\mu i}{G}$ ← loop geometry factor

$$K = \frac{AG}{\mu}$$

Factors:

1) Stator flux density!

2) loop flux density which is proportional to i!

3) $\measuredangle$ between loop + stator flux

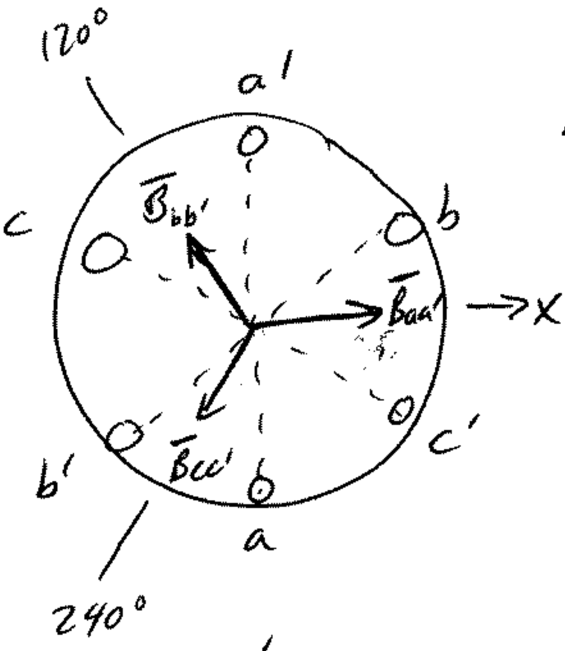
4) Machine geometry + materials (μ)

4.2 Rotating Magnetic Field

→ Problem w/ simple loop ⇒ torque varies as loop rotating ☹

⇓ Solution

→ W/ three windings carrying 3ϕ power it is possible to produce a rotating magnetic field w/ constant magnitude ⇒ constant torque!



Note that $\bar{B}_{aa'}$, $\bar{B}_{bb'}$, + $\bar{B}_{cc'}$ are all 120° wrt one another

Show example

Note: $\bar{B}_{aa'}$ → physical surface normal @ $\theta = 0^\circ$

$\bar{B}_{bb'}$ → " @ $\theta = 120^\circ$

$\bar{B}_{cc'}$ → " @ $\theta = 240^\circ$

Three-phase power & rotating magnetic field example

Cover one period *T* (2π radians) $n := 0 .. 360$ $wt_n := n \cdot \frac{2\cdot\pi}{360}$

$Baa_n := 1\cdot\sin(wt_n)$

Compute magnetic fields for each phase.

Note: In addition to 120 deg or π/3 phase shift due to electrical signals, there is a 120 deg or π/3 physical rotation the the locations of the coils.

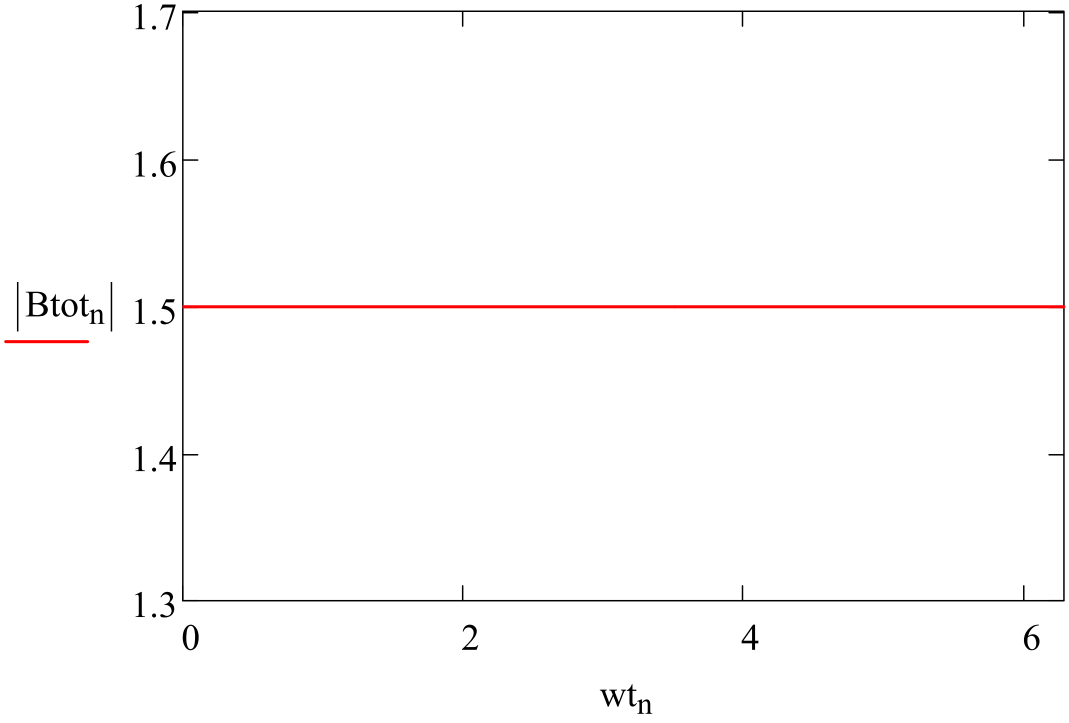
$$Bbb_n := 1\cdot\sin\left(wt_n - \frac{\pi}{3}\right)\cdot e^{j\cdot\frac{\pi}{3}}$$

