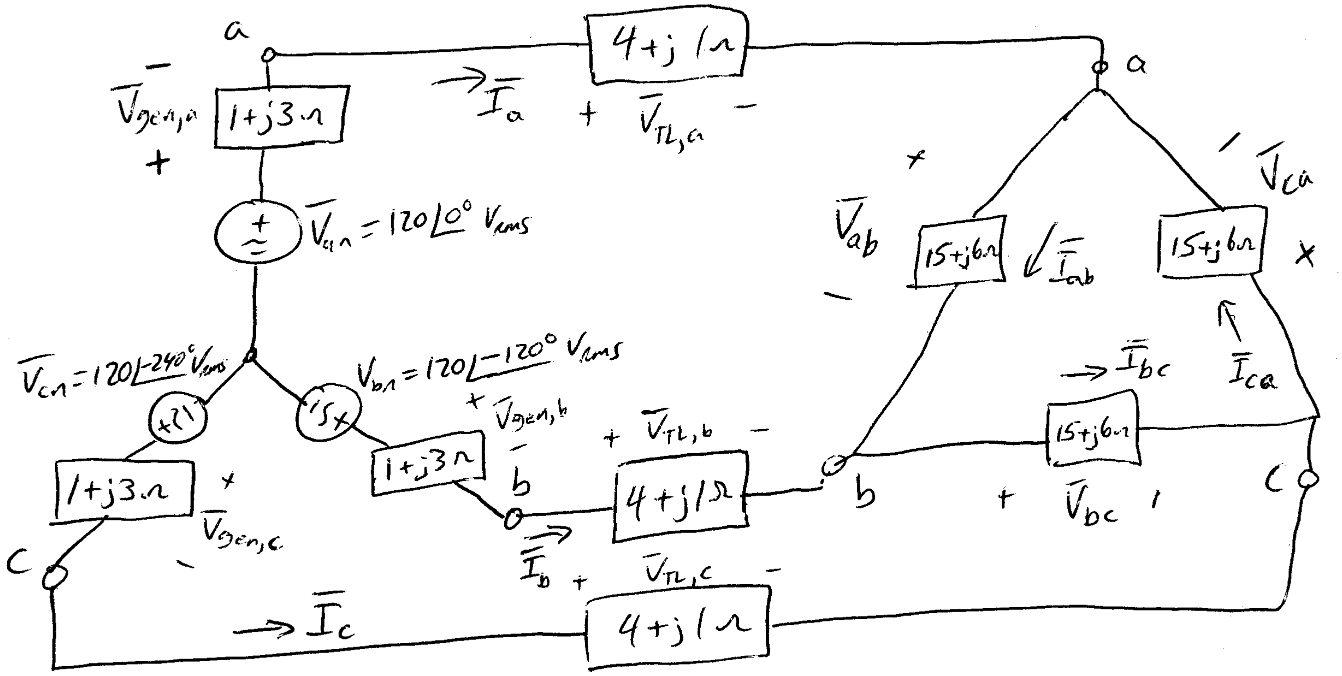
# Three-Phase Circuit Power Example

ex. A balanced Y-config. 3-$\phi$ power source has a phase voltage of $120 V_{rms}$ and impedance of $1+j3\,\Omega$. It is connected to a balanced $\Delta$-config. load, $\bar{Z}_\Delta = 15+j6\,\Omega$, by transmission lines with line impedances of $4+j1\,\Omega$. Calculate all currents, voltages, and power quantities of interest.

