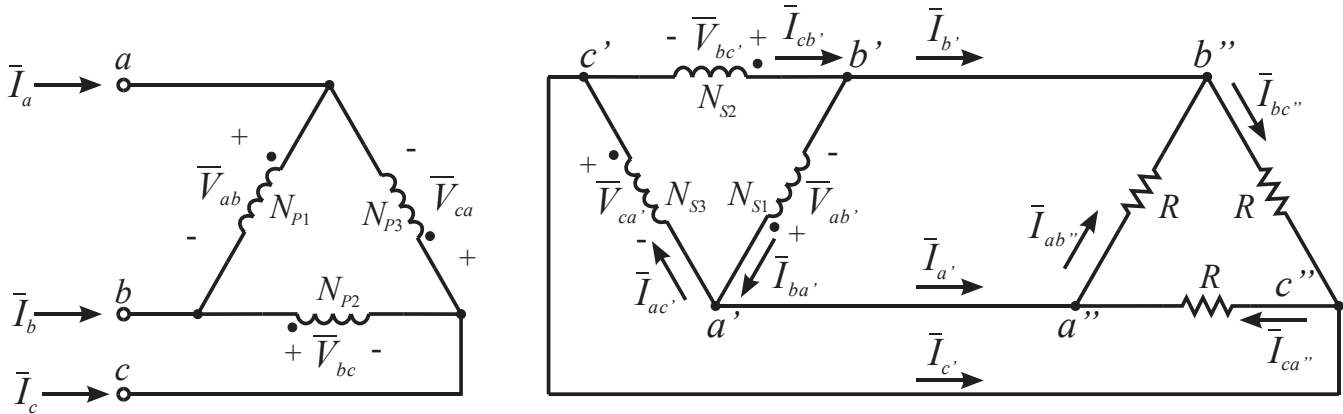


Example- Δ-Δ Three-Phase Transformer w/ Balanced Resistive Loads

On secondary side, let $\underline{\bar{V}_{ab'}} = V_\phi \angle 150^\circ$, $\underline{\bar{V}_{bc'}} = V_\phi \angle 30^\circ$, & $\underline{\bar{V}_{ca'}} = V_\phi \angle -90^\circ$.

By Ohm's Law, the **load** phase currents are:

$$\underline{\bar{I}_{ab''}} = \frac{\bar{V}_{ab'}}{R} = \frac{V_\phi \angle 150^\circ}{R} \Rightarrow \underline{\bar{I}_{ab''}} = I_\phi \angle 150^\circ, \quad \underline{\bar{I}_{bc''}} = \frac{\bar{V}_{bc'}}{R} = I_\phi \angle 30^\circ,$$

$$\text{and } \underline{\bar{I}_{ca''}} = \frac{\bar{V}_{ca'}}{R} = I_\phi \angle -90^\circ.$$

where the phase current magnitude is defined as $I_\phi = \frac{V_\phi}{R}$.

By Kirchoff's Current Law (KCL), the secondary-side **line** currents are:

$$\underline{\bar{I}_{a'}} = \bar{I}_{ab''} - \bar{I}_{ca''} = I_\phi \angle 150^\circ - I_\phi \angle -90^\circ = I_\phi [(1 \angle 150^\circ) - (1 \angle -90^\circ)] \Rightarrow \underline{\bar{I}_{a'}} = \sqrt{3} I_\phi \angle 120^\circ,$$

$$\underline{\bar{I}_{b'}} = \bar{I}_{bc''} - \bar{I}_{ab''} = I_\phi \angle 30^\circ - I_\phi \angle 150^\circ \Rightarrow \underline{\bar{I}_{b'}} = \sqrt{3} I_\phi \angle 0^\circ, \text{ and}$$

$$\underline{\bar{I}_{c'}} = \bar{I}_{ca''} - \bar{I}_{bc''} = I_\phi \angle -90^\circ - I_\phi \angle 30^\circ \Rightarrow \underline{\bar{I}_{c'}} = \sqrt{3} I_\phi \angle -120^\circ.$$

By KCL, the secondary **transformer** phase currents are:

$$\bar{I}_{ba'} = \bar{I}_{ac'} + \bar{I}_{a'}, \quad \bar{I}_{cb'} = \bar{I}_{ba'} + \bar{I}_{b'}, \quad \text{and} \quad \bar{I}_{ac'} = \bar{I}_{cb'} + \bar{I}_{c'}.$$

These equations can be arranged as

$$\begin{aligned}
 (1)\bar{I}_{ba'} + (-1)\bar{I}_{ac'} + (0)\bar{I}_{cb'} &= \bar{I}_{a'} = \sqrt{3} I_{\phi} \angle 120^{\circ} , \\
 (-1)\bar{I}_{ba'} + (0)\bar{I}_{ac'} + (1)\bar{I}_{cb'} &= \bar{I}_{b'} = \sqrt{3} I_{\phi} \angle 0^{\circ} , \& \\
 (0)\bar{I}_{ba'} + (1)\bar{I}_{ac'} + (-1)\bar{I}_{cb'} &= \bar{I}_{c'} = \sqrt{3} I_{\phi} \angle -120^{\circ} .
 \end{aligned}$$

In matrix form, these equations become

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \bar{I}_{ba'} \\ \bar{I}_{ac'} \\ \bar{I}_{cb'} \end{bmatrix} = I_{\phi} \begin{bmatrix} \sqrt{3} \angle 120^{\circ} \\ \sqrt{3} \angle 0^{\circ} \\ \sqrt{3} \angle -120^{\circ} \end{bmatrix} \text{ and} \\
 \begin{bmatrix} \bar{I}_{ba'} \\ \bar{I}_{ac'} \\ \bar{I}_{cb'} \end{bmatrix} = I_{\phi} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} \sqrt{3} \angle 120^{\circ} \\ \sqrt{3} \angle 0^{\circ} \\ \sqrt{3} \angle -120^{\circ} \end{bmatrix} .$$

Solving, we determine the secondary **transformer** phase currents to be:

$$\underline{\bar{I}_{ba'} = I_{\phi} \angle 150^{\circ}}, \quad \underline{\bar{I}_{cb'} = I_{\phi} \angle 30^{\circ}}, \quad \text{and} \quad \underline{\bar{I}_{ac'} = I_{\phi} \angle -90^{\circ}} .$$

The complex power delivered to the resistive loads is equal to the sum of the power delivered from each transformer pair:

$$\begin{aligned}
 \bar{S}_{\Delta-\Delta} &= \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = \bar{V}_{ab'} \bar{I}_{ba'}^* + \bar{V}_{bc'} \bar{I}_{cb'}^* + \bar{V}_{ca'} \bar{I}_{ac'}^* \\
 &= (V_{\phi} \angle 150^{\circ})(I_{\phi} \angle -150^{\circ}) + (V_{\phi} \angle 30^{\circ})(I_{\phi} \angle -30^{\circ}) + (V_{\phi} \angle -90^{\circ})(I_{\phi} \angle 90^{\circ}) \\
 &= V_{\phi} I_{\phi} + V_{\phi} I_{\phi} + V_{\phi} I_{\phi}
 \end{aligned}$$

$$\underline{\bar{S}_{\Delta-\Delta} = P_{\Delta-\Delta} = 3V_{\phi} I_{\phi}} .$$

Note that the power delivered from each of the three individual transformer pairs is entirely real as well as the total power from the three combined.