$$Bcc_n := 1\cdot\sin\left(wt_n - \frac{2\cdot\pi}{3}\right)\cdot e^{j\cdot\frac{2\pi}{3}}$$

$Btot_n := Baa_n + Bbb_n + Bcc_n$

Add up magnetic fields for each phase to get total magnetic field.

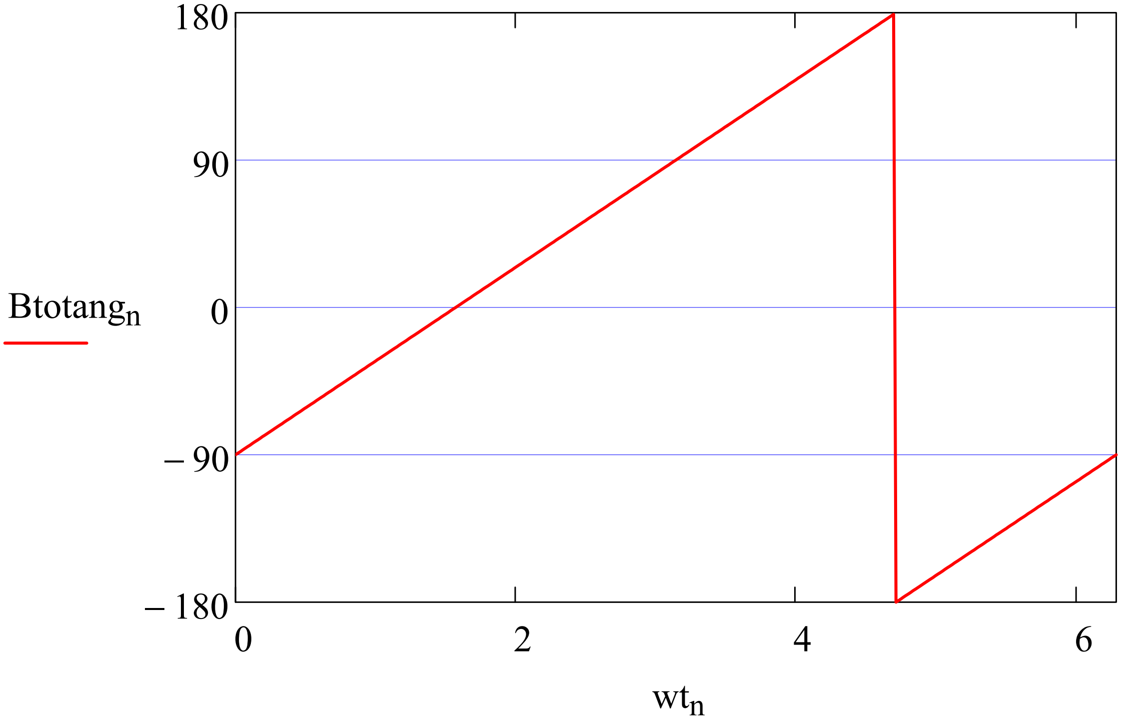
Note magnitude of Btot is constant



What about the angle/phase of B_{tot}?

