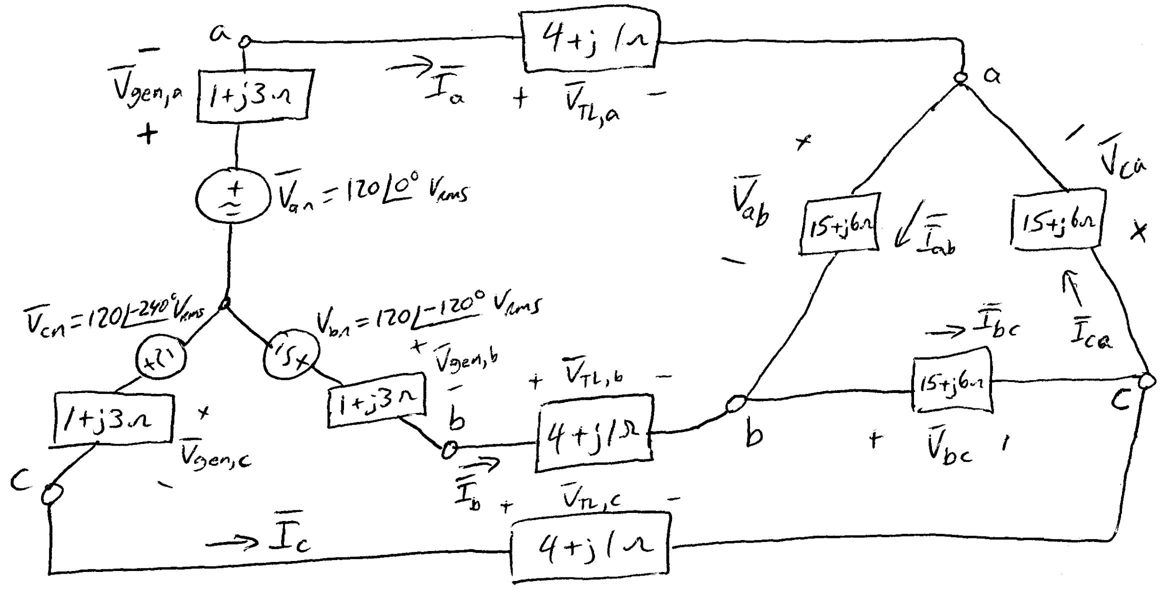
Three-Phase Circuit Power Example

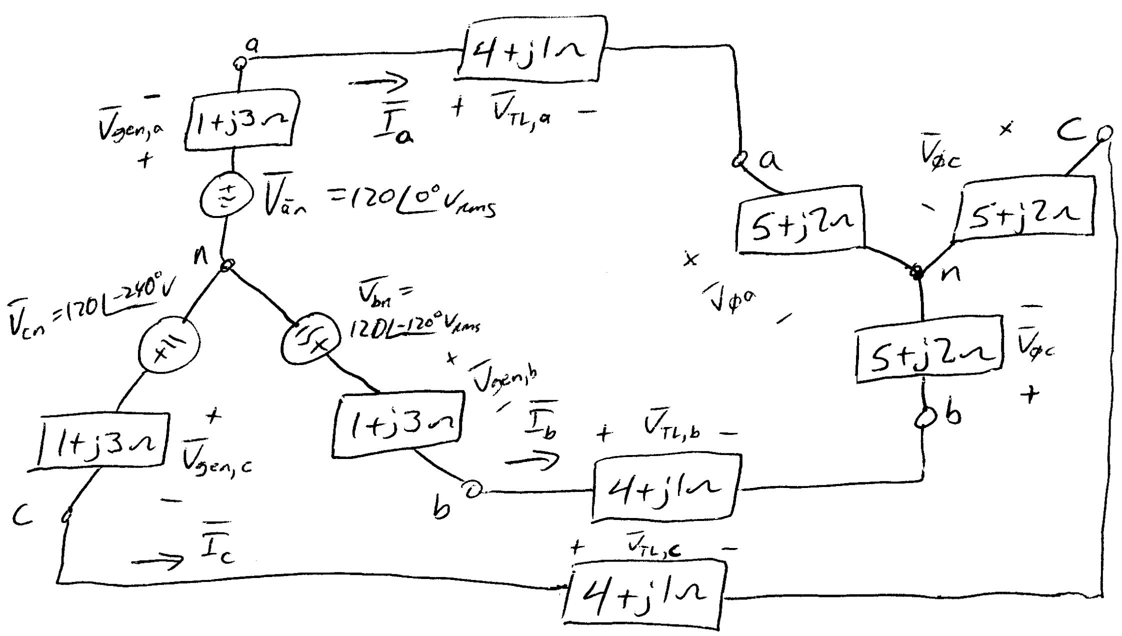
A balanced Y-configuration source (generator) has phase voltages of 120 V_{rms} and an impedance of 1 + *j* 3 Ω per phase. The generator is connected to a balanced Δ-configuration load ($\overline{Z}_{\Delta} = 15 + j6\,\Omega$) by balanced transmission lines (TLs) each with a line impedance of 4 + *j* 1 Ω. Calculate all currents, voltages, and power quantities.



