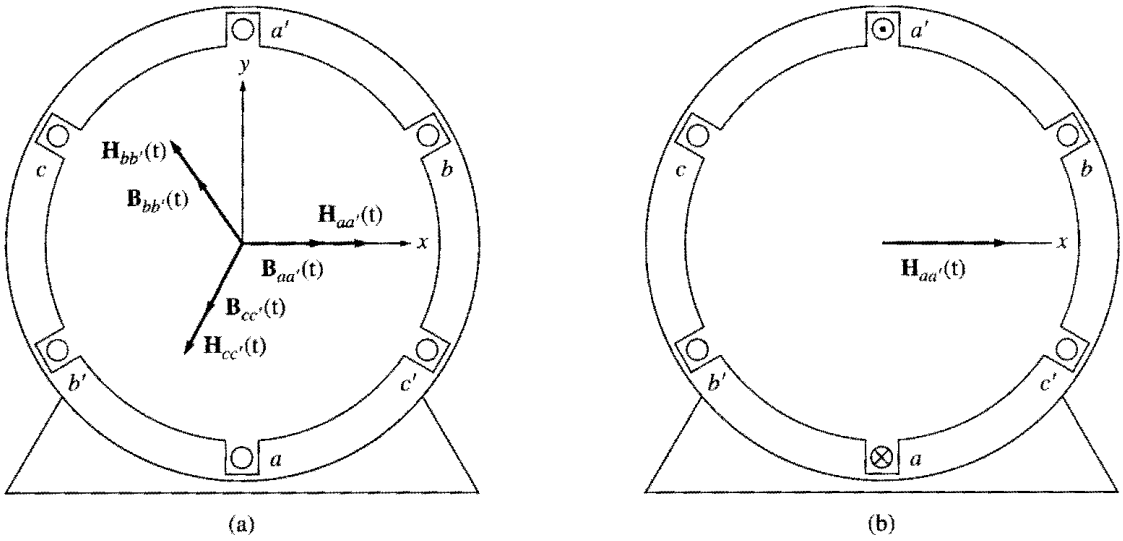
Three-Phase Magnetic Fields

Consider the simple three-phase stator shown in Figure 4-1.

Figure 4–1 (a) A simple three-phase stator. Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils. The magnetic field intensity (**H**) and flux density (**B**) produced by each coil are also shown. (b) The magnetic field intensity vector $\mathbf{H}_{aa'}(t)$ produced by a current flowing in coil *aa'*.



- Note that each coil is physically situated at 120◦ intervals on the stator.
- The current flowing through each coil generates a magnetic field (e.g., $\overline{H}_{aa'}$ for coil *aa'* associated with phase *a*) and magnetic flux density (e.g., $\overline{B}_{aa'}$) vector normal to the surface bounded by the coil (use RHR).
- Assume the three phase currents are:

$$i_{aa'}(t) = I_M \sin(\omega t),$$
$$i_{bb'}(t) = I_M \sin(\omega t - 120°), \text{ and}$$
$$i_{cc'}(t) = I_M \sin(\omega t - 240°).$$

- These currents generate magnetic field vectors pointing at the spatial angles 0°, 120°, & 240° whose <u>amplitude</u> is time-varying in synch with the respective currents:

$$\overline{H}_{aa'}(t) = H_M \sin(\omega t)\angle 0°,$$
$$\overline{H}_{bb'}(t) = H_M \sin(\omega t - 120°)\angle 120°, \text{ and}$$
$$\overline{H}_{cc'}(t) = H_M \sin(\omega t - 240°)\angle 240°.$$

- Using the constitutive relationship, $\overline{B} = \mu \overline{H}$, the corresponding magnetic flux density vectors for the three phases are:

$$\overline{B}_{aa'}(t) = B_M \sin(\omega t)\angle 0°,$$
$$\overline{B}_{bb'}(t) = B_M \sin(\omega t - 120°)\angle 120°, \text{ and}$$
$$\overline{B}_{cc'}(t) = B_M \sin(\omega t - 240°)\angle 240°.$$

- We can find the overall net magnetic flux density vector by adding those from the three phases, i.e., $\overline{B}_{\text{net}} = \overline{B}_{aa'} + \overline{B}_{bb'} + \overline{B}_{cc'}$.
- Using MathCAD, we can shown that $\left|\overline{B}_{\text{net}}\right|$ is constant wrt time while its spatial angle changes linearly, i.e., it rotates!

MathCad 3ϕ power & rotating magnetic field example

Cover one period T (2π rad) $\quad n := 0 .. 360 \quad wt_n := n \cdot \frac{2 \cdot \pi}{360}$

Compute magnetic flux density for each phase.

