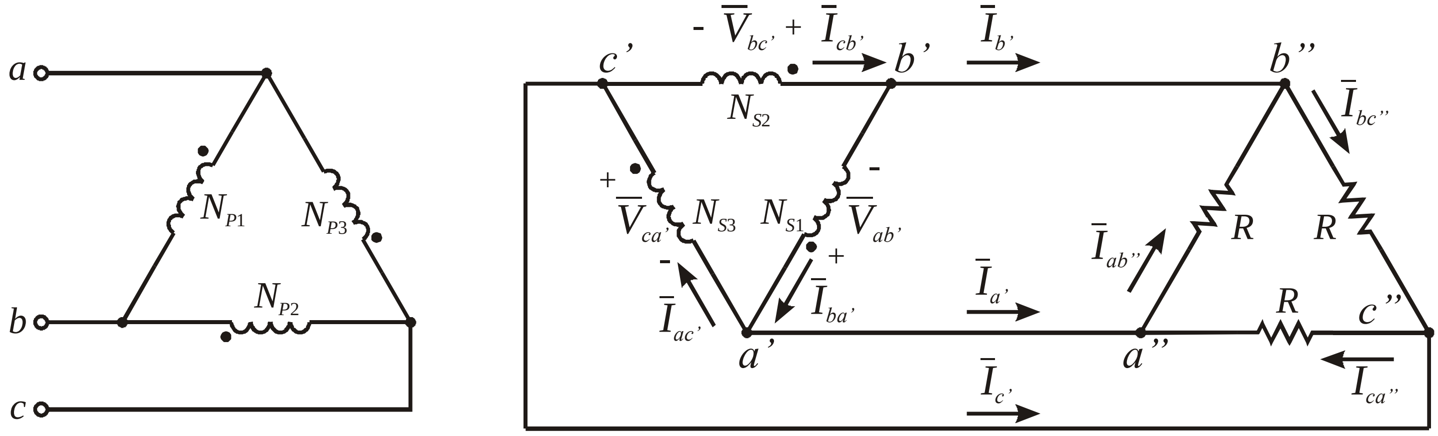
Balanced Δ-Δ 3-phase transformer w/ balanced resistive loads (see Fig. 2-40a)

We'll use this as a basis for comparisons of currents, voltages, and powers with the V-V 3-phase transformer connection.



Make arbitrary assumption that: $\bar{V}_{ab'} = V_\phi \angle 150°$, $\bar{V}_{bc'} = V_\phi \angle 30°$, and $\bar{V}_{ca'} = V_\phi \angle -90°$.

By Ohm's Law, the load phase currents are:

$$\bar{I}_{ab"} = \frac{\bar{V}_{ab'}}{R} = \frac{V_\phi \angle 150°}{R} = I_\phi \angle 150°,\ \bar{I}_{bc"} = \frac{\bar{V}_{bc'}}{R} = I_\phi \angle 30°,\ \text{and}\ \bar{I}_{ca"} = \frac{\bar{V}_{ca'}}{R} = I_\phi \angle -90°$$

where the phase current magnitude is defined as $I_\phi = \dfrac{V_\phi}{R}$.

By Kirchoff's Current Law (KCL), the line currents are:

$$\begin{aligned}\bar{I}_{a'} &= \bar{I}_{ab"} - \bar{I}_{ca"} = I_\phi \angle 150° - I_\phi \angle -90° = I_\phi[(1\angle 150°) - (1\angle -90°)] = I_\phi(\sqrt{3}\angle 120°) \\ &= \sqrt{3}\, I_\phi \angle 120°\end{aligned},$$

$$\bar{I}_{b'} = \bar{I}_{bc"} - \bar{I}_{ab"} = I_\phi \angle 30° - I_\phi \angle 150° = \sqrt{3}\, I_\phi \angle 0°,\ \text{and}$$

$$\bar{I}_{c'} = \bar{I}_{ca"} - \bar{I}_{bc"} = I_\phi \angle -90° - I_\phi \angle 30° = \sqrt{3}\, I_\phi \angle -120°.$$

By Kirchoff's Current Law (KCL), the transformer secondary phase currents are:

$$\bar{I}_{ba'} = \bar{I}_{ac'} + \bar{I}_{a'},\quad \bar{I}_{cb'} = \bar{I}_{ba'} + \bar{I}_{b'},\ \text{and}\ \bar{I}_{ac'} = \bar{I}_{cb'} + \bar{I}_{c'}.$$

These equations can be arranged as

$$(1)\overline{I}_{ba'} + (-1)\overline{I}_{ac'} + (0)\overline{I}_{cb'} = \overline{I}_{a'} = \sqrt{3}\, I_{\phi}\angle 120°$$

$$(-1)\overline{I}_{ba'} + (0)\overline{I}_{ac'} + (1)\overline{I}_{cb'} = \overline{I}_{b'} = \sqrt{3}\, I_{\phi}\angle 0°$$

$$(0)\overline{I}_{ba'} + (1)\overline{I}_{ac'} + (-1)\overline{I}_{cb'} = \overline{I}_{c'} = \sqrt{3}\, I_{\phi}\angle -120°.$$

Solving, we determine the transformer secondary phase currents to be:

