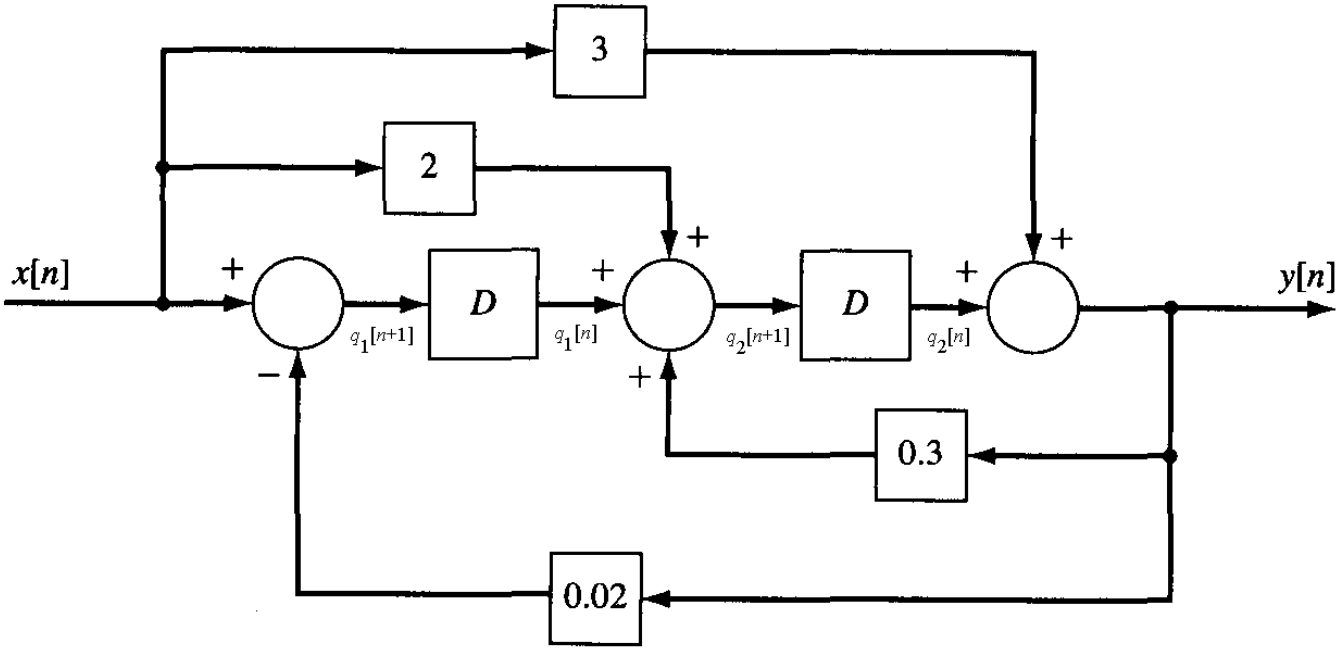
7.32 Consider the discrete-time system shown in Figure P7.32.

(a) Determine the transfer function $H(z)$ of the system.

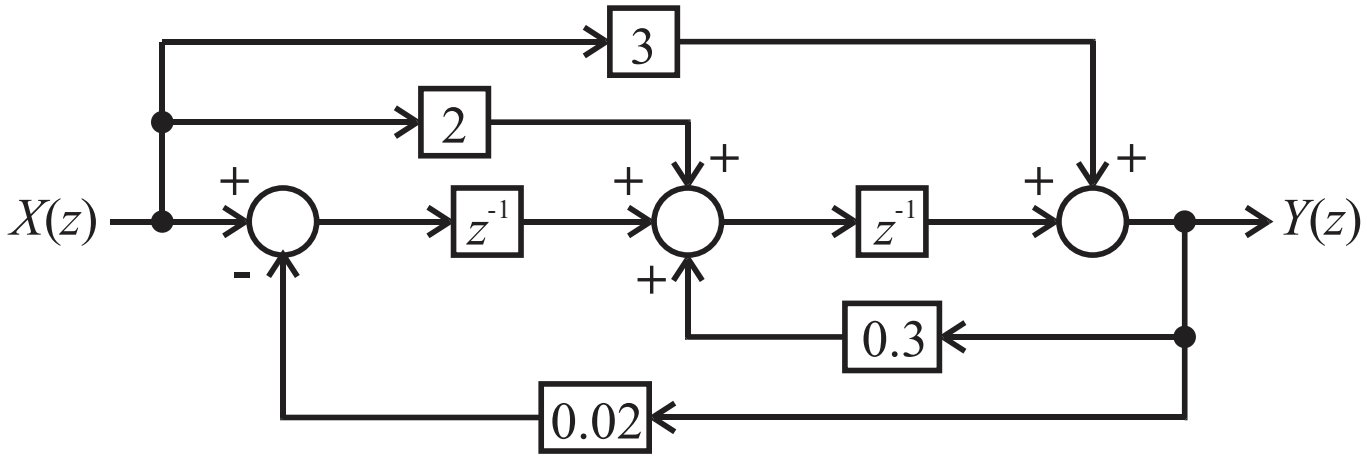
(b) Determine the system's input/output difference equation.

