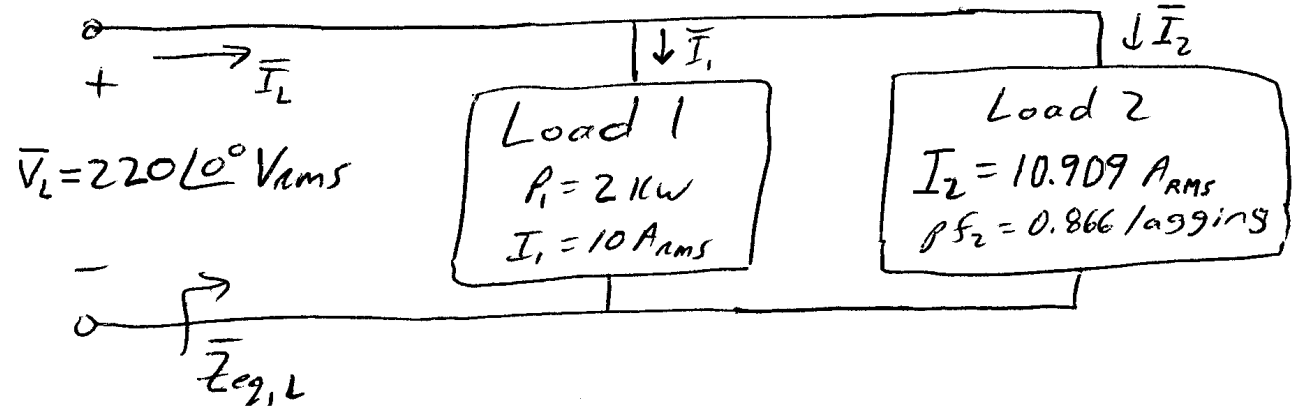
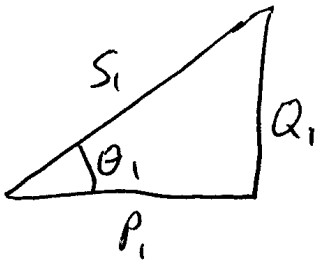
Example- A 220 V_{RMS} power line supplies two loads connected in parallel. Load 1 is an AC motor drawing a current of 10 A_{RMS} (measured using a multimeter) and power of 2 kW (measured using a wattmeter). Load 2 is a combination of lights and AC motors drawing 10.909 A_{RMS} with a lagging power factor of 0.866. Analyze each load as well as the overall equivalent load on the circuit. [Hint: Motors are resistive-inductive loads.]

⇒ Assume power line is @ 0° (reference)



Load 1

motor ⇒ lagging pf



$S_1 = V_{1,RMS} I_{1,RMS} = 220(10) = 2200\ VA$

$P_1 = 2000\ W$ (given)

$PF_1 = \cos\theta_1 = \frac{P_1}{S_1} = \frac{2000}{2200} = 0.9091$ lagging

$\theta_1 = \cos^{-1} 0.9091 = 24.62° = \theta_{V_1} - \theta_{I_1}$

$\hookrightarrow \theta_{I_1} = -24.62°$

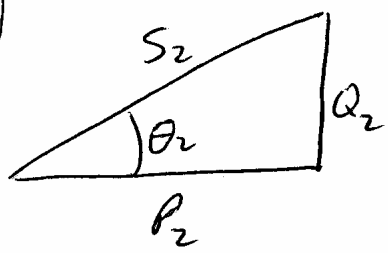
$\bar{I}_1 = 10\angle -24.62°\ A_{rms}$

$Q_1 = S_1 \sin\theta_1 = 2200 \sin 24.62° = 916.516\ VAR$

$\bar{Z}_1 = \frac{\bar{V}_1}{\bar{I}_1} = \frac{220\angle 0°}{10\angle -24.62°} = 22\angle 24.62°\ \Omega = 20 + j9.16516\ \Omega$

$\bar{S}_1 = P_1 + jQ_1 = 2000 + j916.516\ VA = 2200\angle 24.62°\ VA$

Load 2



$S_2 = 220(10.909) = 2400\ VA$

$PF_2 = 0.866\ lagging = \cos\theta_2$

$\hookrightarrow \theta_2 = +30^\circ = \phi_{V_2} - \theta_{I_2}$ ($\phi_{V_2} \to 0$; $+$ because lagging)

$\theta_{I_2} = -30^\circ$

$I_2 = 10.909\ A_{rms}$

$\bar{I}_2 = 10.909\angle -30^\circ\ A_{rms}$

$\bar{S}_2 = \bar{V}_2 \bar{I}_{2rms}^{*} = (220\angle 0^\circ)(10.909\angle +30^\circ)$

$= 2078.4 + j\,1200\ VA$

(P_2 (W), Q_2 (VAR))

$\bar{Z}_2 = \dfrac{\bar{V}_2}{\bar{I}_2} = \dfrac{220\angle 0^\circ}{10.909\angle -30^\circ} = 20.167\angle 30^\circ\ \Omega$

$= 17.465 + j\,10.083\ \Omega$

$\bar{S}_{\substack{power\\line}} = \bar{S}_1 + \bar{S}_2 = 2200\angle 24.62^\circ + 2400\angle 30^\circ$

$= 4594.94\angle 27.427^\circ\ VA = 4078.46 + j\,2116.516\ VA$

($P_{power\ line}$, $Q_{power\ line}$)

$PF_L = \cos 27.427^\circ = 0.8876\ lagging$

$\bar{I}_L = \bar{I}_1 + \bar{I}_2 = 10\angle -24.62^\circ + 10.909\angle -30^\circ = 20.886\angle -27.427^\circ\ A_{RMS}$

$\bar{Z}_L = \bar{Z}_1 // \bar{Z}_2 = \dfrac{\bar{V}_L}{\bar{I}_L} = 10.53\angle 27.427^\circ\ \Omega = 9.35 + j\,4.85\ \Omega$