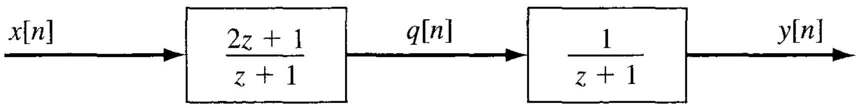
7.36 A linear time-invariant discrete-time system is given by the cascade connection shown in Figure P7.36.

(a) Compute the unit-pulse response of the overall system.

(b) Compute the input/output difference equation of the overall system.



- For part a), do partial fractions on $H(z)/z$ to find $h[n]$. Using MATLAB, plot analytical $h[n]$ solution for $0 \le n \le 5$ and on another stem plot show the answer found using long division. Label stems. Place the partial fractions solution plot on the top and the long division answer plot on the bottom of the same page. Attach m-file. Are the plots the same? Is the system stable or not? Why?

a) Using series rule, $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{2z+1}{(z+1)^2}$

$$\frac{H(z)}{z} = \frac{2z+1}{z(z+1)^2} = \frac{C_0}{z} + \frac{C_1}{z+1} + \frac{C_2}{(z+1)^2}$$

$$C_0 = H(0) = \frac{2(0)+1}{(0+1)^2} = \underline{1}$$

$$C_2 = \left[(z+1)^2 \frac{2z+1}{z(z+1)^2}\right]\Bigg|_{z=-1} = \frac{2(-1)+1}{-1} = \underline{1}$$

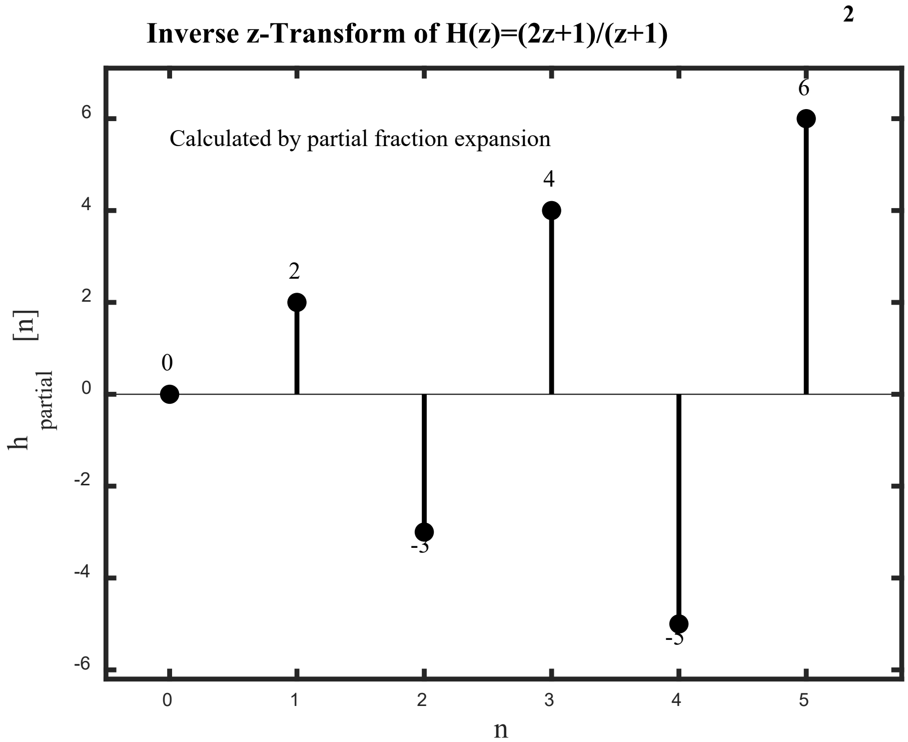
$$C_1 = \left[\frac{d}{dz}\left(\frac{2z+1}{z}\right)\right]\Bigg|_{z=-1} = \frac{z(2)-(2z+1)1}{z^2}\Bigg|_{z=-1} = \frac{-2-(-2+1)}{(-1)^2} = \underline{-1}$$