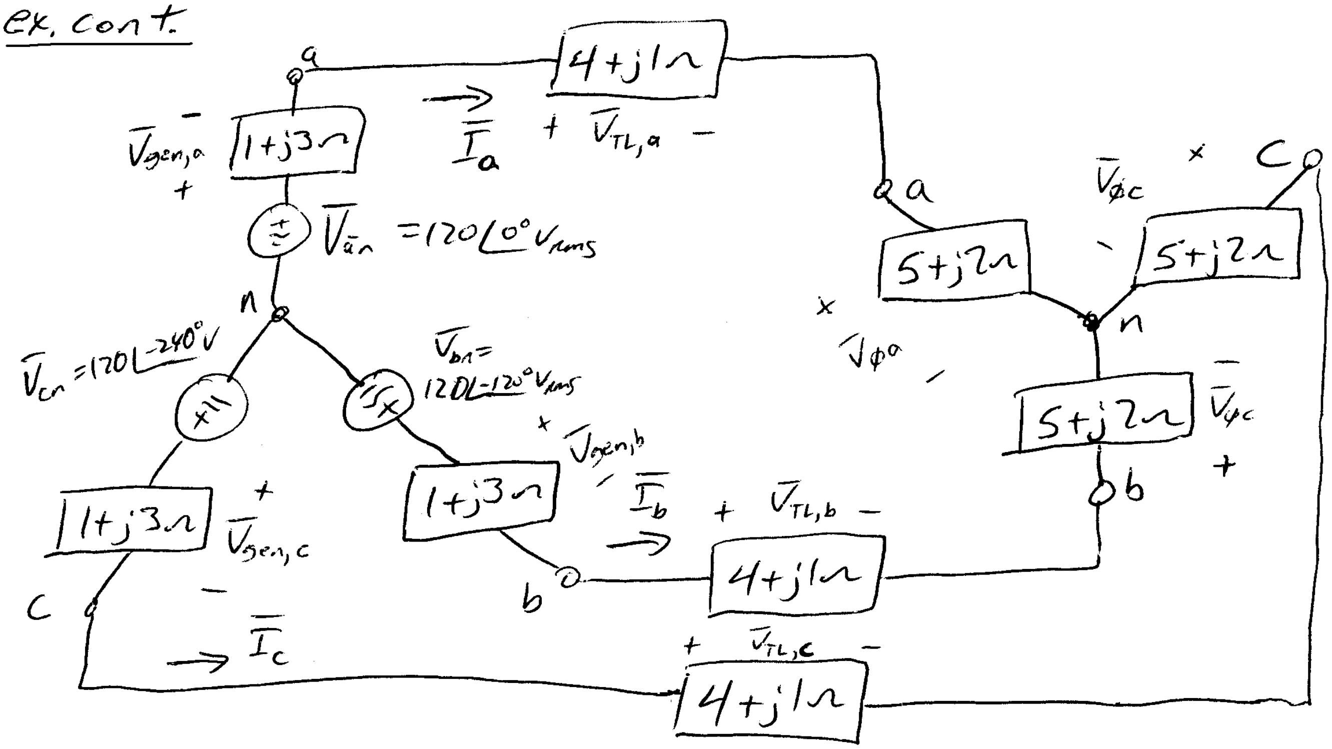



Step 1 – convert $\Delta$-load to Y-load

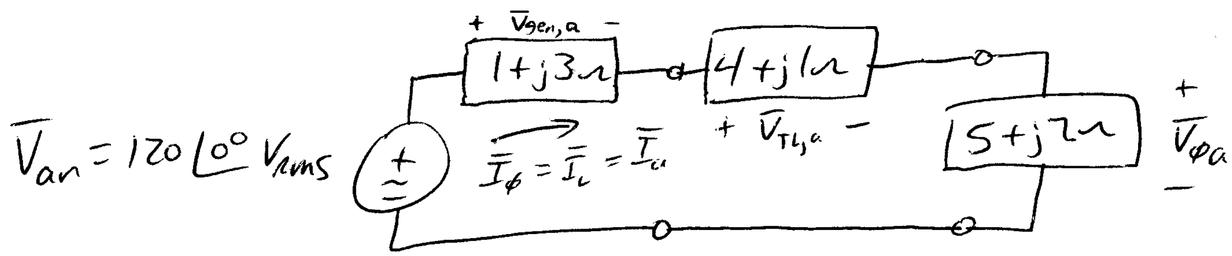
$$\bar{Z}_Y = \frac{\bar{Z}_\Delta}{3} = \frac{15+j6}{3} = 5+j2\,\Omega$$

Step 2 – Redraw circuit (Note that all impedances are now in <u>series</u>!).

ex. cont.




Step 3 – Draw per-phase equivalent circuit (I'll choose phase a).




$$\bar{I}_a = \frac{120\angle 0^\circ}{(1+j3)+(4+j1)+(5+j2)} = \frac{120\angle 0^\circ}{11.6619\angle 30.9638^\circ}$$

$10+j6\,\Omega$

$$\underline{\underline{\bar{I}_a = 10.29\angle -30.9638^\circ \text{ A}_{rms}}}$$

$I_\phi = 10.289915 \text{ A}_{rms}$

$$\underline{\underline{\bar{I}_b = 10.29\angle -150.9638^\circ \text{ A}_{rms}}}$$ (−120° phase shift)

$$\underline{\underline{\bar{I}_c = 10.29\angle -270.9638^\circ \text{ A}_{rms}}}$$ (−240° phase shift)

ex. cont.

