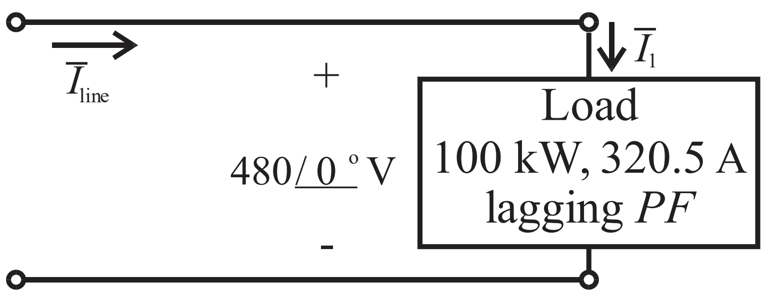
Example- A U.S. industrial plant ($f = 60$ Hz) is supplied by a 480 V power line. Without PF correction, the plant uses 100 kW (measured using wattmeter) and draws a current of 320.5 A (measured using ammeter). Given the equipment in the plant, the power factor is known to be lagging. For the best electrical rates, a power factor of $pf \geq 0.95$ is required by the utility company. Find the range of capacitances, connected in parallel, at the plant service entrance that will work. Also, determine the capacitance that will yield a $PF = 1$. To demonstrate why we do PF correction, calculate the input line current to the plant for the various cases.

- Arbitrarily make the input voltage the phase reference (i.e., 0°).

No PF correction



- The apparent power for the load (industrial plant) is

$$S_1 = V\,I_1 = 480\,(320.5) \quad \Rightarrow \quad \underline{\boldsymbol{S_1 = 153{,}840 \text{ VA} = 153.84 \text{ kVA}}}.$$

- The power factor for the load is

$$PF_1 = \frac{P_1}{S_1} = \frac{100 \text{ kW}}{153.84 \text{ kVA}} \quad \Rightarrow \quad \underline{\boldsymbol{PF_1 = 0.65 \text{ lagging}}}.$$

- This leads to a power angle for the load of

$$PF_1 = \frac{100 \text{ kW}}{153.84 \text{ kVA}} = \cos(\theta_1) \quad \Rightarrow \quad \theta_1 = \cos^{-1}(0.65) \quad \Rightarrow \quad \underline{\boldsymbol{\theta_1 = 49.4564^\circ}}.$$

- The load power angle can then be used to determine the current phase angle

$$\theta_1 = \theta_V - \theta_I = 0^\circ - \theta_I = 49.4564^\circ \quad \Rightarrow \quad \underline{\boldsymbol{\theta_I = -49.4564^\circ}}.$$

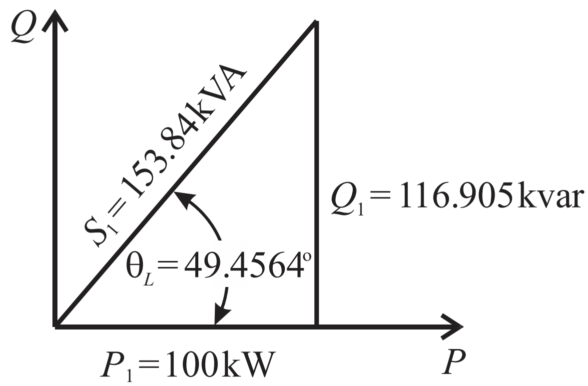
- The line/load current with no PF correction is then

$$\boxed{\overline{I}_{\text{line}} = \overline{I}_1 = I_1\angle\theta_I \quad \Rightarrow \quad \underline{\overline{I}_{\text{line}} = \overline{I}_1 = 320.5\angle -49.4564^\circ \text{ A}_{\text{rms}}}}.$$

- To complete the load power triangle, compute the load reactive power

$$Q_1 = P_1 \tan(\theta_1) = 100 \tan(49.4564°) \quad \Rightarrow \quad \underline{\boldsymbol{Q_1 = 116.9047 \text{ kvar}}}.$$

Load power triangle



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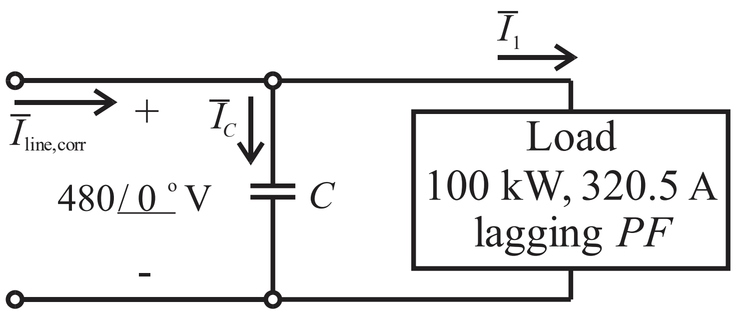
Power Factor correction (i.e., add a capacitor).



where $C = \dfrac{P_1[\tan\theta_1 - \tan\theta_2]}{\omega V^2}$.