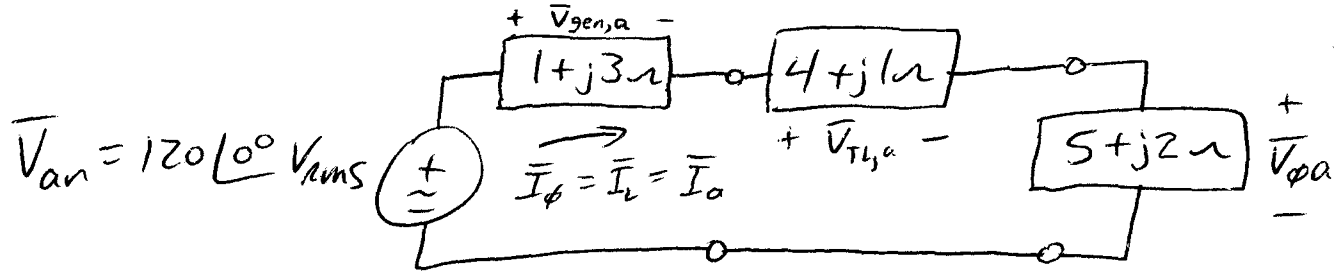
Step 1- Convert the Δ-load to an equivalent Y-load.

$$\boxed{\overline{Z}_Y = \frac{\overline{Z}_{\Delta}}{3} = \frac{15 + j6}{3} \quad \Rightarrow \quad \overline{Z}_Y = 5 + j2\,\Omega}.$$

Step 2- Redraw the circuit. Note that all the impedances are now in **series**.



Step 3- Draw a per-phase equivalent circuit for phase *a*.



Step 4- Compute the equivalent impedance as well as phase and line currents.

The equivalent impedance 'seen' by source is

$$\overline{Z}_{eq} = (1+j3)+(4+j1)+(5+j2) = 10+j6 = 11.6619\angle 30.9638°\ \Omega.$$

The phase/line current is

$$\overline{I}_a = \frac{\overline{V}_{an}}{\overline{Z}_{eq}} = \frac{120\angle 0°}{11.6619\angle 30.9638°} \Rightarrow \underline{\overline{I}_a = 10.290\angle -30.9638°\ \ \mathrm{A_{rms}}},$$

add -120° phase shift to get phase *b* $\Rightarrow \underline{\overline{I}_b = 10.290\angle -150.9638°\ \ \mathrm{A_{rms}}}$,

add -240° phase shift to get phase *c* $\Rightarrow \underline{\overline{I}_c = 10.290\angle -270.9638°\ \ \mathrm{A_{rms}}}$,

and note the phase current magnitude is $\underline{I_\phi = 10.290\ \mathrm{A_{rms}}}$.

Step 5- Compute various power quantities per-phase and overall.

The single-phase **powers** are

$P_\phi = P_a = P_b = P_c = V_\phi I_\phi \cos(\theta) = 120(10.29)\cos(30.9638°) \Rightarrow$ $\underline{\boldsymbol{P_\phi = 1058.8\ \mathrm{W}}}$**.**

The overall power from the ideal 3ϕ voltage source is

$$P = 3P_\phi = 3(1058.8) \qquad \Rightarrow \underline{\boldsymbol{P = 3176.5\ \mathrm{W}}}.$$

The single-phase **reactive powers** are

$Q_\phi = Q_a = Q_b = Q_c = V_\phi I_\phi \sin(\theta) = 120(10.29)\sin(30.9638°) \Rightarrow$ $\underline{\boldsymbol{Q_\phi = 635.3\ \mathrm{var}}}$**.**

The overall reactive power from the ideal 3ϕ voltage source is

$$Q = 3Q_\phi = 3(635.3) \qquad \Rightarrow \underline{\boldsymbol{Q = 1905.9\ \mathrm{var}}}.$$

The single-phase **complex powers** are

$$\overline{S}_\phi = \overline{S}_a = \overline{S}_b = \overline{S}_c = P_\phi + jQ_\phi$$

$$\Rightarrow \underline{\overline{S}_\phi = 1058.8 + j635.3 \text{ VA} = 1234.8\angle 30.964° \text{ VA}}.$$

The overall **complex power** from the ideal 3ϕ voltage source is

$$\overline{S} = 3\overline{S}_\phi = 3(1058.5 + j635.3)$$

$$\Rightarrow \underline{\overline{S} = 3176.5 + j1905.9 \text{ VA} = 3704.4\angle 30.964° \text{ VA}}.$$

