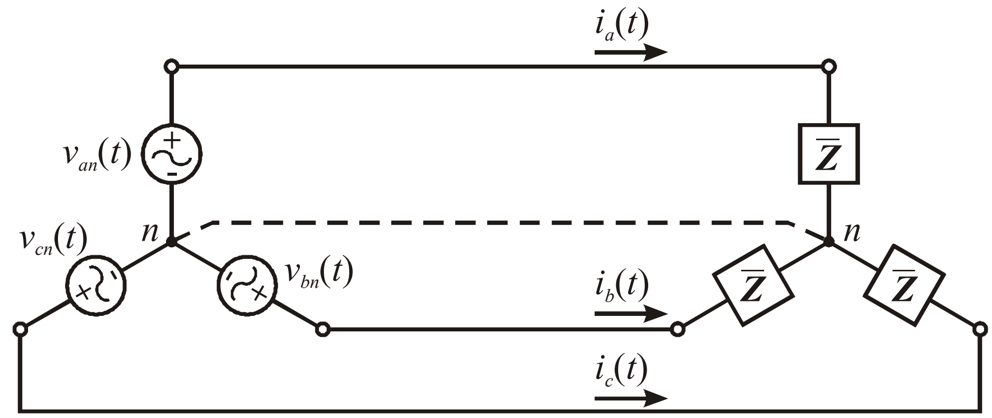
Balanced Three-phase Wye-source and Wye-load power example



Define some constants for the balanced sources & loads ($\boldsymbol{Z} = 20/30°\ \Omega$).

$V := 120$ V_{rms} $\qquad Z := 20$ Ω $\qquad \theta := 30\cdot\dfrac{\pi}{180}$ $\qquad I := \dfrac{V}{Z}$

$I = 6$ A_{rms} $\qquad n := 0..229$ $\qquad wt_n := n\cdot\dfrac{2\cdot\pi}{180}$ $\qquad m := 0..7$

Define phase voltages and currents.

$$van_n := \sqrt{2}\cdot V\cdot\cos(wt_n) \qquad ia_n := \sqrt{2}\cdot I\cdot\cos(wt_n - \theta)$$

$$vbn_n := \sqrt{2}\cdot V\cdot\cos\left(wt_n - 120\,\frac{\pi}{180}\right) \qquad ib_n := \sqrt{2}\cdot I\cdot\cos\left(wt_n - 120\,\frac{\pi}{180} - \theta\right)$$

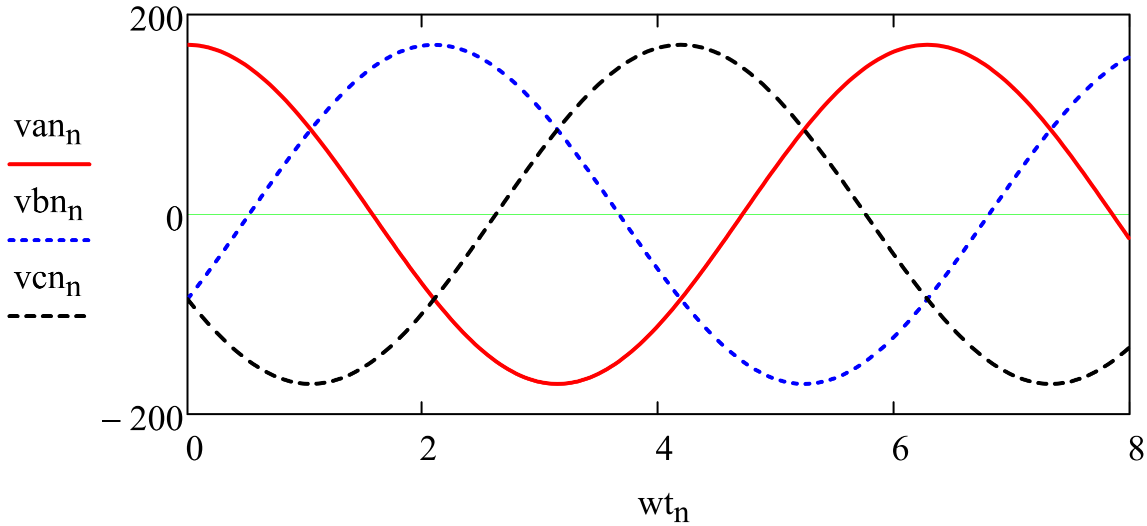
$$vcn_n := \sqrt{2}\cdot V\cdot\cos\left(wt_n - 240\,\frac{\pi}{180}\right) \qquad ic_n := \sqrt{2}\cdot I\cdot\cos\left(wt_n - 240\,\frac{\pi}{180} - \theta\right)$$