$$Btotang_n := \arg(Btot_n)\cdot\frac{180}{\pi}$$

Note angle/phase changes linearly



The linear variation of the angle/phase of the total magnetic flux density vector $\mathbf{B}_{\mathbf{tot}}$ over a total of 360 degrees in one period *T* (2π radians) tells us that the direction of $\mathbf{B}_{\mathbf{tot}}$ is rotating (with respect to time) at a rate of **one** rotation per period *T* of the three-phase power.

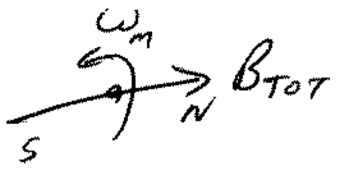
4.2 cont.

For this configuration, we get one rotation of $\overline{B}_{tot}$ per electrical period.

$\Rightarrow \; f_e = f_m$

$\omega_e = \omega_m$

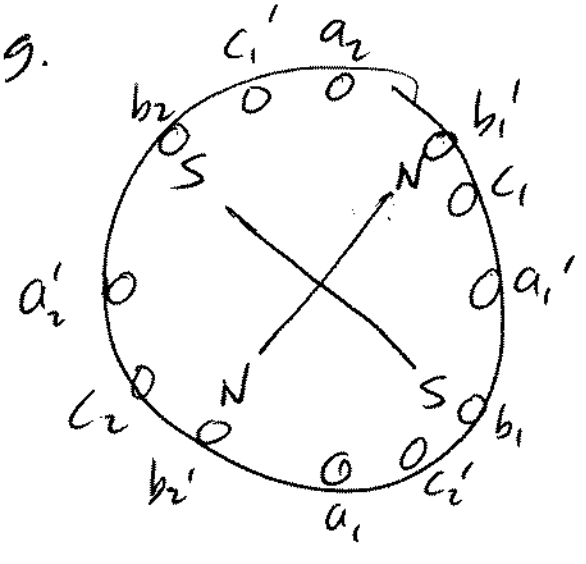
60Hz → 3600 rpm $= n_m$



2-poles

If we use multiple windings for each of the 3-phases, spaced evenly we can increase the rotation frequency of $\overline{B}_{TOT}$

e.g.



creates 4-poles

$\theta_e = 2\theta_m$

$f_e = 2f_m$

$\omega_e = 2\omega_m$

$f_e =$ 60Hz → 1800 rpm $= n_m$

4.2 cont.

In general, if we create P poles, we get $P/2$ repetitions of the 3ϕ windings and

$$\theta_e = \frac{P}{2}\,\theta_m$$

$$f_e = P/2\, f_m$$

$$\omega_e = P/2\, \omega_m$$

$$\hookrightarrow f_e = \frac{\overset{\text{rpm}}{n_m}\, P}{120} \Rightarrow \underline{\underline{n_m = f_e\,\frac{120}{P}}}$$

Reversing Direction of Magnetic Field Rotation

Why? Reverse Motor direction!

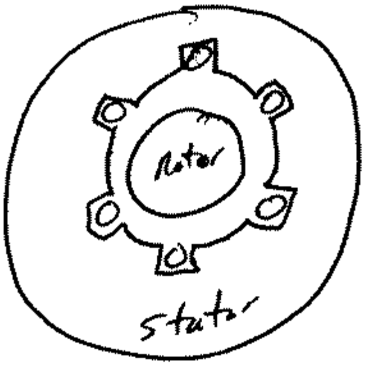
⇒ Swap any 2 of 3 coils' currents

i.e. trade bb' w/ cc'