Note: In addition to the 120° or $2\pi/3$ phase shifts in magnitude due to the electrical currents, there is a 120° or $2\pi/3$ shift in spatial rotation due to the the physical orientations of the coils.

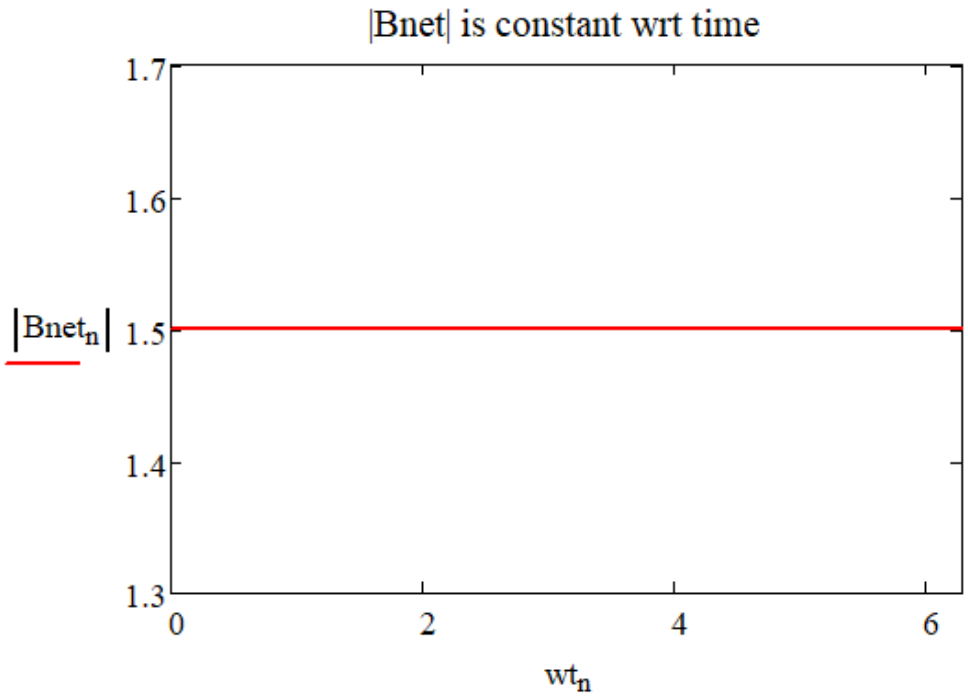
$$Baa_n := 1 \cdot \sin(wt_n) \cdot e^{j \cdot 0}$$

$$Bbb_n := 1 \cdot \sin\left(wt_n - \frac{2\pi}{3}\right) \cdot e^{j \cdot \frac{2\pi}{3}}$$

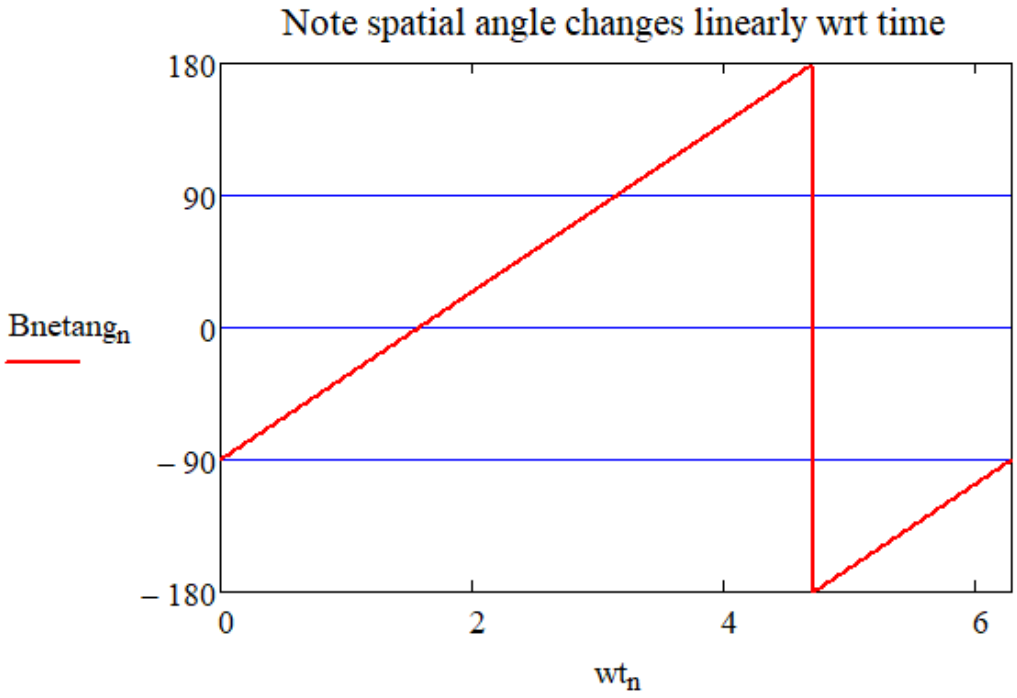
$$Bcc_n := 1 \cdot \sin\left(wt_n - \frac{4 \cdot \pi}{3}\right) \cdot e^{j \cdot \frac{4\pi}{3}}$$