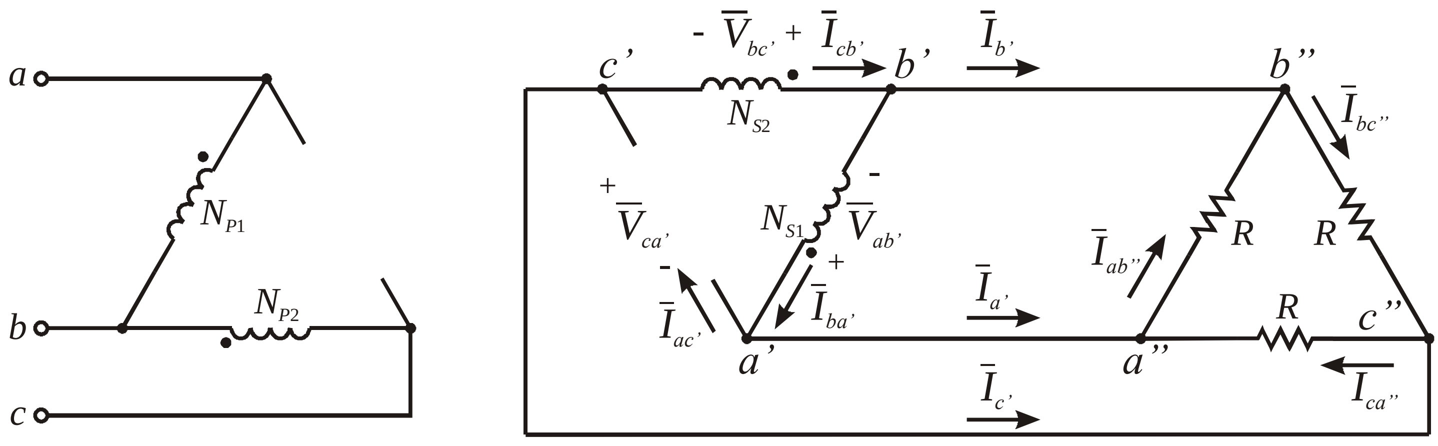
$$\overline{I}_{ba'} = I_{\phi}\angle 150°,\ \ \overline{I}_{cb'} = I_{\phi}\angle 30°,\ \text{and}\ \overline{I}_{ac'} = I_{\phi}\angle -90°.$$

The complex power delivered to the resistive loads is then equal to the sum of the power delivered from each transformer:

$$\begin{aligned}
\overline{S}_{\Delta-\Delta} &= \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = \overline{V}_{ab'}\overline{I}_{ba'}{}^* + \overline{V}_{bc'}\overline{I}_{cb'}{}^* + \overline{V}_{ca'}\overline{I}_{ac'}{}^* \\
&= (V_{\phi}\angle 150°)(I_{\phi}\angle -150°) + (V_{\phi}\angle 30°)(I_{\phi}\angle -30°) + (V_{\phi}\angle -90°)(I_{\phi}\angle 90°) \\
&= V_{\phi}I_{\phi} + V_{\phi}I_{\phi} + V_{\phi}I_{\phi} \\
\overline{S}_{\Delta-\Delta} &= P_{\Delta-\Delta} = 3V_{\phi}I_{\phi}
\end{aligned}$$

Note that the power delivered from each of the individual three transformers is entirely real as well as the total power from the three combined.

V-V 3-phase transformer w/ balanced resistive loads (see Fig. 2-40b)



Again arbitrary assumption that: $\bar{V}_{ab'} = V_\phi\angle 150°$ and $\bar{V}_{bc'} = V_\phi\angle 30°$.

By KVL, $\bar{V}_{ca'} = -(\bar{V}_{ab'} + \bar{V}_{bc'}) = -(V_\phi\angle 150° + V_\phi\angle 30°) = V_\phi\angle -90°$ (**SAME!**).

However, now $\bar{I}_{ac'} = 0$. In turn, this means that $\bar{I}_{a'} = \bar{I}_{ba'}$ and $\bar{I}_{c'} = -\bar{I}_{cb'}$ which tells us that the magnitude of the phase currents in the two transformers must equal the magnitude of the line currents. In order for this to hold true, the line/phase currents experience a phase shift as well as the magnitude reduction (compared to the Δ–Δ case) and are:

$$\bar{I}_{a'} = \bar{I}_{ba'} = I_\phi\angle 120° \; ,$$

$$\bar{I}_{b'} = \bar{I}_{cb'} - \bar{I}_{ba'} = I_\phi\angle 0°, \text{ and}$$

$$\bar{I}_{c'} = -\bar{I}_{cb'} = I_\phi\angle -120° .$$

The complex power delivered to the resistive loads is then equal to the sum of the power delivered from each transformer:

$$\begin{aligned}\bar{S}_{V-V} &= \bar{S}_1 + \bar{S}_2 + 0 = \bar{V}_{ab'}\bar{I}_{ba'}{}^* + \bar{V}_{bc'}\bar{I}_{cb'}{}^* + \bar{V}_{ca'}(0)\\ &= (V_\phi\angle 150°)(I_\phi\angle -120°) + (V_\phi\angle 30°)(I_\phi\angle -60°) + 0\\ &= V_\phi I_\phi(\frac{\sqrt{3}}{2} + j0.5) + V_\phi I_\phi(\frac{\sqrt{3}}{2} - j0.5)\end{aligned}$$

$$\bar{S}_{V-V} = P_{V-V} = \sqrt{3}V_\phi I_\phi$$

Note that the transformers are 'trading' reactive power back-n-forth to the amount of

$$Q = \pm 0.5 V_\phi I_\phi$$

and that

$$\frac{P_{V-V}}{P_{\Delta-\Delta}} = \frac{\sqrt{3} V_\phi I_\phi}{3 V_\phi I_\phi} = \frac{1}{\sqrt{3}} = 0.57735 \text{ or } 57.735\%$$

which is substantially less the 2/3 or 66.66% one would guess as the output with 2 of 3 transformers. Another way to express this is that the power factor will be a minimum of

$$pf_{V-V} = \cos(\pm 30°) = \frac{\sqrt{3}}{2} = 0.866$$

and note that $\frac{\sqrt{3}}{2} * \frac{2}{3} = 0.57735 \text{ or } 57.735\%$.