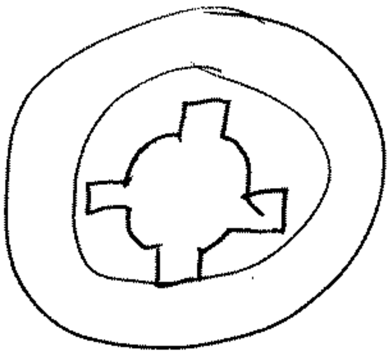
or

⋮

4.3 MMF and Flux Distribution on AC Machines



Non-salient Rotor (less air resistance)



Salient Rotor (easier to put windings on rotor)

⇒ Common issues are air gaps + how $\bar{B}_{rot}$ will not rotate perfectly smoothly in actual practice

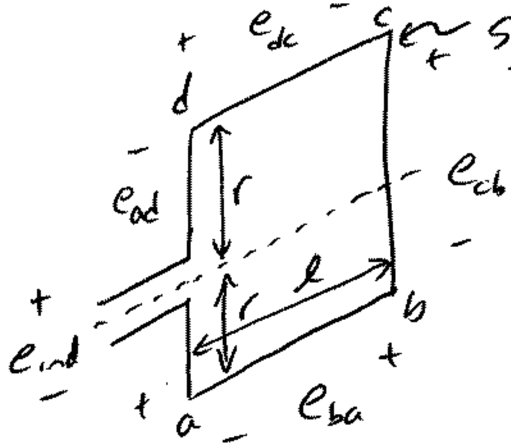
⇒ Get some ripple in B + ϕ as rotor rotates

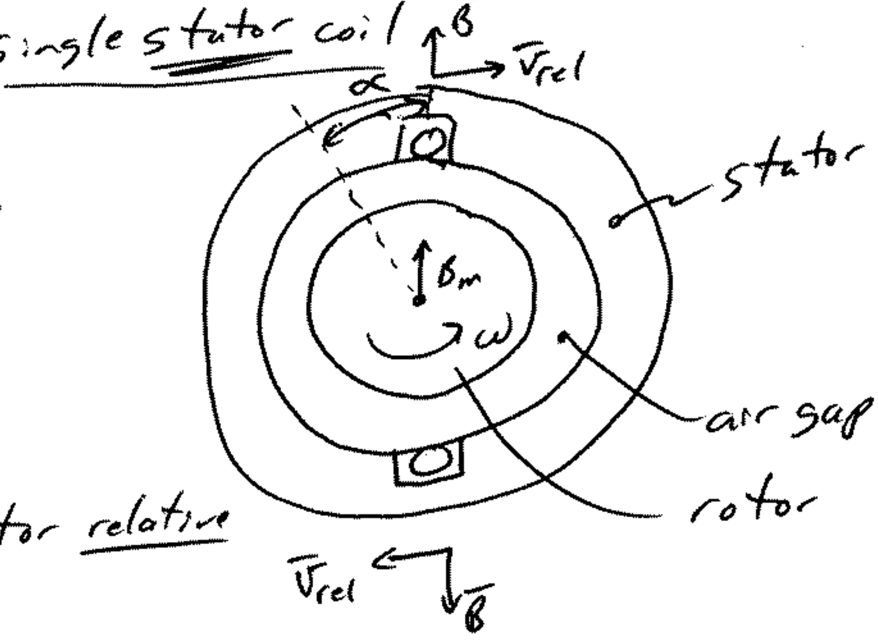
⇒ more slots / more windings → less ripple → practical limits

4.4 Induced Voltage in AC Machines

So far - 3ϕ stator ac power produced rotating magnetic field

Now, how does the magnetic field, produced by the rotor, induce a 3ϕ voltage in the stator





$\bar{v}_{rel}$ = velocity of stator relative to rotor

$\bar{B} \equiv$ rotor mag. flux density

$B = B_m \cos(\omega t - \alpha)$ ← arbitrary choice (straight up)

$\bar{l}$ = length of wire segment, points in direction making smallest ∡ wrt $\bar{v} \times \bar{B}$

Use $e = (\bar{v} \times \bar{B}) \cdot \bar{l}$

1) Side ab $\quad e_{ba} = vBl$

$= -v B_m l \cos(\omega t - 180°)$

2) side bc $\quad e_{cb} = 0$

4.4 cont.