Add up magnetic fields for each phase to get total magnetic field.

$$Bnet_n := Baa_n + Bbb_n + Bcc_n$$



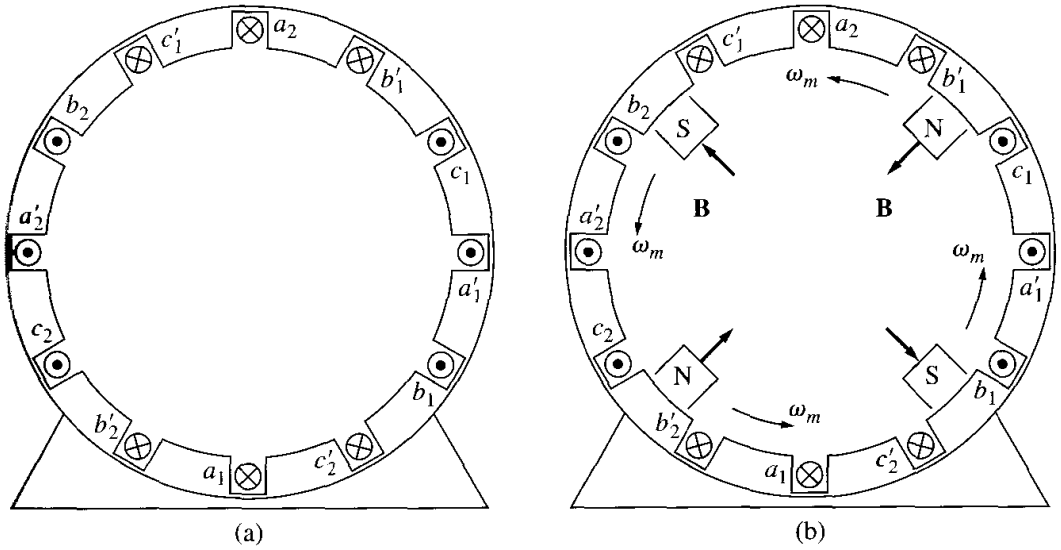
What about angle/phase of $\mathbf{B_{net}}$? $\text{Bnetang}_n := \arg(\text{Bnet}_n) \cdot \frac{180}{\pi}$



The linear variation of the spatial angle of the net magnetic flux density vector $\mathbf{B_{net}}$ over a total of 360° in one period T (2π radians) tells us that the direction of $\mathbf{B_{net}}$ is rotating (wrt time) at a rate of **one** rotation per period T of the three-phase currents.

- For this configuration (two poles per phase, i.e., N & S), we get one rotation of $\overline{B}_{\text{net}}$ per electrical period T, i.e., $\theta_e = \theta_m$. Therefore, the mechanical speed of rotation is $f_e = f_m$ (Hz) or $\omega_e = \omega_m$ (rad/s) for two poles.
- If we create multiple sets of windings per electrical phase, we can change the mechanical speed of rotation.
- For example, a four pole stator is created by having two sets of coils per electrical phase (2 coils × two poles/coil = 4 poles per phase) as shown in Figure 4-4. For phase a, we have two sets of coils a_1a_1' (lower right) and and a_2a_2' (upper left) each producing a N & S pole for a total of 4 poles with alternating polarity at 90° intervals.

Figure 4–4 | (a) A simple four-pole stator winding. (b) The resulting stator magnetic poles. Notice that there are moving poles of alternating polarity every 90° around the stator surface.



- For this four poles configuration, we get half a rotation of $\overline{B}_{\text{net}}$ per electrical period T, i.e., $\theta_e = 2\theta_m$. Then, the mechanical speed of rotation is related to the electrical by $f_e = 2f_m$ (Hz) or $\omega_e = 2\omega_m$ (rad/s) for four poles, i.e., machine rotates slower.
- In general, if we create P poles (must be multiple of 2 as we get a N & S for each coil), we will have $P/2$ coils per electrical phase and $\theta_e = (P/2)\theta_m$, $f_e = (P/2) f_m$, and $\omega_e = (P/2)\,\omega_m$. In terms of RPM, we have $f_e = n_m P / 120$.