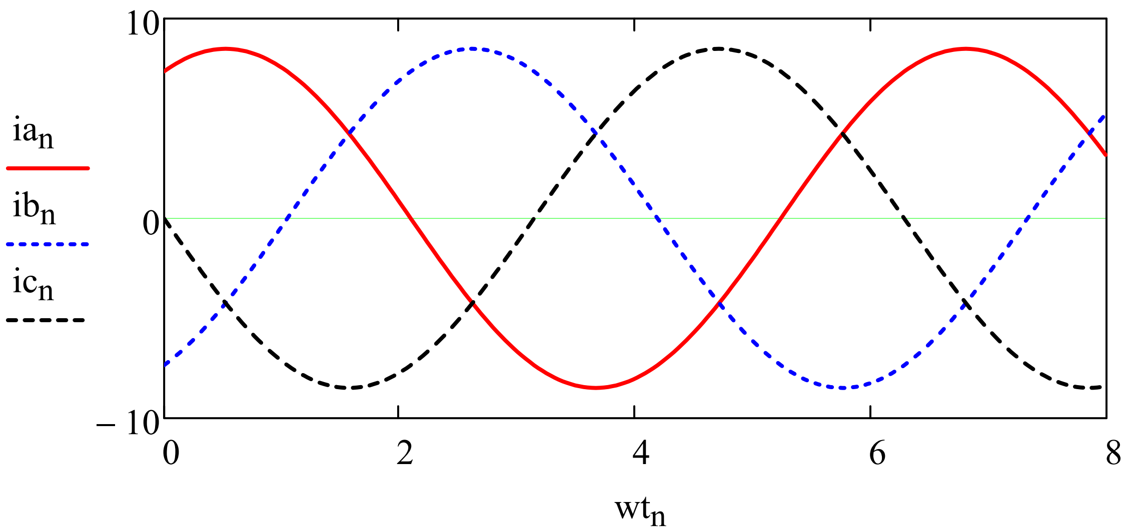
Calculate instantaneous power for each phase as well as total power (2 ways).

$$pa_n := van_n\,ia_n \qquad pb_n := vbn_n\,ib_n \qquad pc_n := vcn_n\,ic_n$$

$ptot_n := pa_n + pb_n + pc_n$ **or** $ptot1_m := 3\cdot V\cdot I\cdot\cos(\theta)$

