7.39 A discrete-time system is given by the input/output difference equation

$$y[n+2] - y[n+1] + y[n] = x[n+2] - x[n+1]$$

Is the system stable, marginally stable, or unstable? Justify your answer.

➢ First, find $H(z)$.

Re-index $n \to n-2$

$$y[n] - y[n-1] + y[n-2] = x[n] - x[n-1]$$

Assume causal system and input. Use,

from Table 7.2, $x[n-1] \leftrightarrow z^{-1}X(z)$

$x[n-2] \leftrightarrow z^{-2}X(z)$

$\Downarrow$ z-transform

$$Y(z) - z^{-1}Y(z) + z^{-2}Y(z) = X(z) - z^{-1}X(z)$$

$$Y(z)\left[1 - z^{-1} + z^{-2}\right] = X(z)\left[1 - z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - z^{-1} + z^{-2}} = \frac{z^2 - z}{z^2 - z + 1}$$

$$\underline{H(z) = \frac{z^2 - z}{z^2 - z + 1}}$$

The roots/poles of the denominator are:

$$\text{roots} = p_i = 1\angle \pm 60^\circ = 0.5 \pm j0.866$$

$\Rightarrow$ Marginally stable from pole locations (p.390)