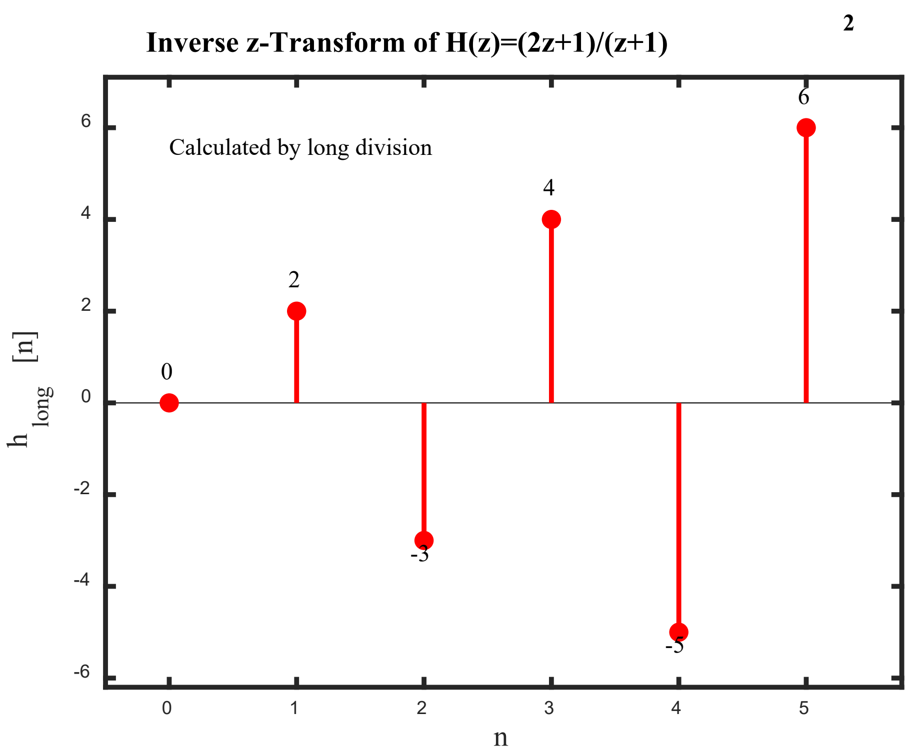
$$H(z) = 1 + \frac{-z}{z+1} + \frac{z}{(z+1)^2} = 1 - \frac{z}{z+1} - \frac{(-1)z}{(z+1)^2}$$

Use Table 7.3, $\delta[n] \leftrightarrow 1$, $a^n u[n] \leftrightarrow \dfrac{z}{z-a}$, and

$$n\,a^n u[n] \leftrightarrow \frac{az}{(z-a)^2}$$

$$\underline{\underline{h[n] = \delta[n] - (-1)^n u[n] - n(-1)^n u[n]}}$$





Are the plots the same? **Yes!**

Is the system stable or not? **No!**

Why? **Repeated roots @ $z = -1$ leads to $n(-1)^n\ u[n]$ in $h[n]$.**

```
% p7_36a.m
% Plot unit pulse response for
%   H(z) = (2z+1)/(z+1)^2 = (2z+1)/(z^2+2z+1)
%   H(z)/z = (2z+1)/(z^3+2z^2+z+0)
% using long division and by partial fractions solution
% h[n] = d[n] -(-1)^n u[n] - n(-1)^n u[n]
%
clc; clear; close all;
num = [2,1];     % Input coefficients of numerator polynomial
den = [1,2,1]; % Input coeff. of denominator polynomial
% Calculate first 6 values of h[n] by long division
hlong = dimpulse(num,den,6);  n = 0:1:5;
h_part = -1*(-1).^n - n.*(-1).^n;
h_part(1) = h_part(1)+1;
stem(n,h_part,'k.','linewidth',2,'markersize',20),
axis([-0.5 5.75 -6.2 7.1]),
ylabel('h_{partial }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=(2z+1)/(z+1)^2',...
    'fontsize',16,'fontname','times'),
text(0,5.5,'Calculated by partial fraction expansion',...
    'fontsize',16,'fontname','times')
for m=1:length(h_part),
    if(h_part(m)<0),
        text(n(m),h_part(m)-0.1,[num2str(h_part(m),2)],...
            'horizontalalignment','center','verticalalignment','top')
    else
        text(n(m),h_part(m)+0.1,[num2str(h_part(m),2)],...
            'horizontalalignment','center','verticalalignment','bottom')
    end
end
figure, stem(n,hlong,'r.','linewidth',2,'markersize',20),
axis([-0.5 5.75 -6.2 7.1]),
ylabel('h_{long }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=(2z+1)/(z+1)^2',...
    'fontsize',16,'fontname','times'),
text(0,5.5,'Calculated by long division',...
    'fontsize',16,'fontname','times')
for m=1:length(hlong),
    if(hlong(m)<0),
        text(n(m),hlong(m)-0.1,[num2str(hlong(m),2)],...
            'horizontalalignment','center','verticalalignment','top')
    else
        text(n(m),hlong(m)+0.1,[num2str(hlong(m),2)],...
            'horizontalalignment','center','verticalalignment','bottom')
    end
end
set(findobj('type','line'),'linewidth',1.5,'markersize',18)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontsize',13,'fontname','times')
```

b) $$H(z) = \frac{Y(z)}{X(z)} = \frac{2z+1}{(z+1)^2} = \frac{2z+1}{z^2+2z+1} = \frac{2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}$$

$$Y(z)\left[1+2z^{-1}+z^{-2}\right] = X(z)\left[2z^{-1}+z^{-2}\right]$$

Assuming causal system and input, use from Table 7.2

$$x[n-1] \leftrightarrow z^{-1}X(z)$$

$$x[n-2] \leftrightarrow z^{-2}X(z)$$

$\Downarrow$ inverse z-transform

$$\underline{\underline{y[n] + 2y[n-1] + y[n-2] = 2x[n-1] + x[n-2]}}$$