3) side cd

$$e_{dc} = v B_m l \cos(\omega_m t)$$

4) side da $e_{da} = 0$

$$e_{ind} = -v B_m l \underbrace{\cos(\omega t - 180^\circ)}_{-\cos(\omega t)} + v B_m l \cos(\omega_m t)$$

$$= 2 v B_m l \cos(\omega_m t)$$ where $v = \omega_m r$

$$= \underbrace{\underbrace{2rl}_{\text{Area}} B_m}_{\max \phi} \omega_m \cos(\omega_m t)$$

$$\underline{\underline{e_{ind} = \phi \omega_m \cos(\omega_m t)}}$$ ← single turn

for multiple turns (N_c)

$$\underline{\underline{e_{ind} = N_c \phi \omega_m \cos(\omega_m t)}}$$

✓ could have been sin() if α chosen differently

STATOR

4.4 cont.

For 3 sets of coils → 3ϕ machine

$$e_{aa'}(t) = N_c \phi \omega \sin(\omega t) \ V$$

$$e_{bb'}(t) = N_c \phi \omega \sin(\omega t - 120°) \ V$$

$$e_{cc'}(t) = \underbrace{N_c \phi \omega}_{E_{max}} \sin(\omega t - 240°) \ V$$

Assumes each coil is identical

$$\omega_m = \omega_e = \omega = 2\pi f$$ (Two-pole case)

$$E_{max} = N_c \phi \omega$$

$$E_{RMS} = \frac{N_c \phi \omega}{\sqrt{2}} = \frac{N_c \phi 2\pi f}{\sqrt{2}} = \sqrt{2} \pi N_c \phi f$$

(per-phase (line-to-line depends on whether Δ or Y wiring)

4.5 Induced Torque in AC Machine

→ due to interaction of rotor mag. field + stator mag. field

$$\boxed{\begin{aligned}\bar{\tau}_{ind} &= K\,\bar{B}_r \times \bar{B}_{net} \\ \tau_{ind} &= K B_r B_{net} \sin\delta\end{aligned}}$$

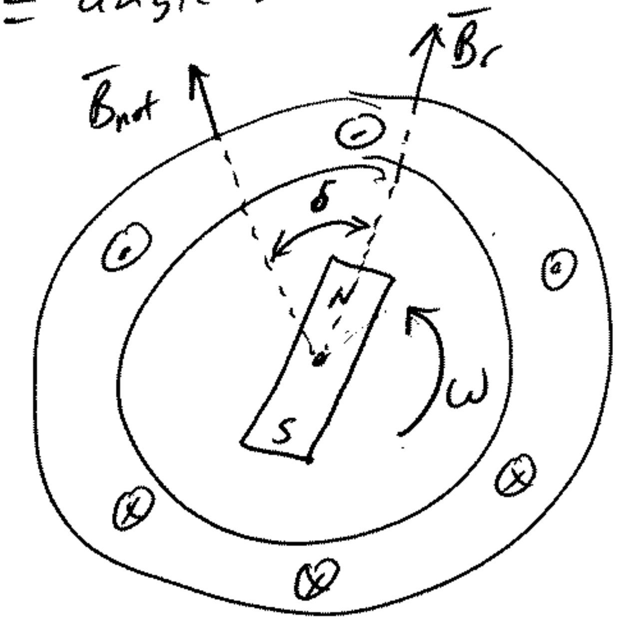
$\bar{B}_r$ = rotor mag. flux density vector

$\bar{B}_{net} = \bar{B}_r + \bar{B}_s \equiv$ net rotor + stator mag. flux density vector

$K \equiv$ machine constant, depends on geometry and materials

$\delta \equiv$ angle between $\bar{B}_{net}$ + $\bar{B}_r$ (smallest)

ex. Motor or Generator?