$\boxed{ptot_{22} = 1870.615}$ W **or** $\boxed{ptot1_1 = 1870.615}$ W


200
0
– 200
0
2
4
6
8
wt_n
van_n
vbn_n
vcn_n


10
0
– 10
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2
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6
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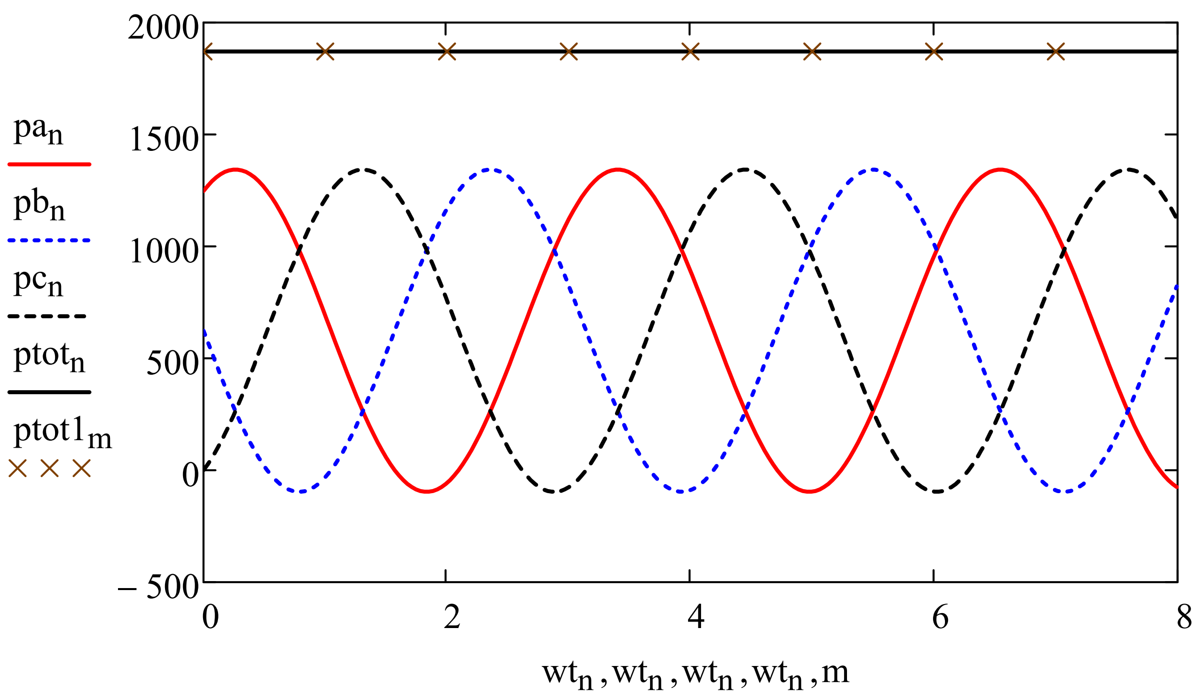
wt_n
ia_n
ib_n
ic_n


2000
1500
1000
500
0
– 500
0
2
4
6
8
wt_n , wt_n , wt_n , wt_n , m
pa_n
pb_n
pc_n
ptot_n
ptot1_m

Calculate apparent power *S* to each phase.

$Sa := V \cdot I$	$Sa = 720$	VA
$Sb := V \cdot I$	$Sb = 720$	VA
$Sc := V \cdot I$	$Sc = 720$	VA
$Stot := 3 \cdot V \cdot I$	$Stot = 2160$	VA

Calculate time-average real power *P* to each phase.

$Pa := V \cdot I \cdot \cos(\theta)$	$Pa = 623.538$	W
$Pb := V \cdot I \cdot \cos(\theta)$	$Pb = 623.538$	W
$Pc := V \cdot I \cdot \cos(\theta)$	$Pc = 623.538$	W
$Ptot := 3 \cdot V \cdot I \cdot \cos(\theta)$	$Ptot = 1870.61$	W

Calculate reactive power *Q* to each phase.

$Qa := V \cdot I \cdot \sin(\theta)$	$Qa = 360$	var
$Qb := V \cdot I \cdot \sin(\theta)$	$Qb = 360$	var
$Qc := V \cdot I \cdot \sin(\theta)$	$Qc = 360$	var
$Qtot := 3\, V \cdot I \cdot \sin(\theta)$	$Qtot = 1080$	var

Calculate power factor (PF) for each phase.

$PF\_a := \dfrac{Pa}{Sa}$	$\boxed{PF\_a = 0.866}$	**lagging** since $Q_a > 0$ & $\theta > 0$
$PF\_b := \cos(\theta)$	$PF\_b = 0.866$	lagging since $Q_b > 0$ & $\theta > 0$
$PF\_c := \cos(\theta)$	$PF\_c = 0.866$	lagging since $Q_c > 0$ & $\theta > 0$
$PF := \dfrac{Ptot}{Stot}$	$PF = 0.866$	lagging since $Q > 0$ & $\theta > 0$