The single-phase **apparent powers** are

$$S_\phi = S_a = S_b = S_c = |\overline{S}_\phi| \qquad \Rightarrow \underline{S_\phi = 1234.8 \text{ VA}}.$$

The overall **apparent power** from the ideal 3ϕ voltage source is

$$S = 3S_\phi = 3(1234.8) \qquad \Rightarrow \underline{S = 3704.4 \text{ VA}}.$$

The **power factor (PF)** overall and per-phase is

PF = $P/S = P_\phi/S_\phi = \cos(\theta) = \cos(30.964°)$ ⇒ **<u>PF = 0.8575 lagging</u> ($Q > 0$).**

Step 6- Compute voltage drops & power quantities for **generator** impedances.

The voltage drops are found using Ohm's Law

$$\overline{V}_{gen,a} = \overline{I}_a \overline{Z}_{gen} = (10.29\angle -30.964°)(1 + j3) \Rightarrow \underline{\overline{V}_{gen,a} = 32.54\angle 40.6° \text{ V}_{\text{rms}}},$$

$$\overline{V}_{gen,b} = \overline{I}_b \overline{Z}_{gen} = (10.29\angle -150.964°)(1 + j3) \Rightarrow \underline{\overline{V}_{gen,b} = 32.54\angle -79.4° \text{ V}_{\text{rms}}}, \ \&$$

$$\overline{V}_{gen,c} = \overline{I}_c \overline{Z}_{gen} = (10.29\angle -270.964°)(1 + j3) \Rightarrow \underline{\overline{V}_{gen,c} = 32.54\angle -199.4° \text{ V}_{\text{rms}}}.$$

The **complex power** per phase is

$$\overline{S}_{gen,\phi} = \overline{V}_{gen,a} \overline{I}_a{}^* = (32.54\angle 40.6°)(10.29\angle +30.964°)$$

$$\Rightarrow \underline{\overline{S}_{gen,\phi} = 105.885 + j317.65 \text{ VA}}.$$

Overall **power** and **reactive power** for generator impedances are

$P_{\text{gen}} = 3P_{\text{gen},\phi} = 3(105.885) \Rightarrow$ **<u>P_{gen} = 317.7 W</u> [Note: P_{gen}/P = 10% of total.]**

$Q_{\text{gen}} = 3Q_{\text{gen},\phi} = 3(317.65) \Rightarrow$ **<u>Q_{gen} = 953 var</u> [Note: Q_{gen}/Q = 50% of total.]**

[**Yikes!** Not good. I need to work on my generators.]

Step 7- Compute voltage drops & power quantities for **TL impedances**.

The voltage drops are found using Ohm's Law

$$\overline{V}_{TL,a} = \overline{I}_a \overline{Z}_{TL} = (10.29\angle -30.964°)(4 + j1) \;\Rightarrow\; \underline{\overline{V}_{TL,a} = 42.426\angle -16.93°\ \mathrm{V_{rms}}},$$

$$\overline{V}_{TL,b} = \overline{I}_b \overline{Z}_{TL} = (10.29\angle -150.964°)(4 + j1) \Rightarrow \underline{\overline{V}_{TL,b} = 42.43\angle -136.9°\ \mathrm{V_{rms}}}, \ \&$$

$$\overline{V}_{TL,c} = \overline{I}_c \overline{Z}_{TL} = (10.29\angle -270.964°)(4 + j1) \Rightarrow \underline{\overline{V}_{TL,c} = 42.43\angle -256.9°\ \mathrm{V_{rms}}}.$$

The **complex power** per phase is

$$\overline{S}_{TL,\phi} = \overline{V}_{TL,a} \overline{I}_a{}^* = (42.426\angle -16.93°)(10.29\angle 30.964°)$$

$$\Rightarrow \underline{\overline{S}_{TL,\phi} = 423.53 + j105.88\ \mathrm{VA}}.$$

Overall **power** and **reactive power** for TL impedances are

$P_{TL} = 3P_{TL,\phi} = 3(423.53) \Rightarrow$ **$\underline{P_{TL} = 1270.6\ \mathrm{W}}$** **[Note: P_{TL}/P = 40% of total.]**

$Q_{TL} = 3Q_{TL,\phi} = 3(105.88) \Rightarrow$ **$\underline{Q_{TL} = 317.7\ \mathrm{var}}$** **[Note: Q_{TL}/Q = 16.7% of total.]**

[**Yikes!** These are horrific TL losses. I need to buy/use heftier TLs.]

Step 8- Compute voltage drops & power quantities for **loads**.