$$P_\phi = P_a = P_b = P_c = V_\phi I_\phi \cos\theta = (120)(10.29)\cos 30.9638^\circ$$

$$\underline{P_\phi = 1058.8\ W}$$

$$\underline{P = 3P_\phi = 3176.5\ W}$$ ← overall power from ideal $3\phi$ voltage sources

$$Q_\phi = Q_a = Q_b = Q_c = V_\phi I_\phi \sin\theta = (120)(10.29)\sin 30.9638^\circ$$

$$\underline{Q_\phi = 635.3\ VAR}$$

$$\underline{Q = 3Q_\phi = 1905.9\ VAR}$$ ← overall reactive power from ideal $3\phi$ voltage sources

$$\bar{S}_\phi = \bar{S}_a + \bar{S}_b + \bar{S}_c = P_\phi + jQ_\phi = \underline{1058.8 + j635.3\ VA}$$

$$\bar{S} = P + jQ = \underline{3176.5 + j1905.9\ VA}$$

$$S_\phi = |\bar{S}_\phi| = \underline{1234.8\ VA} = V_\phi I_\phi = 120(10.29)$$

$$S = |\bar{S}| = \underline{3704.4\ VA}$$

$$pf = \frac{P}{S} = \cos(30.9638^\circ) = \underset{Q>0}{\underline{0.8575\ \text{lagging}}}$$

ex. cont. Find how the various quantities divide up.

Generator Impedances -

$\overline{V}_{gen,a} = \overline{I}_a(1+j3) = (10.29\angle -30.964^\circ)(1+j3) = \underline{32.54\angle 40.60^\circ \ V_{rms}}$

$\overline{V}_{gen,b} = \overline{I}_b(1+j3) = 32.54\angle 40.6^\circ - 120^\circ = \underline{32.54\angle -79.4^\circ \ V_{rms}}$

$\overline{V}_{gen,c} = \overline{I}_c(1+j3) = 32.54\angle 40.6^\circ - 240^\circ = \underline{32.54\angle -199.4^\circ \ V_{rms}}$

$\overline{S}_{gen,a} = \overline{V}_{gen,a}\overline{I}_a^* = (32.54\angle 40.6^\circ)(10.29\angle +30.96^\circ)$

$\underline{\overline{S}_{gen,a} = 105.885 + j\,317.65 \text{ VA} = \overline{S}_{gen,b} = \overline{S}_{gen,c}}$

all $3\phi$ $\begin{cases} P_{gen} = 3(105.885) = 317.7 \text{ W} & \frac{P_{gen}}{P} = \underline{10\%} \\ Q_{gen} = 3(317.65) = 953 \text{ VAR} & \frac{Q_{gen}}{Q} = \underline{50\%} \end{cases}$

Transmission Line Impedances -

$\overline{V}_{TL,a} = \overline{I}_a(4+j1) = (10.29\angle -30.964^\circ)(4+j1) = \underline{42.426\angle -16.93^\circ \ V_{rms}}$

$\overline{V}_{TL,b} = \overline{I}_b(4+j1) = 42.426\angle -16.93^\circ - 120^\circ = \underline{42.426\angle -136.93^\circ \ V_{rms}}$

$\overline{V}_{TL,c} = \overline{I}_c(4+j1) = 42.426\angle -16.93^\circ - 240^\circ = \underline{42.426\angle -256.93^\circ \ V_{rms}}$

$\overline{S}_{TL,a} = \overline{V}_{TL,a}\overline{I}_a^* = (42.426\angle -16.93^\circ)(10.29\angle +30.964^\circ)$

$\underline{\overline{S}_{TL,a} = \overline{S}_{TL,b} = \overline{S}_{TL,c} = 423.53 + j\,105.88 \text{ VA}}$

all $3\phi$ - $\begin{cases} P_{TL} = 3(423.5) = \underline{1270.6 \text{ W}} & \underline{P_{TL}/P = 40\%} \\ Q_{TL} = 3(105.88) = \underline{317.7 \text{ VAR}} & \underline{\frac{Q_{TL}}{Q} = 16.67\%} \end{cases}$

ex. cont.