By RHR

$\bar{B}_r$ curls toward $\bar{B}_{net}$ in Same direction as ω

⇒ Motor

$\tau_{ind} = K B_r B_{net} \sin\delta$ (CCW)

4.6 Winding Insulation in AC Machine

→ Bearings + winding insulation most common failure causes in AC Machines

→ Usually fails due to heat degradation of insulation (as opposed to dielectric breakdown)

→ ratings based on temperature

class A, class B, ... class H

80°C 125°C

$ → $$$

See Fig. 4-20

↳ plot of life expectancy for each class of insulation vs. temperature.

4.7 AC Machine Power Flows + Losses

$$\eta \equiv \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$$

Motors $P_{in} \equiv$ electrical $P_{out} \equiv$ mechanical

Generators $P_{in} \equiv$ mechanical $P_{out} \equiv$ electrical

Losses

similar to transformers

1) Copper loss (I^2R)
2) Core loss (V^2/R_{core}) hysteresis + eddy currents
3) Mechanical losses (friction, air resist.)
4) Stray loads (grab bag for losses above that can't be pinned down)

Rule-of-thumb ~1% of full load

4.7 cont'

Copper Losses

→ Broken down further into those on stator (SCL) and those on rotor (RCL)

AC (RMS)

$P_{SCL} = 3 I_A^2 R_A$

$I_A \equiv$ current mag. on phase A

R_A = resistance of wires on phase A

⇒ Assume all 3φ identical

$P_{RCL} = I_F^2 R_F$ ← synchronous machine field winding on rotor (get to induction machines later)

$I_F \equiv$ current mag. of field winding (DC)

R_F = resistance of field winding

Core Losses

→ hysteresis + eddy current $\propto B^2$

$\propto (RPM)^{1.5}$

Mechanical Losses

Friction – bearings, brushes, slip rings, ... (moving parts in contact)

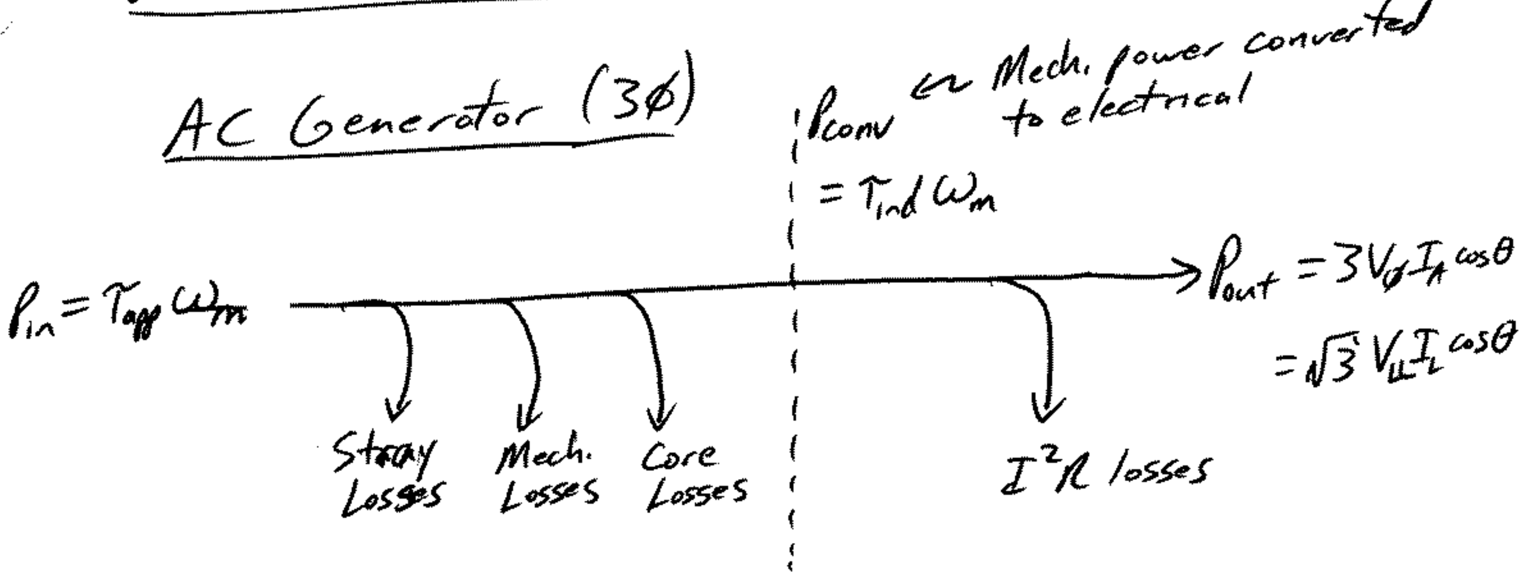
Windage – air resistance/drag on rotor $\propto RPM^3$