The voltage drops for the **Y-**configuration equivalent are

$$\overline{V}_{\phi,a} = \overline{I}_a \overline{Z}_Y = (10.29\angle -30.964°)(5 + j2) \;\Rightarrow\; \underline{\overline{V}_{\phi,a} = 55.413\angle -9.16°\ \mathrm{V_{rms}}},$$

$$\overline{V}_{\phi,b} = \overline{I}_b \overline{Z}_Y = (10.29\angle -150.964°)(5 + j2) \Rightarrow \underline{\overline{V}_{\phi,b} = 55.4\angle -129.2°\ \mathrm{V_{rms}}}, \ \&$$

$$\overline{V}_{\phi,c} = \overline{I}_c \overline{Z}_Y = (10.29\angle -270.964°)(5 + j2) \Rightarrow \underline{\overline{V}_{\phi,c} = 55.4\angle -249.2°\ \mathrm{V_{rms}}}.$$

The voltage drops for the actual **Δ-**configured loads are found using KVL

$$\overline{V}_{ab} = \overline{V}_{\phi,a} - \overline{V}_{\phi,b} = (55.41\angle -9.16°) - (55.41\angle -129.16°) \Rightarrow \underline{\overline{V}_{ab} = 95.98\angle 20.84°\ \mathrm{V_{rms}}},$$

$$\overline{V}_{bc} = \overline{V}_{\phi,a} - \overline{V}_{\phi,b} \Rightarrow \underline{\overline{V}_{bc} = 95.98\angle -99.16°\ \mathrm{V_{rms}}}, \ \&$$

$$\overline{V}_{ca} = \overline{V}_{\phi,c} - \overline{V}_{\phi,a} \Rightarrow \underline{\overline{V}_{ca} = 95.98\angle -219.16°\ \mathrm{V_{rms}}}.$$

We see the line-to-line voltage for the Δ-configured loads is **$\underline{V_{LL} = 95.98\ \mathrm{V_{rms}}}$** [or calculate using $V_{LL} = \sqrt{3}\, V_\phi = \sqrt{3}\,(55.413) = 95.98\ \mathrm{V_{rms}}$]. This is a far cry from the $\sqrt{3}\, V_\phi = \sqrt{3}\,(120) = 207.85\ \mathrm{V_{rms}}$ that left the ideal 3ϕ voltage sources.

The **currents** through the actual **Δ-configured** loads are found using Ohm's Law

$$\overline{I}_{ab} = \overline{V}_{ab} / \overline{Z}_{\Delta} = (95.98\angle 20.84°) / (15 + j6) \quad \Rightarrow \quad \underline{\overline{I}_{ab} = 5.941\angle -0.96° \text{ A}_{\text{rms}}},$$

$$\overline{I}_{bc} = \overline{V}_{bc} / \overline{Z}_{\Delta} = (95.98\angle -99.16°) / (15 + j6) \quad \Rightarrow \quad \underline{\overline{I}_{bc} = 5.941\angle -120.96° \text{ A}_{\text{rms}}} \ \&$$

$$\overline{I}_{ca} = \overline{V}_{ca} / \overline{Z}_{\Delta} = (95.98\angle -219.16°) / (15 + j6) \quad \Rightarrow \quad \underline{\overline{I}_{ca} = 5.941\angle -240.96° \text{ A}_{\text{rms}}}.$$

The **complex power** for the loads per-phase (using equivalent circuit)

$$\overline{S}_{LD,\phi} = \overline{S}_{LD,a} = \overline{V}_{\phi,a}\overline{I}_a{}^* = (55.413\angle -9.16°)(10.29\angle 30.964°)$$

$$\Rightarrow \quad \underline{\overline{S}_{LD,\phi} = 529.4 + j211.8 \text{ VA}},$$

or we can compute using voltage and current for the Δ-configuration

$$\overline{S}_{LD,\phi} = \overline{S}_{LD,ab} = \overline{V}_{ab}\overline{I}_{ab}{}^* = 95.98\angle 20.84°)(5.941\angle +0.96°)$$

$$\Rightarrow \quad \underline{\overline{S}_{LD,\phi} = 529.4 + j211.8 \text{ VA}}$$

which yields the same answer.

The overall **power** and **reactive power** for the loads are

$P_{LD} = 3P_{LD,\phi} = 3(529.4) \Rightarrow$ **$\underline{P_{LD} = 1588.2 \text{ W}}$** **[Note: P_{LD}/P = 50% of total.]**

$Q_{LD} = 3Q_{LD,\phi} = 3(211.8) \Rightarrow$ **$\underline{Q_{TL} = 635.3 \text{ var}}$** **[Note: Q_{TL}/Q = 33.3% of total.]**

In summary, the generator and TLs used in this example are awful**!** I need to spend some money ($$$) to buy/design (or choose more realistic values) a better generator and TLs. General Electric, Siemens, … do far better with their generators while Belden, Southern Wire, … do far better with their TLs.