- The goal of a $PF \geq 0.95$ leads to a desired power angle range of

$$\theta_2 = \pm\left|\cos^{-1}(0.95)\right| \quad \Rightarrow \quad \underline{\boldsymbol{\theta_2 = \pm 18.1949°}}.$$

- The **minimum** needed parallel capacitance, for a power factor of 0.95 **lagging** (i.e., $\theta_2 = +18.1949°$, undercorrection), is

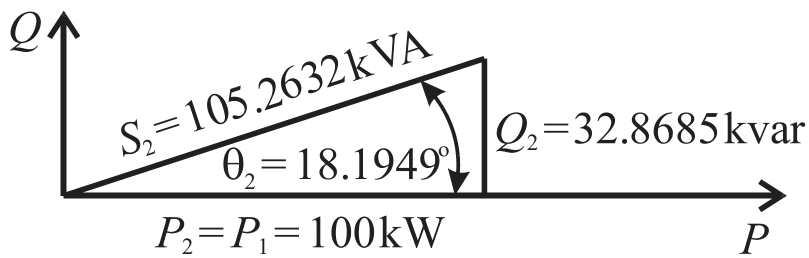
$$C_{\min} = \frac{100 \bullet 10^3 [\tan(49.4564°) - \tan(18.1949°)]}{2\pi(60)480^2} \quad \Rightarrow \quad \underline{\boldsymbol{C_{\min} = 0.9676 \text{ mF}}}.$$

- For a power factor of 0.95 **lagging** (i.e., $\theta_2 = 18.1949°$), the apparent and reactive powers are

$$S_2 = \frac{P_2}{\cos(\theta_2)} = \frac{100 \bullet 10^3}{\cos(18.1949°)} = 105.2632 \text{ kVA} \text{ and}$$

$$Q_2 = P_2 \tan(\theta_2) = 100 \bullet 10^3 \tan(18.1949°) = 32.8685.2632 \text{ kvar}.$$

Power triangle w/ *PF* = 0.95 lagging



- The associated phasor rms current for a power factor of 0.95 **lagging** is -

$$\boxed{\overline{I}_{\text{line, corr}} = \frac{S_2}{V}\angle -\theta_2 = \frac{105.263{\bullet}10^3}{480}\angle -18.195^\circ \quad \Rightarrow \underline{\overline{I}_{\text{line, corr}} = 219.3\angle -18.195^\circ \ \mathrm{A_{rms}}}}$$

- The **maximum** required parallel capacitance, for a power factor of 0.95 **leading** ($\theta_2 = -18.1949^\circ$, overcorrection), is -

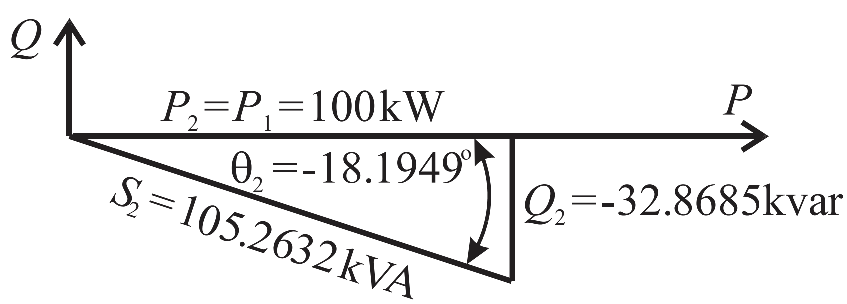
$$C_{\max} = \frac{100{\bullet}10^3\,[\tan(49.4564^\circ) - \tan(-18.1949^\circ)]}{2\pi(60)480^2} \quad \Rightarrow \quad \underline{\boldsymbol{C_{\max} = 1.7243\ \mathrm{mF}}}.$$

- For a power factor of 0.95 **leading** (i.e., $\theta_2 = -18.1949^\circ$), the apparent and reactive powers are

$$S_2 = \frac{P_2}{\cos(\theta_2)} = \frac{100{\bullet}10^3}{\cos(-18.1949^\circ)} = 105.2632\ \text{kVA} \ \text{ and}$$

$$Q_2 = P_2\tan(\theta_2) = 100{\bullet}10^3\tan(-18.1949^\circ) = -32.8685.2632\ \text{kvar}.$$

Power triangle w/ *PF* = 0.95 leading



- The associated phasor rms current for a power factor of 0.95 **leading** is

$$\boxed{\overline{I}_{\text{line, corr}} = \frac{S_2}{V_{\text{rms}}}\angle -\theta_2 = \frac{105.263{\bullet}10^3}{480}\angle 18.195^\circ \quad \Rightarrow \underline{\overline{I}_{\text{line, corr}} = 219.3\angle 18.195^\circ \ \mathrm{A_{rms}}}}.$$

- The overall capacitance range to meet the $PF \geq 0.95$ goal is

$$\boxed{C_{\min} = 0.9676 \text{ mF} \ \leq \ C \ \leq \ C_{\max} = 1.7243 \text{ mF}}.$$

- For perfect correction, i.e., $PF = 1$ and $\theta_2 = 0°$, the required capacitance is

$$C_{PF=1} = \frac{100 \bullet 10^3 [\tan(49.4564°) - \tan(0°)]}{2\pi(60)480^2} \quad \Rightarrow \quad \underline{\boldsymbol{C_{PF=1} = 1.346 \text{ mF}}}.$$

- The associated phasor rms current for a $PF = 1$ (now $S_2 = P_2 = P_1$) is

$$\boxed{\overline{I}_{\text{line, corr}} = \frac{S_2}{V_{\text{rms}}} \angle -\theta_2 = \frac{100 \bullet 10^3}{480} \angle 0° \quad \Rightarrow \ \underline{\overline{I}_{\text{line, corr}} = 208.3 \angle 0° \text{ A}_{\text{rms}}}}.$$

- The rms line currents associated with the power factors after correction, i.e., 219.3 A w/ $PF = 0.95$ lagging, 208.3 A w/ $PF = 1$, & 219.3 A w/ $PF = 0.95$ leading, are **far smaller** than the uncorrected rms line current of 320.5 A.

- This would translate to **significant** reductions in the transmission line losses ($I_{\text{line}}^2 R_{TL}$). For example, with a PF = 0.95, the losses would be

$$219.3^2 / 320.5^2 = 0.4619 = \underline{\mathbf{46.19\%}}$$

of the uncorrected line losses!