→ get values from No-load test on motor

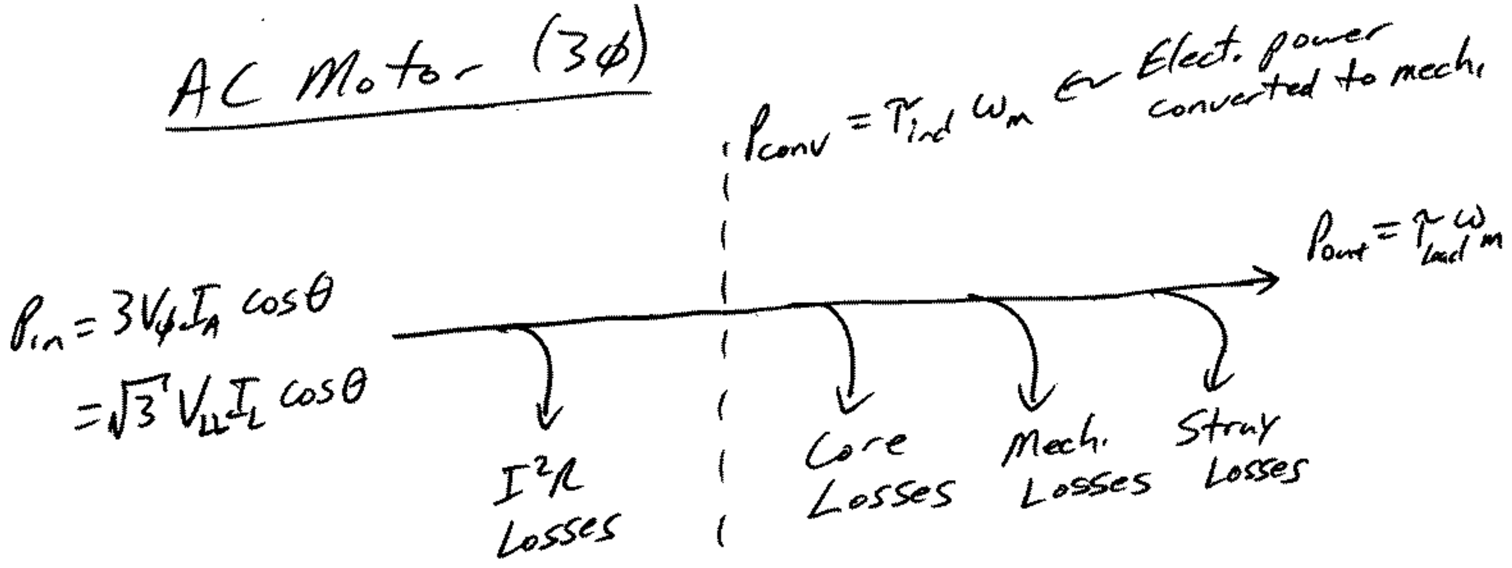
4.7 cont.

Power Flow Diagrams

AC Generator (3ϕ)



AC Motor (3ϕ)



4.8 Voltage Regulation and Speed Regulation

VR → generators use this as a figure-of-merit

→ how much does output voltage vary as load increased from no-load to full-load

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$

SR - motors use this as a figure-of-merit

→ can motor maintain constant speed as load increased to full load?

$$SR = \frac{n_{NL} - n_{FL}}{n_{FL}} \times 100\% \qquad (RPM)$$

$$SR = \frac{\omega_{NL} - \omega_{FL}}{\omega_{FL}} \times 100\% \qquad (\text{rad}/s)$$

Usually want VR & $SR \simeq 0$