How can we reverse our magnetic field rotation, i.e., reverse the motor?

Answer: Swap connections on any two coil currents.

- For example, while keeping the physical orientation of the coils shown in Figure 4-1 the same, we swap the currents for phase *b* & phase *c*. Now

$$i_{aa'}(t) = I_M \sin(\omega t), \qquad \overline{B}_{aa'}(t) = B_M \sin(\omega t)\angle 0°,$$

$$i_{bb'}(t) = I_M \sin(\omega t - 240°), \ \& \quad \text{and} \quad \overline{B}_{bb'}(t) = B_M \sin(\omega t - 240°)\angle 120°, \ \&$$

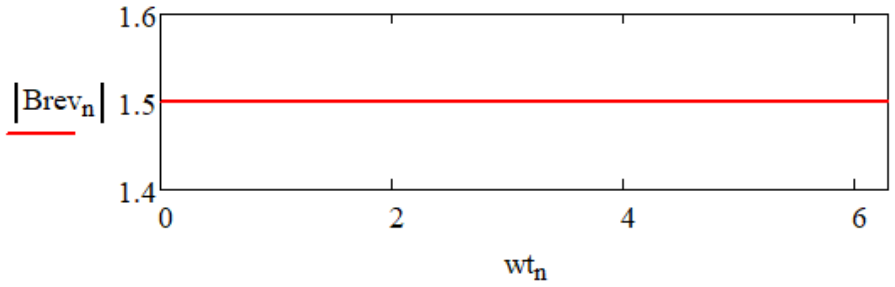
$$i_{cc'}(t) = I_M \sin(\omega t - 120°) \qquad \overline{B}_{cc'}(t) = B_M \sin(\omega t - 120°)\angle 240°.$$

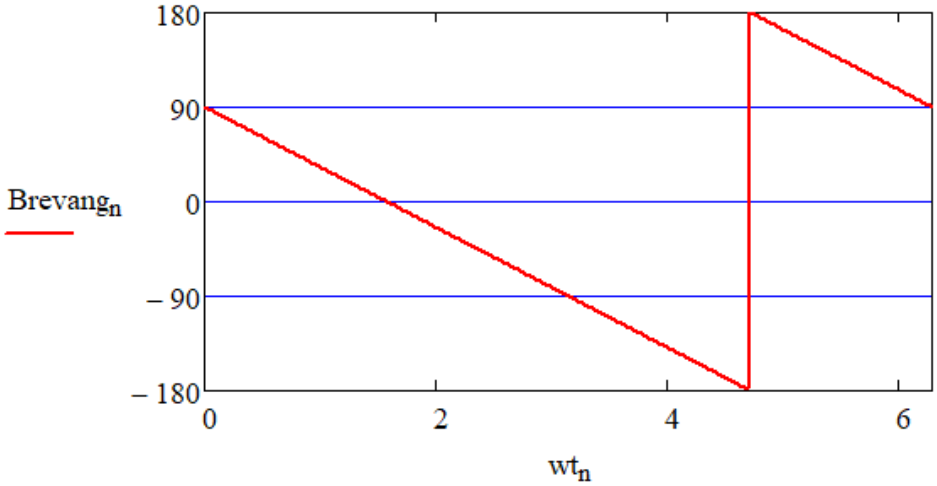
MathCad 3ϕ power & reversed rotating magnetic field example

$n := 0..360 \qquad wt_n := n \cdot \frac{2 \cdot \pi}{360} \qquad Baa_n := 1 \cdot \sin(wt_n - 0) \cdot e^{j \cdot 0}$

$$Bbb_n := 1 \cdot \sin\left(wt_n - \frac{4\pi}{3}\right) \cdot e^{j \cdot \frac{2\pi}{3}} \qquad Bcc_n := 1 \cdot \sin\left(wt_n - \frac{2\pi}{3}\right) \cdot e^{j \cdot \frac{4\pi}{3}}$$

$$Brev_n := Baa_n + Bbb_n + Bcc_n \qquad Brevang_n := \arg(Brev_n) \cdot \frac{180}{\pi}$$





- Slope of the spatial angle of $\overline{B}$ is now <u>negative</u> $\Rightarrow$ rotation direction for $\overline{B}$ is **reversed**!