Load Impedances -

From the per-phase equivalent model

$\overline{V}_{\phi a} = \overline{I}_a (5+j2) = 55.4129 \angle -9.16^\circ$ Vrms

$\overline{V}_{\phi b} = \overline{I}_b (5+j2) = 55.4129 \angle -9.16^\circ - 120^\circ = 55.4129 \angle -129.16^\circ$ Vrms

$\overline{V}_{\phi c} = \overline{I}_c (5+j2) = 55.4129 \angle -9.16^\circ - 240^\circ = 55.4129 \angle -249.16^\circ$ Vrms

Find line-to-line voltage $V_{LL} = \sqrt{3}\, V_\phi = \sqrt{3}(55.4) = 95.98$ Vrms which is not very good compared to $\sqrt{3}\,120 = 207.85$ Vrms that left the ideal $3\phi$ Y-sources.

Powers to load (using per-phase equivalent circuit) -

$\overline{S}_{LD,a} = \overline{V}_{\phi a} \overline{I}_a^* = (55.4129 \angle -9.16^\circ)(10.29 \angle +30.964^\circ) \overset{\text{OR}}{=} I_a^2 \overline{Z}_{\phi a}$

$\overline{S}_{LD,a} = 529.4 + j\,211.8$ VA $= \overline{S}_{LD,b} = \overline{S}_{LD,c}$

Overall $3\phi$ Load $\begin{cases} P_{LD} = 3(529.4) = 1588.2 \text{ W} \\ Q_{LD} = 3(211.8) = 635.3 \text{ VAR} \end{cases}$

$P_{LD}/P = 50\%$

$Q_{LD}/Q = 33.3\%$

$\Rightarrow$ Looks like I better spend some \$\$ on better transmission line and generator!!

ex. cont.

Lastly, we need to find the phasor voltages and currents for the original balanced Δ-load.

By KVL (on circuit on second page of example) –

$$\bar{V}_{ab} = \bar{V}_{\phi a} - \bar{V}_{\phi b} = (55.4129\angle -9.16°) - (55.4129\angle -129.16°)$$

$$\bar{V}_{ab} = 95.98\angle 20.84° \text{ } V_{rms}$$

$$\bar{V}_{bc} = \bar{V}_{\phi b} - \bar{V}_{\phi c} = (55.4129\angle -129.16°) - (55.4129\angle -249.16°)$$

$$\bar{V}_{bc} = 95.98\angle -99.16° \text{ } V_{rms}$$

$$\bar{V}_{ca} = \bar{V}_{\phi c} - \bar{V}_{\phi a} = (55.4\angle -249.2°) - (55.4\angle -9.16°) = 95.98\angle 140.84° \text{ } V_{rms}$$

$$\bar{V}_{ca} = 95.98\angle -219.16° \text{ } V_{rms}$$

As a check, do KVL around upper LH loop of original circuit –

$$-\bar{V}_{an} + \bar{V}_{gen,a} + \bar{V}_{TL,a} + \bar{V}_{ab} - \bar{V}_{TL,b} - \bar{V}_{gen,b} + \bar{V}_{bn} = 0$$

$$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{gen,a} - \bar{V}_{TL,a} + \bar{V}_{TL,b} + \bar{V}_{gen,b} - \bar{V}_{bn}$$

$$= (120\angle 0°) - (32.54\angle 40.6°) - (42.426\angle -16.93°) + (42.426\angle -136.93°)$$
$$+ (32.54\angle -79.4°) - (120\angle -120°)$$

$$\bar{V}_{ab} = 95.98\angle 20.84° \text{ } V_{rms} \Leftarrow \text{same answer}$$

ex. cont.

By Ohm's Law $\bar{I}_{ab} = \dfrac{\bar{V}_{ab}}{15+j6} = \dfrac{95.98\angle 20.84^\circ}{15+j6} = \underline{5.941\angle -0.96^\circ \; A_{rms}}$

$$\underline{\bar{I}_{bc} = \frac{\bar{V}_{bc}}{15+j6} = 5.941\angle -120.96^\circ \; A_{rms}}$$

$$\underline{\bar{I}_{ca} = \frac{\bar{V}_{ca}}{15+j6} = 5.941\angle -240.96^\circ \; A_{rms}}$$

As a check, compute the complex power to each phase of the Δ-loads & compare w/ per-phase equivalent values

$$\bar{S}_{L\emptyset,ab} = \bar{V}_{ab}\,\bar{I}_{ab}^{*} = (95.98\angle 20.84^\circ)(5.941\angle +0.96^\circ)$$

$$= \underline{529.4 + j\,211.8 \text{ VA}} \Leftarrow \text{Same answer!}$$