(c) Compute the output response $y[n]$ when $x[n] = 4u[n]$ with zero initial conditions.

- For part a), use Mason's Theorem to find $H(z)$.



z-transform



a) Find $H(z)$ using Mason's Thm

$P_1(z) = 3 \qquad L_1(z) = 0.3z^{-1}$

$P_2(z) = 2z^{-1} \qquad L_2(z) = 0.02z^{-2}$ } Loops touch each other

$P_3(z) = z^{-2}$

$\Delta(z) = 1 - (L_1(z) + L_2(z)) = 1 - 0.3z^{-1} + 0.02z^{-2}$

$\Delta_1(z) = \Delta_2(z) = \Delta_3(z) = 1$ all paths touch both loops

$$H(z) = \frac{\sum P_i \Delta_i}{\Delta(z)} = \frac{3(1) + 2z^{-1}(1) + z^{-2}(1)}{1 - 0.3z^{-1} + 0.02z^{-2}}$$

$$H(z) = \frac{3 + 2z^{-1} + z^{-2}}{1 - 0.3z^{-1} + 0.02z^{-2}} = \frac{3z^2 + 2z + 1}{z^2 - 0.3z + 0.02}$$

b) $$\frac{Y(z)}{X(z)} = \frac{3z^2 + 2z + 1}{z^2 - 0.3z + 0.02} = \frac{3 + 2z^{-1} + z^{-2}}{1 - 0.3z^{-1} + 0.02z^{-2}}$$

$$Y(z) - 0.3z^{-1}Y(z) + 0.02z^{-2}Y(z) = 3X(z) + 2z^{-1}X(z) + z^{-2}X(z)$$

⇓ inverse z-transform (assume zero I.C.)

$$y[n] - 0.3y[n-1] + 0.02y[n-2] = 3x[n] + 2x[n-1] + x[n-2]$$

c) $Y(z) = H(z)X(z)$ w/ $X(z) = 4\left(\frac{z}{z-1}\right)$

$$= \frac{3z^2+2z+1}{z^2-0.3z+0.02}\left(\frac{4z}{z-1}\right) = \frac{12z^3+8z^2+4z}{(z-1)(z-0.1)(z-0.2)}$$

$$\frac{Y(z)}{z} = \frac{(12z^2+8z+4)\cancel{z}}{\cancel{z}(z-0.1)(z-0.2)(z-1)} = \frac{12z^2+8z+4}{(z-0.1)(z-0.2)(z-1)}$$

$$= \frac{C_1}{z-0.1} + \frac{C_2}{z-0.2} + \frac{C_3}{z-1}$$

$$C_1 = \left[(z-0.1)\frac{Y(z)}{z}\right]_{z=0.1} = \frac{12(0.1)^2+8(0.1)+4}{(0.1-0.2)(0.1-1)} = \underline{54.\overline{6}}$$

$$C_2 = \left[(z-0.2)\frac{Y(z)}{z}\right]_{z=0.2} = \frac{12(0.2)^2+8(0.2)+4}{(0.2-0.1)(0.2-1)} = \underline{-76}$$

$$C_3 = \left[(z-1)\frac{Y(z)}{z}\right]_{z=1} = \frac{12(1)^2+8(1)+4}{(1-0.1)(1-0.2)} = \underline{33.\overline{3}}$$

$$Y(z) = \frac{54.\overline{6}\,z}{z-0.1} - \frac{76z}{z-0.2} + \frac{33.\overline{3}z}{z-1}$$

using Table 7.3 transform pairs $a^n u[n] \leftrightarrow \frac{z}{z-a}$ and linearity. $u[n] \leftrightarrow \frac{z}{z-1}$

$$\underline{y[n] = 54.\overline{6}(0.1)^n u[n] - 76(0.2)^n u[n] + 33.\overline{3}u[n]}$$